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Recoil corrections in antikaon-deuteron scattering

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Abstract. Using the non-relativistic effective field theory approach for K^-d scattering, it is demonstrated that a systematic *perturbative* expansion of the recoil corrections in the parameter $\xi = M_K/m_N$ is possible in spite of the fact that K^-d scattering at low energies is inherently non-perturbative due to the large values of the $\bar{K}N$ scattering lengths. The first order correction to the K^-d scattering length due to single insertion of the retardation term in the multiple-scattering series is calculated. The recoil effect turns out to be reasonably small even at the physical value of $M_K/m_N \simeq 0.5$.

1 Introduction

Antikaon-nucleon scattering is an excellent testing ground for understanding of the $SU(3)$ QCD dynamics at low energies in the one-baryon sector. Starting from the seminal paper [1], it is often described within the so-called unitarized Chiral Perturbation Theory (ChPT), which uses the chiral potential calculated at a certain order. The common feature of such approaches is a relatively large number of free parameters, which are fixed from the fit to the experimental data, see for recent developments, e.g., Refs. [2–5]. An essential part of the input is coming from the S-wave $\bar{K}N$ scattering lengths, which "nail down" the amplitudes at the $\bar{K}N$ threshold and thus impose stringent constraints both on the scattering in the $\bar{K}N$ channel as well as the sub-threshold behavior of the amplitudes.

The experiments with kaonic atoms have been carried out in order to extract the precise values of the S-wave $\bar{K}N$ scattering lengths. Recently, the energy shift and width of kaonic hydrogen were measured very accurately in the SIDDHARTA experiment at DAΦNE [6]. These two quantities can be related to the K^-p scattering lengths via the so-called modified Deser-type formula, see Refs. [7, 8]. The same experimental collaboration has made an attempt to measure the energy and the width of the ground state of the kaonic deuterium as well. What makes the experiments with kaonic deuterium extremely important is the fact that the S-wave $\bar{K}N$ scattering lengths are complex-valued. Therefore, extracting two complex scattering lengths $a_0,\ a_1$ corresponding to the total isospin $I = 0, 1$ in the $\bar{K}N$ system implies the determination of four real quantities and thus requires measurements of four

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independent observables. Two observables are provided by the kaonic hydrogen, and the remaining two can come from, e.g., the kaonic deuterium. Therefore, it remains to derive an explicit relation between the $\bar{K}d$ and $\bar{K}N$ scattering lengths. This can be done assuming that the nucleons are infinitely heavy, leading to the type of Brueckner formula [9, 10]. However, there exists no *a priori* reason to believe that this is a good approximation in view of the fact that the mass ratio $M_K/m_N \simeq 0.5$ is not small. The goal of the present work is to continue the work of Ref. [11] with the main goal to formulate a procedure for including the recoil corrections *perturbatively* into the multiple-scattering series, in which the static interactions are summed up *to all orders*.

2 Framework

Our approach relies on the existence of two distinct momentum scales. The nucleon-nucleon and three-particle interactions are characterized by a low scale (of the order of the pion mass) and are described by non-local, energy-independent potentials $V_{NN}(\mathbf{p}, \mathbf{q})$ and $V_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$, respectively. On the contrary, $\bar{K}N$ interactions are characterized by a heavier scale (numerically of the order of the mass of the $\rho, \omega, \cdots$ resonances). We shall describe these interactions by a tower of local terms in the Lagrangian with zero, two,... space derivatives. The couplings emerging in these terms are expressed through the $\bar{K}N$ scattering lengths, effective radii and so on in a standard manner. For this reason, a *perturbative* expansion in such an effective theory automatically yields the *multiple-scattering series*, known from the potential scattering framework.

A generic term in the multiple-scattering expansion contains the diagrams in which the kaons are exchanged between two nucleons as well as kaons hopping on the same nucleon (the self-energy-type diagrams). Since NN interactions are non-perturbative, they have to be included to all orders. This is normally done by solving the Lippmann-Schwinger type equations which yield the NN amplitudes at low-energies. The NN amplitude in its turn is to be included in each intermediate state of the $\bar{K}NN - \bar{K}NN$ Feynman diagrams. In the vicinity of the static limit ($\xi \to 0$), only diagrams where the kaon is exchanged between two nucleons survive. In this limit the series in the diagrams can be summed up to all orders. Taking the Fourier transform and folding the result with the deuteron wave function in the isospin limit, we finally arrive at

$$\mathcal{A}_{\rm st} = \frac{1}{1+\xi/2} \int d^3\mathbf{r}\, |\Psi(\mathbf{r})|^2 \, \frac{3\tilde{a}_1 r^2 + r\tilde{a}_0(4\tilde{a}_1 + r)}{\tilde{a}_0(r - 2\tilde{a}_1) + r(2r - \tilde{a}_1)}\,, \tag{1}$$

where $r := |\mathbf{r}|$ and $\Psi(\mathbf{r})$ stands for the wave function of the deuteron, normalized to unity. The $\bar{K}N$ scattering lengths of isospin i enter the above equation as $\tilde{a}_i = (1+\xi)\, a_i$. Away from the static limit, the $\bar{K}NN$ Green functions g in the exchange diagram can be decomposed to a static part $g_{\rm st}$ and the recoil correction Δg as $g := g_{\rm st} + \Delta g$. Let $\tilde{a}$ denote a generic $\bar{K}N$ scattering length which is related to the non-derivative coupling in the effective Lagrangian. The multiple-scattering series can be therefore written as

$$\tilde{a} + \tilde{a}g\tilde{a} + \tilde{a}g\tilde{a}g\tilde{a} + \cdots = \left\{\tilde{a} + \tilde{a}^2 g_{\rm st} + \cdots\right\} + \left\{\tilde{a} + \tilde{a}^2 g_{\rm st} + \cdots\right\} (\Delta g) \left\{\tilde{a} + \tilde{a}^2 g_{\rm st} + \cdots\right\} + \cdots . \tag{2}$$

It is seen that the whole multiple-scattering series can be rearranged so that the static contributions are re-summed to all orders (the expression in the first curly bracket in the r.h.s of Eq. (2)), whereas the recoil corrections enter perturbatively, in the form of one, two, ... "recoil insertions", see the terms $\sim (\Delta g)^n$, with $n = 1, \cdots$. The contributions from the self-energy-type diagrams and those including NN interactions are parts of the recoil insertions, because they are absent in the static case. Moreover, in the original publication [12] we show that each insertion counts as $O(\xi^{1/2})$ (or, in some cases, as $O(\xi)$) and, consequently, to carry out calculations at a given order in the expansion parameter ξ, it suffices to consider a finite number of insertions.

3 Single recoil insertion

The antikaon-deuteron scattering length can be written as $\mathcal{A} = \mathcal{A}_{\text{st}}+\mathcal{A}^{(1)}+\mathcal{A}^{(2)}+\ldots$. In this section, we present compact explicit expression for the one recoil insertion correction $\mathcal{A}^{(1)}$ as derived in Ref. [12]. At this order, one has two diagrams, where the kaon scatters either on the same nucleon or on different nucleons, as well as one diagram which takes into account the intermediate NN interactions to all orders. Following [12], it is advantageous to combine all three contributing diagrams to two blocks of a definite isospin of the intermediate NN pair, $\mathcal{A}^{(1)} = (\mathcal{A}_1 + \Delta\mathcal{A}_{\text{st},1}) + (\mathcal{A}_0 + \mathcal{A}^{(c)} + \Delta\mathcal{A}_{\text{st},0})$. The single terms read ($\mathbf{p}_{1/2} := \mathbf{p} \pm \mathbf{l}/2$)

$$
\begin{aligned}
\mathcal{A}_0 &= \frac{1}{2}\frac{1}{1+\xi/2}\int \frac{d^3\mathbf{p}d^3\mathbf{l}}{(2\pi)^6}\, G_{\text{r}}(\mathbf{p},\mathbf{l})\Big(\Phi^+(\mathbf{p}_1)\Big(\Phi^+(\mathbf{p}_1)+\Phi^+(\mathbf{p}_2)\Big)\Big),\\
\mathcal{A}_1 &= \frac{1}{2}\frac{1}{1+\xi/2}\int \frac{d^3\mathbf{p}d^3\mathbf{l}}{(2\pi)^6}\, G_{\text{r}}(\mathbf{p},\mathbf{l})\Big(\Phi^-(\mathbf{p}_1)\Big(\Phi^-(\mathbf{p}_1)-\Phi^-(\mathbf{p}_2)\Big)+2\Phi_x(\mathbf{p}_1)\Big(\Phi_x(\mathbf{p}_1)-\Phi_x(\mathbf{p}_2)\Big)\Big),\\
\mathcal{A}^{(c)} &= \frac{\xi}{8\pi m_N}\frac{1}{1+\xi/2}\int \frac{d^3\mathbf{p}d^3\mathbf{l}d^3\mathbf{q}}{(2\pi)^9}\, G(\mathbf{p},\mathbf{l})\, M_{NN}(\mathbf{p},\mathbf{q},\mathbf{l})\, G(\mathbf{q},\mathbf{l})\Big(\Phi^+(\mathbf{p}_1)\Phi^+(\mathbf{q}_1)\Big),\\
\Delta\mathcal{A}_{\text{st},0} &= -\frac{\xi}{2(1+\xi)(1+\xi/2)}\int d^3\mathbf{r}\frac{\Psi^2(\mathbf{r})}{r}A^+(r)^2\,,\\
\Delta\mathcal{A}_{\text{st},1} &= \frac{\xi}{2(1+\xi)(1+\xi/2)}\int d^3\mathbf{r}\frac{\Psi^2(\mathbf{r})}{r}\Big(A^-(r)^2+2A_x(r)^2\Big)\,.
\end{aligned}
\tag{3}
$$

Here, the Fourier transform of $\Psi(\mathbf{r})A^{\pm}_{(x)}(r)$ is denoted by $\Phi^{\pm}_{(x)}$, where the rational functions $A^{\pm}(r) := A_p(r) \pm A_n(r)$ and $A_x(r)$ account for static contributions, resummed to all orders. Explicitly, they read

$$
A_p(r) = \frac{r^2 a_1 + r a_0(2a_1+r)}{a_0(r-2a_1)+r(2r-a_1)}\,,\ A_n(r) = \frac{2ra_1(a_0+r)}{a_0(r-2a_1)+(2r-a_1)r}\,,\ A_x(r) = \frac{r^2(a_0-a_1)}{a_0(2a_1-r)+(a_1-2r)r}\,.
$$

The nucleon-nucleon amplitude M_{NN} is determined from a solution of the Lippmann-Schwinger equation for a given two-nucleon potential V_{NN}. For the explicit representation of this as well as Green functions $G_{(r)}(p,l)$ we refer the reader to Ref [12].

The above expression of the single recoil insertion depend on the variable ξ in a non-trivial way. In order to establish systematic power-counting rules, it is therefore necessary to perform an expansion of this expression in ξ. Further, considering such an expansion helps one to reveal the pattern of cancellations of the leading terms, which has been discussed already in Ref. [11]. The uniform expansion method, see Refs. [13, 14], allows one to perform such an expansion by identifying three different momentum regions according to the scales appearing in the problem. The integrand is expanded in each region before performing the integration, and the results are summed up. Utilizing this method and introducing a modified expansion parameter $\tilde{\xi} = \xi/(1+\xi/2)$, we find that the expansion of each contribution $\mathcal{A}_1$, $\mathcal{A}_0$ and $\mathcal{A}^{(c)}$ in powers of $\sqrt{\tilde{\xi}}$ indeed converges quite rapidly. For more details on the expansion, explicit results and discussion of cancellation patterns, the reader is referred to the original publication [12]. Before coming to the final *numerical* results, we also wish to note that these do not rely on the latter expansion of individual contributions.

Since no experimental results on kaon-deuteron scattering length are available yet, in our exploratory study we estimate the size of the recoil correction for a given set of the KN scattering lengths used as input in Ref. [15], i.e. $a_0 = -1.62 + i\,0.78$ fm, $a_1 = 0.18 + i\,0.68$ fm. Moreover, at this stage we use the phenomenological nucleon-nucleon Hulthén and PEST separable potentials instead of a more complicated treatment based completely on chiral EFT. The results of the calculations shown in the following table.

Hulthén	$\mathcal{A}_{\rm st}$	$-1.49 + i\,1.19$	
	$\mathcal{A}^{(1)}$	$\mathcal{A}_1$	$-0.13 + i\,0.16$
		$\Delta\mathcal{A}_{\rm st,1}$	$+0.12 - i\,0.20$
		$\mathcal{A}_0$	$-0.27 + i\,0.87$
		$\Delta\mathcal{A}_{\rm st,0}$	$-0.12 + i\,0.33$
		$\mathcal{A}^{(c)}$	$+0.35 - i\,1.06$
	$\mathcal{A}_{\rm st} + \mathcal{A}^{(1)}$	$-1.52 + i\,1.27$	

PEST	$\mathcal{A}_{\rm st}$	$-1.55 + i\,1.25$	
	$\mathcal{A}^{(1)}$	$\mathcal{A}_1$	$-0.13 + i\,0.18$
		$\Delta\mathcal{A}_{\rm st,1}$	$+0.13 - i\,0.22$
		$\mathcal{A}_0$	$-0.29 + i\,0.97$
		$\Delta\mathcal{A}_{\rm st,0}$	$-0.11 + i\,0.34$
		$\mathcal{A}^{(c)}$	$+0.36 - i\,1.19$
	$\mathcal{A}_{\rm st} + \mathcal{A}^{(1)}$	$-1.59 + i\,1.32$	

We conclude that in the isoscalar channel, the individual contributions, which still contain the dominant $O(\tilde{\xi}^{1/2})$ term, are very large, especially the imaginary parts thereof. However, they undergo significant cancellations, yielding only about a 10% net correction to the imaginary part of the static term. The resulting isovector recoil correction appears to be even smaller providing only about a 3% correction to the static term. Its smallness can be understood from the exact cancellation of the $O(\tilde{\xi}^{1/2})$ term along with some additional cancellations among higher-order terms. Furthermore, the net correction $\mathcal{A}^{(1)}$ which stems from one recoil insertion in the multiple scattering diagrams appears to be quite small, of order of $\simeq 6-8\%$ of $\mathcal{A}_{\rm st}$, despite the large value of ξ. An additional suppression is partly accounted for by cancellations of the isoscalar and isovector recoil corrections.

Finally, it is instructive to compare this results to the result of numerical estimations. In Ref. [15] the value $\mathcal{A} \simeq (-1.47 + i1.11)$ fm was obtained from the solution of the Faddeev equations with the one-channel energy-independent optical potential, which was adjusted to reproduce the same $\bar{K}N$ scattering lengths as used in our calculation. Despite the differences in two calculations, such as different NN models, or the use of the off-shell form factors, needed to regularize the Faddeev calculations, the differences between the static result and the solution of the Faddeev equation are, generally, of the same order as the full recoil correction $\mathcal{A}^{(1)}$.

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