EPJ Web of Conferences **113**, 04024 (2016)
DOI: 10.1051/epjconf/201611304024

Nucleon-nucleon scattering in the $^1S_0$ partial wave in the modified Weinberg approach

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Abstract. Nucleon-nucleon scattering in the $^1S_0$ partial wave is considered in chiral effective field theory within the recently suggested renormalizable formulation based on the Kadyshevsky equation. Contact interactions are taken into account beyond the leading-order approximation. The subleading contact terms are included non-perturbatively by means of subtractive renormalization. The dependence of the phase shifts on the choice of the renormalization condition is discussed. Perturbative inclusion of the subleading contact interaction is found to be justified only very close to threshold. The low-energy theorems are reproduced significantly better compared with the leading order results.

1 Introduction

Recently a novel formulation of the NN scattering problem in EFT has been suggested in Refs. [1–4]. It is based on a modification of the original Weinberg's approach [5]. In the new scheme one solves the relativistic Kadyshevsky equation instead of the Lippmann-Schwinger equation. One of the advantages of such an approach is that ultraviolet divergences reproduce the ones that appear in covariant Feynman diagrams, i.e. there exist only logarithmic or even-power divergences. The odd-power divergences present in iterations of the Lippmann-Schwinger equation are artifacts of the non-relativistic approximation. In particular, the integral equation based on the leading-order potential, which consists of the momentum- and energy-independent contact interaction and the one-pion-exchange potential, possesses only logarithmic divergences and is therefore renormalizable (there are exceptions from such a naive picture, e.g. the case of $^3P_0$ partial wave, see Ref. [1]). Higher-order contributions are included perturbatively in this scheme without spoiling the renormalizability feature. The present approach is alternative to the standard non-relativistic EFT approach based on the Lippmann-Schwinger equation [6, 7] where one necessarily has to introduce a cut off regulator of the order of the hard scale [8, 9].

In the $^1S_0$ channel a large discrepancy between the leading order EFT results and the phase shift of the Nijmegen partial wave analysis [10] is observed, which starts already at rather low energies. This indicates that at least some parts of the higher-order contributions to the effective potential, in contrast with the original formulation, have to be included non-perturbatively. A similar observation

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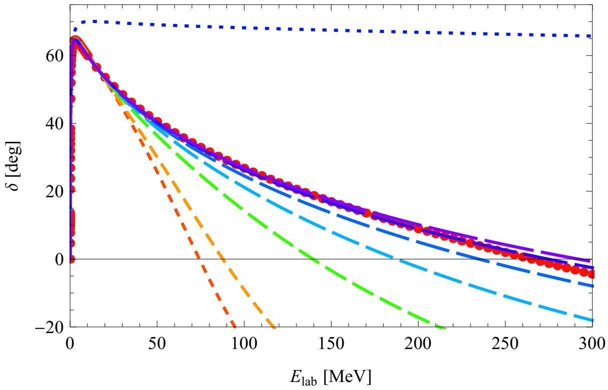

Figure 1. Neutron-proton $^1S_0$ phase shifts versus the energy in the laboratory frame. Circles (red) correspond to the Nijmegen PWA [10]. Dotted line represents the LO result. Curves with different dashing (colors) correspond to non-perturbative inclusion of the NLO contact interaction potential for $\mu = 50, 100, 300, 500, 700, 850$ and 900 MeV respectively.

was made in Ref. [11] in the scheme with an explicit dibaryon field. In this talk we report on the results for the $^1S_0$ phase shifts calculated both with perturbative and non-perturbative inclusion of the next-to-leading-order (NLO) contact interaction and make the comparison between the two options.

2 Formalism

We assume that only the short range part of the next-to-leading-order (NLO) potential (in standard Weinberg power counting [5]) needs to be treated non-perturbatively. It involves only the contact interaction terms quadratic in momenta and the pion mass. This makes it possible to perform the subtractive renormalization explicitly in non-perturbative expressions [12].

The NLO $^1S_0$ partial wave NN scattering amplitude can be obtained by extracting the $^1S_0$ component from the solution to the integral equation for the off-shell amplitude T

$$T\left(p_0, \vec{p}\,', \vec{p}\,\right) = V\left(\vec{p}\,', \vec{p}\,\right) + \frac{m^2}{2} \int \frac{d^3k}{(2\,\pi)^3}\, \frac{V\left(\vec{p}\,', \vec{k}\right)\, T\left(p_0, \vec{k}, \vec{p}\right)}{\left(\vec{k}^2 + m^2\right)\left(p_0 - \sqrt{\vec{k}^2 + m^2} + i\epsilon\right)}\,, \tag{1}$$

with the potential given by

$$V\left(\vec{p}\,', \vec{p}\,\right) \;=\; \left[C + C_2\left(\vec{p}\,'^2 + \vec{p}^{\,2}\right)\right] - \frac{g_A^2 M_\pi^2}{4F_\pi^2}\, \frac{1}{\left(\vec{p}\,' - \vec{p}\,\right)^2 + M_\pi^2}\,. \tag{2}$$

After some algebraic transformations it is possible to rewrite Eq.(1) in terms of two finite renormalized low-energy constants $\tilde{C}^R(\mu)$ and $C_2^R(\mu)$, which depend on the the subtraction momentum μ [12].

3 Results

The renormalized integral equations for the scattering amplitude have been solved numerically and the values of the low-energy constants $\tilde{C}^R(\mu)$ and $C_2^R(\mu)$ were determined from a fit to the phase shifts of the Nijmegen partial-wave analysis [10] for different choices of the subtraction point μ. The resulting phase shifts for different choices of μ are plotted in Fig. 1. The description of the data is significantly improved as compared to the leading order result. On the other hand one can observe some residual μ-dependence of the predicted phase shifts which gets strongly enhanced if one chooses μ of the order or smaller than the pion mass. This supports the fact that the appropriate choice of the subtraction

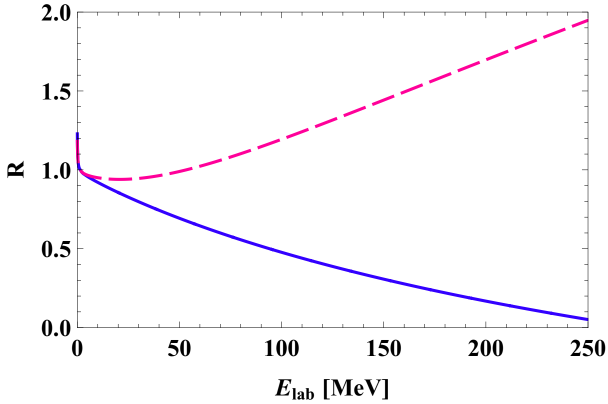

Figure 2. The ratio R of the absolute values of the T-matrices calculated in the leading and next-to-leading order approximation as described in text versus the energy in the laboratory frame. The solid (dashed) line shows the result based on $T^{\rm NLO}$ corresponding to the non-perturbative (perturbative) inclusion of the subleading contact interaction. In both cases, the subtraction scale is set to $\mu = 850$ MeV.

scale for the non-perturbative treatment of C_2^R should be $\mu \sim \Lambda$ (see e.g. Refs. [8, 9]), where the hard scale Λ can be estimated to be of the order of $\Lambda \sim$ 600-700 MeV (masses of the sigma and rho mesons which phenomenologically are known to yield the most important short-range contribution to the nucleon-nucleon potential [13]).

We also address the question of perturbativeness of the subleading contact interaction within our scheme. The ratio $R(E_{\rm lab}) = \frac{\left|T^{\rm NLO}(E_{\rm lab})\right|}{\left|T^{\rm LO}(E_{\rm lab})\right|}$ for the cases when the subleading contact term is included perturbatively and non-perturbatively is plotted in Fig. 2. Note that the ratio R is not equal to 1 at threshold, because in our subtraction scheme diagrams containing NLO contact interactions but no overall divergences are not subtracted. Looking at this figure or based on explicit calculations one can conclude that it is advantageous to include the subleading contact interaction non-perturbatively for energies of about $E_{\rm lab} \sim 50$ MeV and higher. This conclusion is similar to the one made in Ref. [11] but in a different regularization and renormalization scheme.

Finally, we found a clear improvement when going from leading order to next-to-leading order for the coefficients in the effective range expansion, i.e. for the low-energy theorems (LETs) (see Table 1). It is instructive to compare the convergence pattern for the LETs in our approach with the one in the scheme with perturbative pions suggested by Kaplan, Savage and Wise (KSW) [14], where convergence is, at best, very slow as follows from Table 1. On the other hand, the μ dependence of the low-energy coefficients appears to be rather weak because they are mostly determined by the energy region very close to threshold. By the same reason perturbative inclusion of the subleading contact terms leads to nearly the same values for the low-energy coefficients.

4 Summary

We extended the modified Weinberg scheme for the NN-interactions of Ref. [1] by including the subleading contact interaction both perturbatively and non-perturbatively. The perturbative treatment appears to be reasonable for energies below $E_{\rm lab} \sim 50$ MeV whereas the non-perturbative approach leads to a good description of the data up to $E_{\rm lab} \sim 300$ MeV for appropriate choices of the subtraction point. The results of our work allow one to proceed with inclusion of higher-order terms not taken into account in the present work—such as two-pion exchange loop contributions—perturbatively according to the original scheme.

Table 1. Predictions for the coefficients in the effective range expansion of the $^1S_0$ phase shifts (low-energy theorems) at LO and NLO in the modified Weinberg approach in comparison with the NLO and NNLO KSW predictions of Ref. [15] and empirical numbers extracted from the Nijmegen PWA [16, 17]. For the NLO Weinberg results, we show the predictions corresponding to the variation of the subtraction point in the range of $\mu = 500 \ldots 900$ MeV. The errors quoted for the LO predictions refer to the uncertainty in the numerical extraction of the coefficients [1].

	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
LO, Ref. [1]	fit	1.50	−1.9	8.6(8)	−37(10)
NLO, nonperturb. C_2	fit	fit	$-0.55 \ldots -0.61$	$5.1 \ldots 5.5$	$-29.6 \ldots -30.8$
NLO, perturbative C_2	fit	fit	$-0.51 \ldots -0.57$	$4.5 \ldots 4.7$	$-28.8 \ldots -29.8$
NLO KSW, Ref. [15]	fit	fit	−3.3	19	−117
NNLO KSW, Ref. [15]	fit	fit	−3.3	2.9	−0.7
Nijmegen PWA	−23.7	2.67	−0.5	4.0	−20

Acknowledgements

This work is supported by the Deutsche Forschungsgemeinschaft (SFB/TR 16, “Subnuclear Structure of Matter”), by the European Community Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (acronym HadronPhysics3, Grant Agreement n. 283286) under the 7th Framework Programme of the EU, by the European Research Council (acronym NuclearEFT, ERC-2010-StG 259218).

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