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A Λnn three-body resonance

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Abstract. We solved the Faddeev equations in the Λnn system ($J^{\pi} = \frac{1}{2}^{+}, T = 1$). There is no bound state but found a resonance state. Complex Energy Method helps to find the location of the resonance energy in the complex Riemann sheet. We obtained a resonance energy $E = 0.25 - 0.40i$ MeV using the realistic NN and YN Nijmegen potential. As a preliminary result the recent Nijmegen YN potential (NSC97f) gives 0.60± 0.05 - (0.25 ± 0.05)i MeV. We discuss importance of an irreducible three-body force to make it bound.

1 Introduction

The hypertriton of Λnp was observed and the Faddeev calcultions also achived to obtain a precise binding energy [1]. On the other hand, in the Λnn system, because the partial wave $^1S_0$ between two neutrons does not have sufficient attractive force, it has been expected that there is no bound state [1]. However, recently, a bound state was reported [2]. Hiyama *et al.*[3] addressed this issue and investigated it by the variational calculation with the Gauss expansion method. They insisted that there must be no bound state, however, a tiny modification of the tensor force between $\Lambda N - \Sigma N$ coupled channel might make it bound. The Faddeev calculations were confirmed [4] with a quark cluster model potential. From aspect of several hypernuclear systems it was examined [5]. They are consistent with the former work[1].

In this paper we revisit the Λnn system and look for the binding energy or the resonance one by solving the Faddeev equations using the realistic YN Nijmegen potential (NijmYN89) [6] and NN one (Nijm93)[7]. In our last work[1] the recent YN Nijmegen potential (NSC97f) [8] was not applied yet. In addition the Complex Energy Method (CEM) [9] is powerful and useful tool to solve the few-body system for the continuum energy regime. It has been used not only to the Faddeev calculation but to the four-body Yakubovsky calculation[10, 11].

The next section we briefly explain our Faddeev equations and the CEM. The results are demonstrated in sec. 3. Summary and outlook are given in sec. 4.

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2 Faddeev equations and Complex Energy Method

The Faddeev equations of Λnn system consists of the two components $\psi^{(12)}$ and $\psi^{(13)}$ which links to the channel subsystems (*Nucleon*:1,2; *Hyperon*:3) described by the Jacobi coordinates.

$$\begin{aligned} \psi^{(12)} &= G_0\, T_{12}\, (\, 1 - P_{12}\,)\, \psi^{(13)}, \\ \psi^{(13)} &= G_0\, T_{13}\, (\, \psi^{(12)} - P_{12}\psi^{(13)}\,). \end{aligned} \tag{1}$$

where P_{ij} are permutation operators which exchanges particle i and j, and T_{ij} means the two-body transition matrix which couples Λn and Σn channels in case of YN interaction. Detail of the formalism is written in [1] as well as the case of Λpn system. After the partial-wave decompositions and the preparation of integral points, Eq. (1) becomes analogous to a large eigenvalue equation;

$$K(E)\vec{\psi} = \eta(E)\vec{\psi} \quad \text{with} \quad \vec{\psi} = \begin{pmatrix} \psi^{(12)}_{\Lambda nn} \\ \psi^{(12)}_{\Sigma nn} \\ \psi^{(13)}_{\Lambda nn} \\ \psi^{(13)}_{\Sigma nn} \end{pmatrix}. \tag{2}$$

The matrix $K(E)$ is regarded as the integral kernel in Eq.(1). The eigen value $\eta(E)$ must be 1 when the energy reaches to the resonance energy ($E = E_r - i\,\Gamma/2$). In order to solve the eigenvalue equation of the large rank there are 2 difficulties.

- The largest number of the absolute of the eigenvalue is not 1 because the NN and YN realistic potentials have a strong repulsive core.
- For the continuum states ($E > 0$) the equations have singularities of the integral kernel on the real axis of momenta.

Gauss-Seidel method works well to get the largest absolute value of the eigen value $|\eta|$, however, it becomes unphysical negative ($\eta^{neg} < 0$) because of the potential having a repulsive core. Using the negative eigenvalue η^{neg} we could converge the Gauss-Seidel iteration to the physical state as;

$$\vec{\psi}^{(n+1)} = \frac{K\vec{\psi}^{(n)} - \eta^{neg}\vec{\psi}^{(n)}}{1 - \eta^{neg}}, \tag{3}$$

where $\vec{\psi}^{(0)}$ is a nonzero arbitary vector which is of course a finite size. Therefore, the physical eigenvalue η^{phy} is obtained from

$$\eta^{phy} = (1 - \eta^{neg})\, r + \eta^{neg} \quad \text{with} \quad r \equiv \lim_{n\to\infty} \frac{\vec{\psi}^{(n+1)} \cdot \vec{\psi}^{(n)}}{\vec{\psi}^{(n)} \cdot \vec{\psi}^{(n)}}. \tag{4}$$

One expects $r = 1$ at $E = Er - i\,\Gamma/2$. The second difficulty can be avoided by the CEM on the physical Riemann sheet of the complex energy ($E = E_r + i\epsilon$). The typical $\epsilon > 0$ is about 1.0 MeV. Instead of the direct calculation at $E = E_r - i\,\Gamma/2$ some samples $\eta(E_r + i\epsilon)$ are calculated. Both the eigen energy of $E_r + i\epsilon$ and $E_r - i\,\Gamma/2$ are located on the same/close physical Riemann sheet of the complex energy plane. Using these samples $\eta(E_r + i\epsilon)$ we could extrapolate [9] $\eta^{phy}(E_r - i\,\Gamma/2) = 1$ of the resonance state.

3 Results

In case of the total spin ($J^{\pi} = \frac{1}{2}^{+}$) and the isospin ($T = 1$) for the Λnn system we expect a first resonance state (see Fig.4 of [1]). Table 1 shows the number of the coupled equations in Eq. (1). In

Table 1. Number of three-body couple channels and the binding energies E_b in MeV. j_{max} is the total spin in the subsystem. The last column show the Λnn binding energy with the strength factor s=1.20.

j_{max}	$\Lambda\ n$ channel	$\Sigma\ n$ channel	$n\ n$ channel	Sum	E_b
2	10	18	36	64	-0.224
3	12	26	52	90	-0.241
4	18	34	68	120	-0.243
5	20	42	84	146	-0.244

Table 2. The resonance energies $E_r - i\,\Gamma/2$. The factor s in the first column is multiplied to the original YN potential[6] in Eq.(5). Caution that in [3] NN potential was used Argonne V8 model and the factor s was multiplied only with the tensor part of $^3V_{N\Sigma-N\Lambda}$ in NSC97f potential.

s	NijmYN89 + Nijm93	NSC97f+ Nijm93	Hiyama *et al.* [3]
1.00	0.25 -0.40i	0.60± 0.05 -(0.25 ± 0.05)i	
1.05	0.15 -0.20i		
1.10	0.08 -0.15i		
1.20	-0.243 (bound)		-0.054 (bound)

order to obtain 3 digits accuracy for binding energy we need the partial waves up to $j_{max} = 4$ or 5. We take 34 integral points for two Jacobi momenta and 16 points for angle integral.

Before looking into the continuum state we can track the resonance pole from an artificial bound state which is forced by enlarging the strength of the YN potential. We introduced a factor s and define an artificial potential $\tilde{V}_{YN}$ as

$$\tilde{V}_{YN} = sV_{YN}. \tag{5}$$

Of course the potential $\tilde{V}_{YN}$ is switched into the original V_{YN} with $s = 1$. In fact the factor s is multiplied all partial waves of YN potentials except for the case of isospin $3/2$.

We take the realistic YN Nijmegen potential (NijmYN89) [6] and NN one (Nijm93)[7]. The artificial bound state is found at E=-0.244 MeV under the strength factor $s = 1.20$ and $j_{max} = 5$. The convergency of the binding energy E_b is demonstrated in Table 1. Decreasing s and choosing $j_{max} = 4$ we calculate some eigenvalues η at $E = E_r + i\epsilon$. The number of samples η is about 20. We extrapolate the resonance energy out of these sampling $\eta(E_n)$ $(n = 1, 2, 3, \cdots, 20)$.The resonance energies are shown for each s in Table 2. Under the physical condition $s = 1$ we obtain the physical resonance energy E =0.25 -0.40i MeV. As the preliminary result we have E=0.60± 0.05 - (0.25 ± 0.05)i MeV using the recent NSC97f YN potential[8]. Comparing to NijmYN89 case the numerical stability is not good, we estimate the errors about ± 0.05MeV. Fig. 1 shows the position of the resonance poles in the complex energy plane.

4 Summary and Outlook

We solved the Faddeev equations in the Λnn system ($J^\pi = \frac{1}{2}^+, T = 1$). There is no bound state but found a resonance state. The Complex Energy Method helps to find the location of the resonance energy in the complex Riemann sheet. We obtained $E = 0.25 - 0.40i$ MeV using Nijm93[7] and NjimYN89[6] models. Though preliminary, we got a result of E=0.60± 0.05 - (0.25 ± 0.05)i MeV using the recent NSC97f YN potential[8] instead of NijmYN89 model.

Our results suggest that there is no bound state which does not agree with the experimental report [2]. However, the remaining possibility of the exsistance of the bound state may be supported by

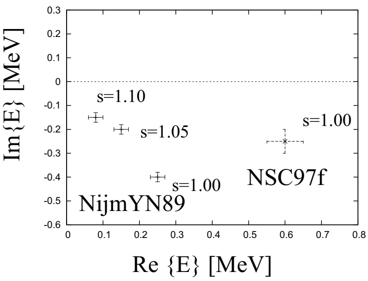

Figure 1. The resonence pole positions in the complex energy plane. The resonance energies from NijmYN89 model are marked with solid errorbars as well as NSC97f's does with dashed one.

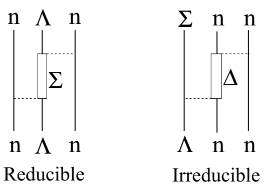

Figure 2. Diagrams for a reducible three-body force and a irreducible three-body force. The process of the irreducible three-body force contains with the Δ isobar particle transition.

the other YN potentials [12–14] or a three-body force. In the sense of the meson theoretical picture Fig. 2 shows diagrams of typical three-body forces. The left diagram of the pictures is a reducible one which is already taken into account in the YN two-body potential. On the other hand the right diagram involved with a Δ isobar excitation process is an irreducible one which might be seriously missing. There is a tiny lack of energy (~ 0.5 MeV) for binding. For example, the case of triton (^{3}H) needs more than 0.5 MeV from a three-body force.

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