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Low-lying ^{12}C continuum states in three-α model

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Abstract. The electromagnetic monopole ($E0$) transition of the ^{12}C nucleus, the 0^+ ground state to three α-particles (3α) continuum states, is calculated by applying the Faddeev three-body formalism in coordinate space accommodating the long range Coulomb potential between α particles. Results show that there are two 0^+ states, ^{12}C(0_3^+) and ^{12}C(0_4^+) states at excitation energies around 10 MeV, which is consistent with results of recent semi-microscopic calculations and analysis of α-^{12}C inelastic scattering.

1 Introduction

Low-lying states in ^{12}C nucleus are interesting subjects to study as a 3α system. For example, it is widely known that the first 0^+ resonant state ^{12}C(0_2^+) at the excitation energy $E_x = 7.65$ MeV (so called the Hoyle state) plays an essential role in the synthesis of ^{12}C from a 3α continuum state (the triple-α process). However, there still have been uncertainties in the ^{12}C energy level structure at these energies ($E_x \approx 10$ MeV): While a 0^+ state was tentatively listed at $E_x = 10.3$ MeV in the compilation of experimental data [1], there is an experimental report that there exist two 0^+ states (0_3^+ and 0_4^+) [2, 3] near $E_x = 10$ MeV. Theoretically, the existing of two 0^+ states in this energy region was predicted by a semi-microscopic model, the orthogonal-condition model (OCM), combined with the complex scaling method [4].

In this paper, I will consider a transition of the ^{12}C ground state ^{12}C(0_1^+) to 3α 0^+ continuum states by the $E0$ operator and calculate the transition strength as a function of 3α energy E at the center of mass (c.m.) system above the 3α threshold. This is an extension of recent works [5, 6], in which 3α continuum states up to $E = 0.6$ MeV were studied to calculate the reaction rate of the triple-α process at stellar temperature $T \sim 10^9$ K, and clarify the decay mode of the Hoyle state.

In Sec. 2, I will give a short description of the formalism to calculate the strength function. In such a treatment, every complicacies arising from nucleon structure of the α-particle are assumed to be incorporated in interaction potentials among the α-particles, which usually consist of two-α potential (2αP) as well as three-α potential (3αP). Such models used in this work and results of the strength function will be presented in Sec. 3. Summary will be given in Sec. 4.

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2 Formalism

We will consider a transition process of the ^{12}C(0_1^+) state $|\Psi_b\rangle$ to 3α continuum states induced by the $E0$ operator (see, *e.g.* [7]),

$$\hat{O}(E0) = \boldsymbol{x}^2 + \frac{4}{3}\boldsymbol{y}^2, \tag{1}$$

where $\boldsymbol{x}$ is the relative coordinate of α-α pair and $\boldsymbol{y}$ is the relative coordinate of the spectator α-particle with respect to the c.m. of the pair.

With the transition amplitude of the reaction, $\langle\Psi^{(-)}_{\boldsymbol{qp}}|\hat{O}(E0)|\Psi_b\rangle$, where $|\Psi^{(-)}_{\boldsymbol{qp}}\rangle$ is an eigen-state of 3α Hamiltonian H, and $\boldsymbol{q}$ and $\boldsymbol{p}$ are Jacobi momenta conjugate to the coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, the $E0$ transition strength function is defined by

$$\frac{dB(E0;E)}{dE} = \int d\boldsymbol{q}d\boldsymbol{p}\left|\langle\Psi^{(-)}_{\boldsymbol{qp}}|\hat{O}(E0)|\Psi_b\rangle\right|^2 \delta\left(E - (E_q + E_p)\right), \tag{2}$$

where $E_q = \frac{\hbar^2}{m_\alpha}\boldsymbol{q}^2$ and $E_p = \frac{3\hbar^2}{4m_\alpha}\boldsymbol{p}^2$ with m_α being the α particle mass.

Let us define a wave function for the transition process by

$$\Psi(\boldsymbol{x},\boldsymbol{y}) = \left\langle \boldsymbol{x},\boldsymbol{y}\left|\frac{1}{E + \imath\epsilon - H}\hat{O}(E0)\right|\Psi_b\right\rangle, \tag{3}$$

which has the asymptotic form [5],

$$\Psi(\boldsymbol{x},\boldsymbol{y}) \underset{\substack{x\to\infty \\ y/x \text{ fixed}}}{\longrightarrow} \frac{e^{\imath(K+\mathcal{O}(1/R))R}}{R^{5/2}} F^{(B)}(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}},E_q), \tag{4}$$

where $K = \sqrt{\frac{m_\alpha}{\hbar^2}E}$, $R = \sqrt{x^2 + \frac{4}{3}y^2}$, $\mathcal{O}(1/R)$ expresses the effects from the long-range Coulomb interaction, and $F^{(B)}(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}},E_q)$ is the breakup amplitude,

$$F^{(B)}(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}},E_q) = e^{\frac{\pi}{4}\imath}\sqrt{\frac{\pi}{2}\frac{m_\alpha}{\hbar^2}}\left(\frac{4K}{3}\right)^{3/2}\langle\Psi^{(-)}_{\boldsymbol{qp}}|\hat{O}|\Psi_b\rangle. \tag{5}$$

The wave function Ψ will be obtained by applying the Faddeev three-body formalism to solve integral equations in coordinate space with accommodating the long range Coulomb force effects. (See Refs. [5, 6, 8] for the details of the calculations.) This procedure provides the amplitude $F^{(B)}(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}},E_q)$, and then the $E0$ strength function is calculated as

$$\frac{dB(E0;E)}{dE} = \frac{1}{2\pi K^3}\left(\frac{3}{4}\right)^2\int d\hat{\boldsymbol{q}}d\hat{\boldsymbol{p}}pqdE_q\left|F^{(B)}(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}},E_q)\right|^2. \tag{6}$$

3 Results

In the present work, I used the model D of the Ali-Bodmer potential AB(D) [9] with the (point) Coulomb potential for the α-α interaction. In addition, 3αP of the following form is introduced:

$$\Delta(a) = W_\Delta \exp\left(-\frac{r_{12}^2 + r_{23}^2 + r_{31}^2}{a^2}\right) + [\text{c.p.}], \tag{7}$$

Table 1. The range and strength parameters of the 3αP models, Eq. (7), used in this work, and calculated energy of the ^{12}C ground state with respect to the 3α threshold, $E[^{12}\mathrm{C}(0_1^+)]$.

Model	a (fm)	W_Δ (MeV)	$E[^{12}\mathrm{C}(0_1^+)]$ (MeV)
Δ(2.4)	2.43	-143.7	-7.546
Δ(2.6)	2.61	-101.22	-7.584
Δ(3.0)	3.00	-52.3	-7.759
Δ(3.4)	3.39	-30.95	-7.789
Δ(4.0)	4.00	-16.18	-7.281
Exp.			-7.275

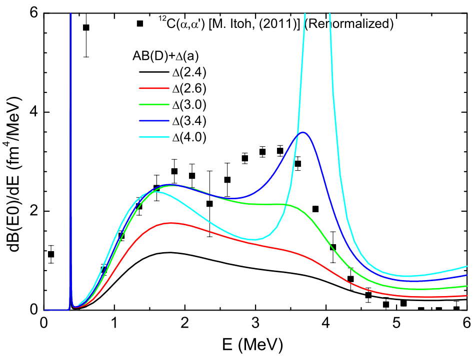

Figure 1. The $E0$ strength function for a transition from ^{12}C ground state to 3α 0^+ continuum states as a function of the final 3α energy in the c.m. system E. The black, red, green, blue, and aqua lines are calculations by Δ(2.4), Δ(2.6), Δ(3.0), Δ(3.4), and Δ(4.0), respectively. Experimental data are taken from Ref. [2, 11] renormalized according to Ref. [10] (see the text).

where [c.p.] denotes the cyclic permutations. Taking into account of uncertainty in the knowledge of 3αP, I examined some different values for the range parameter a from 2.4 fm to 4.0 fm in Eq. (7).

The strength parameter W_Δ for each of $\Delta(a)$ is determined to reproduce the energy of the Hoyle state, and results are shown in Table 1 along with calculated energy of $^{12}\mathrm{C}(0_1^+)$. In the table, calculated energies of $^{12}\mathrm{C}(0_1^+)$ differ from the empirical value by (at most) 10 %, which leaves the construction of a 3αP that reproduces both of bound and continuum states for a future work.

Fig. 1 shows calculated $E0$ transition strength functions for five different models of 3αP denoted as $\Delta(a)$ together with the AB(D) 2αP. In the figure, the experimentally extracted $E0$ strength from $^{12}\mathrm{C}(\alpha,\alpha')$ at $E_\alpha = 386$ MeV [2, 11] is also plotted, but it is renormalized, for a comparison, to reproduce the strength $B(E0, 0_3^+ \to 0_1^+) = |M(E0, 0_3^+ \to 0_1^+)|^2 = (2.9 \pm 0.3\ \mathrm{fm})^2$, which was evaluated by a folding-mode analysis of the inelastic α + ^{12}C scattering data at medium energies [10].

Besides the sharp peak of the Hoyle state $^{12}\mathrm{C}(0_2^+)$, the calculated strength functions reveal two peaks at $E \approx 1.8$ MeV and at $E \approx 3.5$ MeV with rather large widths. These peaks may correspond to $^{12}\mathrm{C}(0_3^+)$ and $^{12}\mathrm{C}(0_4^+)$ resonant states, which were predicted in Ref. [4] and found in Ref. [2]. For the calculations with smaller value of a such as Δ(2.4), two-peak structure of the strength function is not clear, especially for the higher $^{12}\mathrm{C}(0_4^+)$ peak. As a increases, two-peak structure develops gradually, and the peak height of $^{12}\mathrm{C}(0_4^+)$ increases much faster than that of $^{12}\mathrm{C}(0_3^+)$. Comparing the heights of two peaks, calculations with Δ(3.4) and Δ(3.0) look favorable with the experimental data. It is remarkable that $a = 3.4$ fm corresponds to a configuration that two α particles are almost in touch.

Three particles in the final state spread by sharing momenta and energies in variety of manners as far as conservation laws are satisfied in principle. In Ref. [6], it was shown that the decay of the Hoyle state is dominated by a sequential decay (SD) process, in which the decay occurs in two steps: first 3 α-particles separate to 2-α resonant state, $^8\mathrm{Be}(0_1^+)$ [$E_{2\alpha,r} = 92$ keV, $\Gamma_{2\alpha} = 6.8$ eV] and the rest

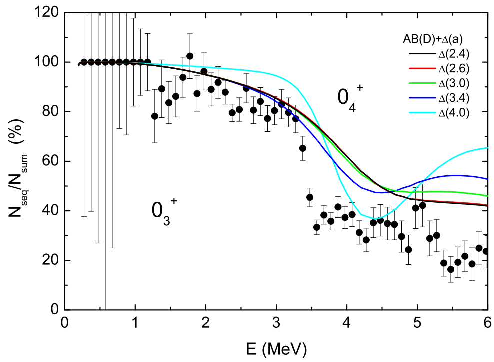

Figure 2. Ratio of the sequential decay component to the total flux for $E0$-disintegration of ^{12}C ground state as a function of the c.m. energy E. The meaning of the lines is the same as in Fig. 1. Experimental data are evaluated from the figure 2 of Ref. [3].

α-particle, and then ^{8}Be(0_1^+) decays to 2-α particles. In Ref. [3], from an analysis of semi-exclusive ^{12}C($\alpha, \alpha'\alpha''$) reaction, contributions of the SD mode were extracted. In the present work, the SD component is evaluated by limiting the E_q-integration in Eq. (6) as $E_{2\alpha,r} - \Delta E \leq E_q \leq E_{2\alpha,r} + \Delta E$, where ΔE is set to be a few times of the width $\Gamma_{2\alpha}$.

In Fig. 2, calculated fraction of the SD component to the total is plotted and compared to the data. While the 3α decay mode for the ^{12}C(0_3^+) state is dominated by the SD process via the ^{8}Be(0_1^+) state, that for the ^{12}C(0_4^+) state is not. The details of the 3α decay mode depend on the choice of the 3αP.

4 Summary

The 3α-model calculations in this work suggest that there are two peaks in the $E0$ strength function from ^{12}C ground state, which may be assigned as ^{12}C(0_3^+) and ^{12}C(0_4^+) states.

While the sequential decay mode is dominant one for ^{12}C(0_3^+) state, it is not for ^{12}C(0_4^+) one. Further and more precision studies of the $E0$ strength function as a function of E as well as those of the decay mode of ^{12}C(0_4^+) state will provide valuable information on the interaction model of α-particles, especially on 3α potential.

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