

## Separable Forces for $(d, p)$ Reactions in Momentum Space

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**Abstract.** Treating  $(d, p)$  reactions in a Faddeev-AGS framework requires the interactions in the sub-systems as input. We derived separable representations for the neutron and proton-nucleus interactions from phenomenological global optical potentials. In order to take into account excitations of the nucleus, excitations need to be included explicitly, leading to a coupled-channel separable representation of the optical potential.

The  $(d, p)$  scattering problem can be viewed as a three-body problem and thus be described exactly by the Faddeev equations, which are more readily solved in momentum space, specifically when transfer or break up are considered. However, when considering  $(d, p)$  reactions involving heavier nuclei, currently employed screening techniques for solving Faddeev equations with charged particles break down. At present, methods are developed to solve the Faddeev equations in the Coulomb basis, however, those rely on the short range forces being separable. For the neutron-proton subsystem separable potentials are readily available in literature. This is not true for nucleon-nucleus subsystems, for which interactions are often described by Woods-Saxon type optical potentials.

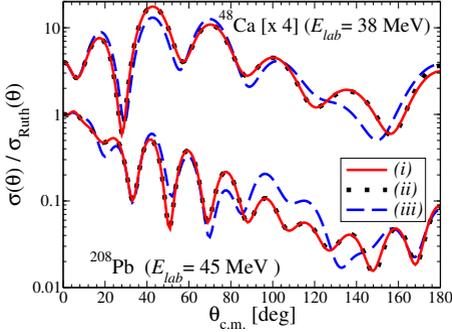
The method of deriving a separable representation of any arbitrary real potential proposed by Ernst, Shakin, and Thaler (EST) [1] is well suited. EST separable potentials have the property that at specific chosen energies the wavefunctions corresponding to the original potential and its separable representation are identical. In order to apply the EST method to optical potentials, it must be generalized to non-Hermitian potentials. Based on this generalized EST scheme neutron-nucleus optical potentials for <sup>48</sup>Ca, <sup>132</sup>Sn, and <sup>208</sup>Pb [2] were derived. Here a rank-5 separable interaction was sufficient to provide a good description of the neutron-nucleus scattering observables from 0 to 50 MeV.

For proton-nucleus optical potentials one has to consider the point Coulomb interaction, which is seen at large distances, and a short range Coulomb potential describing the charged nuclear sphere. In order to extend the EST scheme to include the point Coulomb interaction, one has to recast it in a Coulomb instead of a plane wave basis. This extended EST scheme was used to construct a separable representation of the proton-nucleus optical potential for <sup>48</sup>Ca and <sup>208</sup>Pb [3]. The cross-sections

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corresponding to the CH89 proton-nucleus phenomenological potential [4] and its rank-5 separable representation are shown in Fig. 1. The good agreement between the two results illustrates the success of the EST scheme in reproducing on-shell properties of the proton-nucleus optical potentials.



**Figure 1.** (color online) Unpolarized differential cross section for elastic scattering of protons from  $^{48}\text{Ca}$  (upper) at 38 MeV laboratory kinetic energy and  $^{208}\text{Pb}$  (lower) at 45 MeV divided by the Rutherford cross section as a function of the c.m. angle. The cross section for  $^{48}\text{Ca}$  is multiplied by 4. The solid lines (i) depict the cross section calculated in momentum space based on the rank-5 separable representation of the CH89 [4] optical potential, while the dotted lines (ii) represent the corresponding coordinate space calculations. The dash-dotted lines (iii) show calculations in which the short-ranged Coulomb potential is omitted.

In order to develop a potential for a deformed nucleus, non-spherical contributions (excitations) need to be added. A separable rank-3 representation of the  $n+^{12}\text{C}$  Olsson [5] coupled-channels optical potential including the  $2^+$  and  $4^+$  states of  $^{12}\text{C}$  was constructed. The EST support points in the 0 to 50 MeV energy regime were determined to be 5, 16.5, and 45 MeV. In Fig. 2 elastic and inelastic scattering cross sections computed using the Olsson potential (dashes) and its separable representation (solid line) are shown. We observe that the quality of the coupled-channels separable representation matches that of the single-channel case [2, 3].

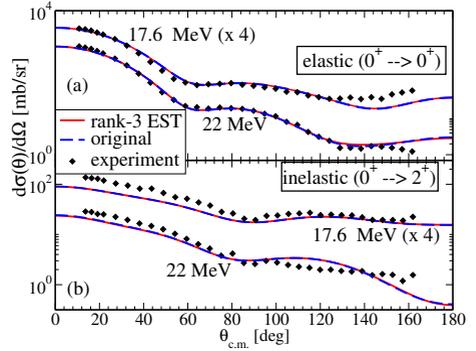
Summarizing, we constructed separable representations of neutron- and proton-nucleus phenomenological optical potentials, including non-spherical contributions, which can serve as input to Faddeev type ( $d, p$ ) or ( $d, n$ ) reaction calculations.

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**Figure 2.** (color online) Unpolarized differential cross section for scattering of neutrons off  $^{12}\text{C}$ . Panel (a) shows the elastic scattering cross section while the inelastic  $0^+ \rightarrow 2^+$  cross section is shown in panel (b). The (blue) dashed line shows results obtained using the Olsson optical potential [5]. The (red) solid line indicates results obtained using a separable representation of the Olsson optical potential with EST points at 5, 16.5, and 45 MeV. Experimental data from Ref. [5] is depicted by black diamonds. The cross sections at 17.6 MeV are scaled up by a factor of four.