

## Three Dimensional SRG Evolution of the $NN$ Interactions Using Picard Iteration

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**Abstract.** The Similarity Renormalization Group (SRG) evolution of nucleon-nucleon ( $NN$ ) interactions is calculated directly as function of momentum vectors for realistic potentials. To overcome the stiffness of the SRG flow equations in differential form for far off diagonal matrix elements, the differential equation is transformed to an integral form without employing a partial wave decomposition.

The SRG evolution of  $NN$  interaction in a three-dimensional representation, without using a partial wave decomposition, can be obtained by solving the differential flow equation [1]

$$\begin{aligned} \frac{dV_{s\Lambda\Lambda'}^{\pi ST}(\mathbf{p}, \mathbf{p}')}{ds} &= -\left(\frac{p^2}{m} - \frac{p'^2}{m}\right)^2 V_{s\Lambda\Lambda'}^{\pi ST}(\mathbf{p}, \mathbf{p}') \\ &+ \frac{1}{4} \sum_{\Lambda''=-1}^{\Lambda''=+1} \int d^3 p'' \left(\frac{p^2}{m} + \frac{p'^2}{m} - \frac{2p''^2}{m}\right) V_{s\Lambda\Lambda''}^{\pi ST}(\mathbf{p}, \mathbf{p}'') V_{s\Lambda''\Lambda'}^{\pi ST}(\mathbf{p}'', \mathbf{p}'). \end{aligned} \quad (1)$$

The input to this formulation is an operator form of the  $NN$  interaction, where the total spin of the system,  $S$ , is treated in a helicity representation [2]. Dependent on the value of  $S$ , one needs to solve a single equation for  $S = 0$  and four coupled equations for  $S = 1$ . The non-linear SRG differential equations can become extremely stiff and take a prohibitively long time to evolve to large values of the flow parameter  $s$ . To overcome this problem, we transform the differential equation to an integral form via a Picard ansatz and solve it iteratively. By integrating both sides of the differential equation (1) over the flow parameter  $s$  from zero to a specific value  $s$ , the SRG flow evolution can be written schematically as

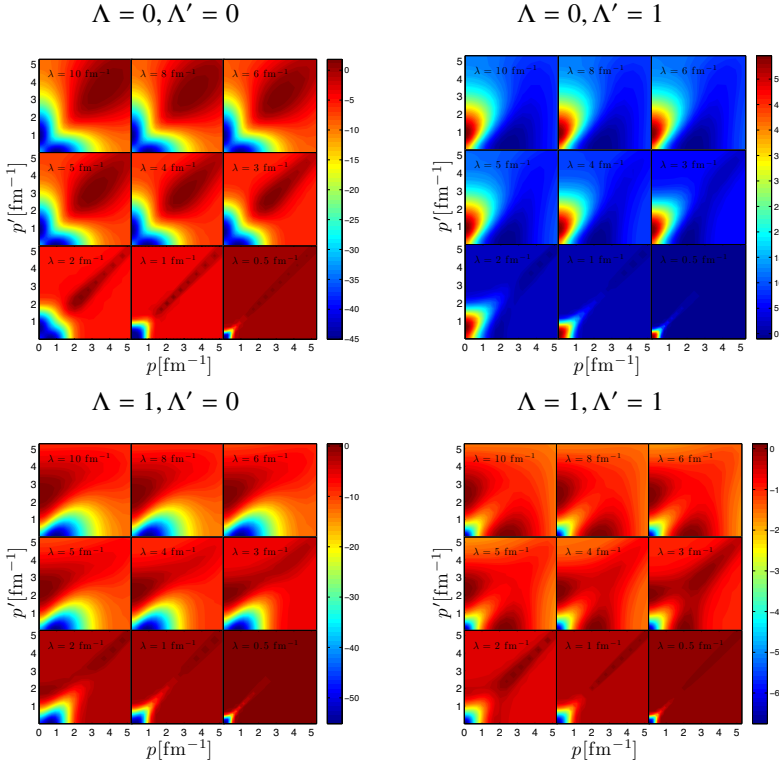
$$V_s = V_{NN} + A \int_0^s ds V_s + B \int_0^s ds V_s V_s, \quad (2)$$

which can be solved iteratively as

$$V_s^{(n)} = V_{NN} + A \int_0^s ds V_s^{(n-1)} + B \int_0^s ds V_s^{(n-1)} V_s^{(n-1)}; \quad n = 1, 2, \dots \quad (3)$$

The iteration is started with the bare  $NN$  interaction  $V_{NN}$ . The SRG integral equations are successfully solved for the Bonn-B and a Chiral N3LO potential and evolved to a large flow parameter  $s$ , or small

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**Figure 1.** The matrix elements of evolved  $NN$  interaction  $V_{s\Lambda\Lambda'}^{\pi ST}(p, p', x = \frac{1}{2})$  (in units  $\text{MeV fm}^3$ ) calculated from Bonn-B potential by solving 3D form of SRG integral equation using Picard iteration method.

$\lambda = s^{-1/4} / \sqrt{\frac{\hbar^2}{m}} = 0.5 \text{ fm}^{-1}$ . In Fig. 1 we show the matrix elements of the evolved Bonn-B interaction,  $V_{s\Lambda\Lambda'}^{\pi ST}(p, p', x = \frac{1}{2})$ , for total spin  $S = 1$ , isospin  $T = 0$ , parity  $\pi = 1$  and different helicity eigenvalues  $\Lambda', \Lambda = 0, 1$ . The successful implementation of the 3D evolution of  $NN$  interactions paves the path to consider a 3D evolution of three-nucleon forces.

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## References

- [1] M. R. Hadizadeh, K. A. Wendt, and Ch. Elster, in preparation.
- [2] I. Fachruddin, Ch. Elster, W. Glöckle, Phys. Rev. C **62**, 044002 (2000).