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Light-quark mass behaviour of the X(3872) as a molecular state

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Abstract.
Chiral extrapolations of the binding energy of the $X(3872)$ molecular state are investigated using an explicitly renormalizable framework free of finite cut-off artefacts. Insights into the binding mechanisms are discussed: if the X is less bound with the growing pion mass, its binding energy is governed by the explicit pion mass dependence from one-pion exchange; an opposite behaviour would indicate the importance of the pion-mass dependent short-range interactions, in addition to pionic effects. The important role of the three-body $D\bar{D}\pi$ dynamics is emphasised.

1 Introduction

The light-quark mass dependence of hadronic observables is of particular importance for understanding various phenomena in modern hadron and nuclear physics. For example, the knowledge of the light-quark or equivalently the pion mass dependence of the nucleon-nucleon (NN) scattering lengths is crucial for understanding certain fine-tunings in nuclear physics such as the formation of the life-important elements in connection to the anthropic principle, see a review [1] for a detailed discussion. In hadron physics the quark-mass dependence of the resonance poles could be useful to investigate the interplay of the long- and short-range forces in the formation of hadronic molecules. Furthermore, this information should be of practical value because it provides the connection between the lattice simulations which are still being carried out at relatively large unphysical pion masses and the physical point. One of the most interesting candidates is an enigmatic meson 1^{++} X(3872) discovered by the Belle collaboration in 2003, see Ref. [2] for a review. The existence of the X was predicted a long time ago [3, 4] based on the similarities between the $\bar{D}D^*$ and NN forces. In particular, it is assumed

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[5–7] that the X is a bound (or virtual) state of $\bar{D}D^*$ because of its close proximity to the $\bar{D}D^*$ threshold and because of its appropriate quantum numbers. In analogy to the deuteron, the X is amenable to the effective field theory treatment, see [8–10] for various studies within pionless framework and [11–17] for pionful approaches.

2 Description of the approach

In ref. [15] the pion mass dependence of the X binding energy was investigated within the EFT framework based on the non-perturbative solution of the non-relativistic Lippmann-Schwinger-type integral equations for the $\bar{D}D^*$ problem with the leading-order (LO) potential consisting of the one-pion-exchange (OPE) and the pion-mass dependent contact interaction. Special care was taken to include non-perturbatively all scales from three-body $D\bar{D}\pi$ intermediate states which play an important role because of a close proximity of the $D^{\pm}D^{\mp}\pi^0$, $D^{\pm}\bar{D}^0\pi^{\mp}$, and $D^0\bar{D}^0\pi^0$ thresholds to the $X(3872)$ pole. Notice that the use of non-perturbative OPE for chiral extrapolations allows one to extend the domain of applicability of the approach to larger pion masses which is important for analysing the results of lattice QCD calculations.

In this contribution we briefly discuss the extension of the work of Ref. [15] to address another important issue which is related to nonperturbative renormalisation of the integral equations for the $\bar{D}D^*$ problem, see a detailed discussion in the full-length publication [18]. The standard nonrelativistic approach to heavy mesons leads to coupled-channel integral equations for the scattering amplitudes which, at leading order in the EFT expansion, are linearly divergent. As a consequence, iterations of the OPE potential within the integral equations generate an infinite series of ultraviolet (UV) divergent higher-order contributions to the amplitude which cannot be absorbed into a finite number of counter terms (contact interactions) included in the truncated potential. To suppress the unwanted higher-order contributions, one usually employs a finite UV cut-off of the order of a natural hard scale in the problem, as advocated in Ref. [19]. This strategy was followed, in particular, in Refs. [9, 10, 14, 15]. Recently, another solution of this problem was proposed within a novel renormalizable approach to NN scattering [20] where exactly the same problem occurs. The core of the problem lies in the non-commutativity of the non-relativistic expansion and the renormalisation procedure [21], although the effect of relativistic corrections at low energies is of course small after renormalisation. Therefore, we derive the system of 3-dimensional dynamical equations for the $\bar{D}D^*$ problem which satisfy the relativistic unitarity, as naturally comes out if one uses time-ordered perturbation theory. This approach is renormalizable at leading order since all iterations of the potential generate only logarithmically divergent contributions which can be absorbed in the redefinition of the contact terms, see Refs. [18, 20]. Then the higher-order contributions to the potential are included perturbatively.

While both approaches described above should agree in general, the explicitly renormalizable approach has an advantage that finite cut-off artefacts are removed which is an important pre-requisite for understanding the binding mechanism. For example, unlike the finite cut-off results of Ref. [15], the pion mass dependence of the binding energy in the explicitly renormalizable framework can be predicted at LO and is entirely governed by the pion mass dependence from OPE. The novel approach should therefore be very useful for carrying out chiral extrapolations and studying correlations in the scattering amplitude induced by the analytic structure of the long-range forces Refs. [22].

3 Discussion of the results

We start from the LO. As was explained above, the relevant potential at this order reads

$$V_{\mathrm{LO}} = V_{\mathrm{OPE}}(\boldsymbol{p}, \boldsymbol{p}', \xi) + C_0, \tag{1}$$

where the first term on the r.h.s. denotes the OPE potential and the contact term C_0 is m_π-independent at this order. Once C_0 is adjusted to reproduce the binding energy at the physical pion mass $m_\pi^{\rm ph}$ (for definiteness we set $E_B(m_\pi^{\rm ph}) = 0.5$ MeV), the scattering amplitude can be calculated for unphysical pion masses without loss of renormalizability of the LO equations. Therefore, at the LO of our EFT, the dependence $E_B(m_\pi)$ can be predicted in a parameter free way—see Fig. 1. At this order, the pion mass dependence of E_B originates only from the pionic effects in the OPE potential and in the renormalised selfenergy loops. The binding energy at the LO demonstrates a clear tendency to disappear with the growing m_π. Furthermore, the slope of the binding energy in m_π at the physical point, $(\partial E_B/\partial m_\pi)\big|_{m_\pi=m_\pi^{\rm ph}}$, exhibits a strong sensitivity to the 3-body $D\bar{D}\pi$ effects. In particular, neglecting the 3-body dynamics (the so-called static OPE) results in a much steeper fall of the binding energy—compare the dotted (blue) line versus the solid (black) in the left panel of Fig. 1.

Since no real experiment is possible for unphysical pion masses, the only source of information on the m_π-dependence of the X pole is provided by lattice simulations. In analogy to the deuteron (see e.g. Ref. [22] for the discussion), the first lattice calculations for the $X(3872)$ evidence a stronger bound X for $m_\pi > m_\pi^{\rm ph}$ [23–25], although these results still suffer from potentially large finite-range corrections, as pointed out in Ref. [13]. To understand this type of behaviour we proceed to the NLO potential which takes into account the short-range corrections quadratic in m_π

$$V_{\rm NLO} = V_{\rm LO} + D\,(\xi^2 - 1), \quad \xi = m_\pi/m_\pi^{\rm ph}. \tag{2}$$

The unknown coefficient D can be fixed to the slope of the binding energy at the physical pion mass, which is therefore considered as an additional input quantity. For example, in Fig. 1 we illustrate the behaviour of the binding energy for the slope of natural size $(\partial E_B/\partial m_\pi^2)\big|_{m_\pi=m_\pi^{\rm ph}} \approx E_B^{\rm ph}/{m_\pi^{\rm ph}}^2$—see the dashed curve in the right panel. Interestingly, in a theory with the same polynomial behaviour of the contact operator but without pions one would observe a much flatter behaviour $E_B(m_\pi)$ for the same slope, as shown by the dashed-dotted curve. The difference between the two curves demonstrates the role of dynamical pions as an explicit long-range degree of freedom. As seen from Fig. 1, the contact interaction provides a smooth background for a rapidly varying pion mass dependence stemming from OPE. Therefore integrating out pions and the corresponding 3-body soft scales while still trying to at least partially compensate for neglecting these long-range effects, one would arrive at unnaturally large m_π-dependent coefficients accompanying short-range operators.

In summary, the explicitly renormalizable approach for $\bar{D}D^*$ scattering is developed and applied to explore the light-quark mass dependence of the $X(3872)$ pole. The latter appears to be strongly affected by the 3-body effects. Our approach should be of practical use to provide extrapolation of the lattice data to the physical limit.

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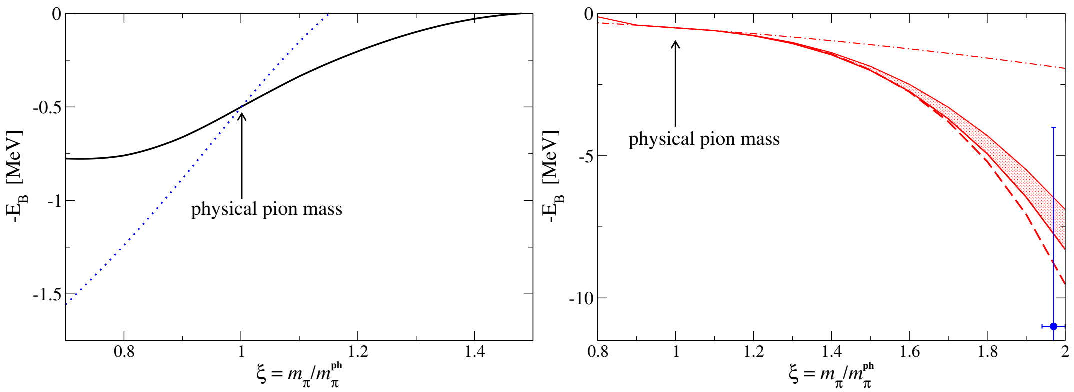

Figure 1. Pion mass dependence of the $X(3872)$ binding energy. Left panel: LO calculation; the results of the full dynamical theory with 3-body effects (black solid curve) vs. static OPE (blue dotted curve). Right panel: NLO calculation with an opposite slope used as input; perturbative inclusion of the contact operator D (red dashed curve) is in agreement with nonperturbative treatment of the same operator employing resonance saturation (red band). The dashed-dotted line corresponds to the calculation without pions, the blue data point is from Ref. [25].

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