

Proton-Deuteron Scattering and Test of Time-Reversal Invariance

Yu.N. Uzikov^{1,2,a}

¹Laboratory of Nuclear Problems, JINR, Dubna, Moscow reg., 141980 Russia

²Department of Physics, Moscow State University, Moscow, 119991 Russia

Abstract. The integrated proton-deuteron scattering cross section $\tilde{\sigma}$ for transversely polarized protons (P_y^p) and tensor polarized deuterons (P_{xz}) constitutes a null test signal for time-reversal invariance violating but P-parity conserving effects. This cross section will be measured at COSY. Using the generalized optical theorem and Glauber theory we study the null-test observable $\tilde{\sigma}$ for different types of T-odd P-even NN-interactions. The formalism includes full spin dependence of elementary pN-amplitudes and S- and D-components of the deuteron wave function.

1 Introduction

Time-invariance-violating (T-odd) P-parity conserving (P-even) (TVPC) interactions do not arise at the fundamental level within the standard model. This type of interaction can be generated by radiative corrections to the T-odd P-odd interaction discovered in the physics of kaons and B-mesons. However, in this case its intensity is too low to be observed in experiments at present [1]. Thus, observation of TVPC effects would be considered as an indication of physics beyond the standard model.

As was shown in Ref. [2], the total polarized cross section $\tilde{\sigma}$ of the proton-deuteron scattering with vector polarization of the proton p_y^p and tensor polarization of the deuteron P_{xz} constitutes a null-test observable for TVPC effects. The dedicated experiment is planned at COSY [3] at proton beam energy 135 MeV. The first analysis of the TVPC null-test signal [4] was done within the nonmesonic deuteron breakup channel $pd \rightarrow ppn$ estimated in the single scattering approximation. Recently we used the spin-dependent formalism [5] of the Glauber theory to calculate the cross section $\tilde{\sigma}$ [6] and "null-combinations" of some differential spin observables of the pd elastic scattering [7] which deviate from zero if the TVPC effects occur. The formalism includes full spin dependence of elementary pN-amplitudes and S- and D-components of the deuteron wave function. This formalism allows one to explain existing data on the non-polarized differential cross section and spin observables of the elastic pd scattering at 135 MeV [8]. Here we consider some qualitative arguments concerning the ρ -meson contribution to $\tilde{\sigma}$ and briefly explain the role of the deuteron D-wave.

^ae-mail: uzikov@jinr.ru

2 Elements of formalism

Time-reversal symmetry conserving and P-parity conserving (TCPC or T-even P-even) interactions lead to the following transition amplitude of the elastic pd scattering at 0 degree [9]

$$e_{\beta}^{\prime*} M(0)_{\alpha\beta}^{TCPC} e_{\alpha} = g_1[\mathbf{e} \mathbf{e}^{\prime*} - (\mathbf{m}\mathbf{e})(\mathbf{m}\mathbf{e}^{\prime*})] + g_2(\mathbf{m}\mathbf{e})(\mathbf{m}\mathbf{e}^{\prime*}) + ig_3\{\boldsymbol{\sigma}[\mathbf{e} \times \mathbf{e}^{\prime*}] - (\boldsymbol{\sigma}\mathbf{m})(\mathbf{m} \cdot [\mathbf{e} \times \mathbf{e}^{\prime*}])\} + ig_4(\boldsymbol{\sigma}\mathbf{m})(\mathbf{m} \cdot [\mathbf{e} \times \mathbf{e}^{\prime*}]), \quad (1)$$

where \mathbf{e} (\mathbf{e}') is the polarization vector of the initial (final) deuteron, \mathbf{m} is the unit vector along the beam momentum, $\boldsymbol{\sigma}$ is the Pauli matrix, g_i ($i = 1, \dots, 4$) are complex amplitudes. To the right-hand side of Eq.(1) one can add the TVPC (T-odd P-even) term in a very general form

$$e_{\beta}^{\prime*} M(0)_{\alpha\beta}^{TVPC} e_{\alpha} = \tilde{g}\{(\boldsymbol{\sigma} \cdot [\mathbf{m} \times \mathbf{e}])(\mathbf{m} \cdot \mathbf{e}^{\prime*}) + (\boldsymbol{\sigma} \cdot [\mathbf{m} \times \mathbf{e}^{\prime*}])(\mathbf{m} \cdot \mathbf{e})\}, \quad (2)$$

where \tilde{g} is the TVPC transition amplitude. The matrix elements of the operators (1), (2) are

$$\langle \mu' = \frac{1}{2}, \lambda' = 1 | M^{TCPC} | \mu = \frac{1}{2}, \lambda = 1 \rangle = g_1 + g_4, \quad (3)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = -1 | M^{TCPC} | \mu = \frac{1}{2}, \lambda = -1 \rangle = g_1 - g_4, \quad (4)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = 0 | M^{TCPC} | \mu = \frac{1}{2}, \lambda = 0 \rangle = g_2, \quad (5)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = 0 | M^{TCPC} + M^{TVPC} | \mu = -\frac{1}{2}, \lambda = 1 \rangle = \sqrt{2}g_3 + i\sqrt{2}\tilde{g}, \quad (6)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = -1 | M^{TCPC} + M^{TVPC} | \mu = -\frac{1}{2}, \lambda = 0 \rangle = \sqrt{2}g_3 - i\sqrt{2}\tilde{g}. \quad (7)$$

where μ (μ') and λ (λ') are spin projections of the initial (final) proton and deuteron on the beam direction, respectively. All diagonal matrix elements of the M^{TVPC} operator are zeros.

The total cross section of the pd scattering has the form [8]

$$\sigma_{tot} = \sigma_0 + \sigma_1 \mathbf{p}^p \cdot \mathbf{p}^d + \sigma_2 (\mathbf{p}^p \cdot \mathbf{m})(\mathbf{p}^d \cdot \mathbf{m}) + \sigma_3 P_{zz} + \tilde{\sigma} p_y^p P_{xz}^d, \quad (8)$$

where \mathbf{p}^p (\mathbf{p}^d) is the vector polarization of the initial proton (deuteron) and P_{zz} and P_{xz} are the tensor polarizations of the deuteron. The OZ axis is directed along the proton beam momentum \mathbf{m} , OY $\uparrow\uparrow \mathbf{p}^p$, OX $\uparrow\uparrow [\mathbf{p}^p \times \mathbf{m}]$. In Eq. (8) the terms σ_i with $i = 0, 1, 2, 3$ are non-zero only for T-even P-even interactions corresponding to Eq. (1) and the last term $\tilde{\sigma}$ constitutes a null-test signal of T-invariance violation with P-parity conservation. Using the generalized optical theorem we find $\tilde{\sigma} = -4\sqrt{\pi}Im\frac{2}{3}\tilde{g}$.

Hadronic amplitudes of pN scattering are taken as [5]

$$M_N(\mathbf{p}, \mathbf{q}; \boldsymbol{\sigma}, \boldsymbol{\sigma}_N) = A_N + C_N \boldsymbol{\sigma} \hat{\mathbf{n}} + C'_N \boldsymbol{\sigma}_N \hat{\mathbf{n}} + \sum_{l=n,q,k} B_N^l(\mathbf{q})(\boldsymbol{\sigma} \hat{\mathbf{l}})(\boldsymbol{\sigma}_N \hat{\mathbf{l}}), \quad (9)$$

where $\hat{\mathbf{q}}$, $\hat{\mathbf{k}}$ and $\hat{\mathbf{n}}$ are defined as unit vectors along the vectors $\mathbf{q} = \mathbf{p} - \mathbf{p}'$, $\mathbf{k} = \mathbf{p} + \mathbf{p}'$ and $\mathbf{n} = [\mathbf{k} \times \mathbf{q}]$, respectively; \mathbf{p} (\mathbf{p}') is the initial (final) proton momentum; $\boldsymbol{\sigma}_N$ is the Pauli matrix acting on the spin state of the nucleon N . We consider the following terms of the TVPC NN interaction which were under discussion in Ref. [4]:

$$t_{pN} = h_N [(\boldsymbol{\sigma} \cdot \mathbf{k})(\boldsymbol{\sigma}_N \cdot \mathbf{q}) + (\boldsymbol{\sigma}_N \cdot \mathbf{k})(\boldsymbol{\sigma} \cdot \mathbf{q}) - \frac{2}{3}(\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma})(\mathbf{k} \cdot \mathbf{q})] / m_p^2 + g_N [\boldsymbol{\sigma} \times \boldsymbol{\sigma}_N] \cdot [\mathbf{q} \times \mathbf{k}] / m_p^2 + g'_N (\boldsymbol{\sigma} - \boldsymbol{\sigma}_N) \cdot i [\mathbf{q} \times \mathbf{k}] [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z / m_p^2. \quad (10)$$

Here $\boldsymbol{\sigma}$ ($\boldsymbol{\sigma}_N$) is the Pauli matrix acting on the spin state of the proton (nucleon $N = p, n$), $\boldsymbol{\tau}$ ($\boldsymbol{\tau}_N$) is the corresponding matrix acting on the isospin state; m_p is the proton mass. In the framework of the phenomenological meson exchange interaction the term g' corresponds to ρ -meson exchange, and h -term provides the axial meson h_1 exchange.

2.1 g' -term

The g' term contributes only to the charge exchange transitions, because the non-zero matrix elements of the isospin-operator connected with the g' term in Eq. (10) are the following

$$\langle n, p | [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z | p, n \rangle = -i2, \quad \langle p, n | [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z | n, p \rangle = i2. \quad (11)$$

The isospin matrix element of the C-odd isospin operator $\mathcal{T}_z = [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z$ in Eq. (11) changes the sign under replacement $p \leftrightarrow n$. In contrast, the similar matrix elements for T-even P-even (strong) NN-interaction are equal one to other. This difference is one cause for the vanishing of the amplitude \tilde{g} for the double scattering mechanism of the process $pd \rightarrow pd$. As was shown in Ref. [6], the g' -term gives zero contribution to \tilde{g} within the Glauber model. Below we discuss this observation briefly.

The TVPC charge-exchange Glauber operator of the double scattering has a form [6]

$$O_{TVPC}^c = -\frac{1}{2} [M_{np \rightarrow pn}(\mathbf{q}_2) t_{pn \rightarrow np}(\mathbf{q}_1) + t_{np \rightarrow pn}(\mathbf{q}_2) M_{pn \rightarrow np}(\mathbf{q}_1)], \quad (12)$$

where $\mathbf{q}_1 = \mathbf{q}/2 + \mathbf{q}'$ is the transferred momentum in the first and $\mathbf{q}_2 = \mathbf{q}/2 - \mathbf{q}'$ in the second collision and \mathbf{q} is the total transferred momentum. For the next step one has to calculate the matrix element of the operator (12) over the deuteron states $\psi(\mathbf{r})$ with the factor $\exp(i\mathbf{q}'\mathbf{r})$ and integrate over \mathbf{q}' . Under the sign of this integral the operator (12) is not changed after the substitution $\mathbf{q}_1 \leftrightarrow \mathbf{q}_2$ [5, 6]. Therefore, one may add to the right side of Eq. (12) the term $O_{TVPC}^c(1 \leftrightarrow 2)$ and divide the obtained sum by a factor of 2. In collinear kinematics ($\mathbf{q} = 0$), this symmetry and linear dependence of g' -term on $[\mathbf{q} \times \mathbf{k}]$ lead to cancellation of the spin-independent term A_N in the transition operator (12). The same is true for the B_N terms in Eq. (9). Thus, only C_N and C'_N terms may contribute as [6]

$$O_{TVPC}^c = \frac{g'}{\Pi} \left[C_n(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p) \cdot \mathbf{n}_1 - C'_n(\boldsymbol{\sigma}_n \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_p \cdot \mathbf{n}_1) + C'_n \mathbf{n}_1 \hat{\mathbf{n}}_1 + \right. \\ \left. C_p(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_p - \boldsymbol{\sigma}_n) \cdot \mathbf{n}_1 - C'_p(\boldsymbol{\sigma}_p \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_n \cdot \mathbf{n}_1) + C'_p \mathbf{n}_1 \hat{\mathbf{n}}_1 \right], \quad (13)$$

where $\mathbf{n}_1 = [\mathbf{k} \times \mathbf{q}']$, $\hat{\mathbf{n}}_1 = \mathbf{n}_1/|\mathbf{n}_1|$ and $\mathbf{q}' = \mathbf{q}_2 = -\mathbf{q}_1$; Π is a constant. In Eq. (13) the C'_N -terms do not contain the proton beam spin $\boldsymbol{\sigma}$. We can show that this is a consequence of Eqs. (11). According to Eqs. (6), (7), it means that the contribution of the $C'_N \times g'$ term to the amplitude \tilde{g} is zero. Furthermore, due to Eqs. (11) the remaining terms with C_N in Eq. (13) contain the difference $\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p$, but not the sum. These terms can be rewritten as $\mathbf{V}_p \boldsymbol{\sigma}_p + \mathbf{V}_n \boldsymbol{\sigma}_n = 2(\mathbf{V}_p + \mathbf{V}_n)(\boldsymbol{\sigma}_p + \boldsymbol{\sigma}_n) \equiv 0$ [6]. Thus, the contribution of the operator (13) to the amplitude \tilde{g} vanishes and this fact is directly connected with Eqs. (11).

Strong suppression of the contribution of the ρ -meson as compared to the axial h_1 meson was found numerically in the Faddeev calculations [10] of the null-test signal for the nd scattering at 100 keV, but no explanation of this result was offered. We suppose that the cause for this suppression is the same spin-isospin structure of the scattering amplitude which leads to the vanishing ρ -meson contribution within the Glauber approach.

2.2 h - and g -terms

For the single scattering mechanism the amplitude \tilde{g} vanishes within the Glauber theory. Using Eqs. (3)-(7) for the double scattering mechanism with pN -amplitudes (9) and (10) we find for the h - and g -terms in Eq.(10) that all T-even P-even amplitudes of the pd -scattering are zeros: $g_1 = g_2 = g_3 = g_4 = 0$. Furthermore, we find for the TVPC amplitude

$$\tilde{g} = \frac{i}{4\pi m_p} \int_0^\infty dq q^2 [S_0^{(0)}(q) - 2\sqrt{2}S_2^{(1)}(q)] [C'_n(q)(g_p - h_p) + C'_p(q)(g_n - h_n)], \quad (14)$$

where $S_0^{(0)} = \int_0^\infty dr u^2(r) j_0(qr)$ and $S_2^{(1)}(q) = 2 \int_0^\infty dr u(r) w(r) j_2(qr)$ are the elastic form factors of the deuteron, and $u(r)$ and $w(r)$ are the S-wave and D-wave $w(r)$ of the deuteron, respectively, [6]. We can show that the S-D wave interference, not considered in Ref.[6], considerably diminishes the null-test signal $\tilde{\sigma}$ at the energies of the planned COSY experiment [3] ~ 100 MeV as compared to the pure S-wave contribution and provides an enhancement at 700-800 MeV.

3 Summary

In contrast to Ref. [4] we show, using the optical theorem, that within the single scattering approximation the null-test observable $\tilde{\sigma}$ is zero. Our result obtained within the Glauber theory is formulated by Eq. (14). Only the amplitude C'_N appears in Eq.(14) whereas other T-even P-even pN amplitudes, which were found in Ref. [4] to contribute to the TVPC null-test signal, are absent in Eq. (14). Furthermore, we find the deuteron D-wave gives a valuable contribution to the null-test signal for the case of the h - and g -type of interaction. The g' -term caused by the ρ - meson exchange in the TVPC NN-interaction makes a zero contribution to $\tilde{\sigma}$ and this result is true in the case when both the S- and D-components of the deuteron wave function are taken into account. We discuss some symmetry arguments to clarify a cause for the vanishing contribution of the g' -term. The g' -optical potential [11] and the corresponding coupling constant of the ρ -meson to the nucleon \bar{g}_ρ is widely used as a measure of intensity of the TVPC effects [12, 13]. Since the g' -term gives zero contribution to $\tilde{\sigma}$ within the Glauber theory, this parameter cannot be applied straightforwardly for the nucleon-deuteron scattering as a scale of the TVPC interactions at large enough energies. However, the g' -term can give contribution to the null-test signal $\tilde{\sigma}$ if this interaction is included into the deuteron bound state [6]. One-pion exchange is excluded from the TVPC NN-interaction [12], however, two-pion exchange probably contributes similarly to the P-violating pp -interaction [14]. Finally, TVPC NN forces can contribute to the electromagnetic p-d interaction due to the toroidal quadrupole form factor of the deuteron [15].

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