EPJ Web of Conferences **113**, 08014 (2016)
DOI: 10.1051/epjconf/201611308014

A New Treatment Below the Three-Body Break up Threshold in the NNπ System

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Abstract. The two-body threshold behavior at NN′ and πD are investigated by using the multi-channel Lippmann-Schwinger equations with an energy dependent two-body quasi potential, which are analytically continued from the three-body Faddeev equations at the three-body break up threshold. Our calculated NN′ and πD scattering lengths show better agreement with the experimental data for NN and πD systems than those from the original NNπ three-body Faddeev equations.

Lovelace's idea in the early 1960s that the two-body N+N→ N+N reaction should be described by the three-body (N$_2\pi$)+N$_1$ → N$_2$ + (πN$_1$) reaction [1], can be accomplished by the introduction of an energy dependent two-body quasi potential (E2Q) below the three-body break up threshold [2]. The three-body Faddeev equations for the NNπ system are analytically continued to the multi-channel two-body Lippmann-Schwinger (MLS) equations by the E2Q [2] where the P_{11} bound state of (Nπ)≡ N′ is normalized to a nucleon N in this approach. The denominator of the Born term in the three-body Faddeev formalism is given by $\omega_j(k_j) = \sqrt{k_j^2 + m_j^2}$ $(j = 1,\ 2,\ 3)$ with the pion mass $m_3 = m_\pi \equiv m$ and the nucleon mass $m_1 = m_2 \equiv M$, and the three-body total energy is $\sqrt{S}$,

$$D_{\rm Fadd} = \sqrt{S} - \omega_1(k_1) - \omega_2(k_2) - \omega_3(k_3) = \sqrt{S} + m - \omega_1(\bar{k}_1) - \omega_2(\bar{k}_2) - \omega_3(\bar{k}_3) \equiv D_{\rm E2Q}, \tag{1}$$

where $D_{\rm E2Q}$ is defined as the denominator of the E2Q. The non relativistic approximation for Eq.(1) gives,

$$D_{\rm E2Q} \approx (E + m) - \bar{k}_1^2/2m_1 - \bar{k}_2^2/2m_2 - (\bar{\mathbf{k}}_1 - \bar{\mathbf{k}}_2)^2/2m_3 \equiv E_{\rm cm} - \bar{k}_{1,2}^2/2\mu_{1,2} - \bar{z}_{1,2} \tag{2}$$

$$D_{\rm Fadd} \approx E - k_1^2/2m_1 - k_2^2/2m_2 - (\mathbf{k}_1 - \mathbf{k}_2)^2/2m_3 \equiv E - k_{1,2}^2/2\mu_{1,2} - z_{1,2} \tag{3}$$

where $\bar{k}_{1,2}$ indicates $\bar{k}_1$ or $\bar{k}_2$, and $E_{\rm cm} = E + m$ is the c.m. energy in the E2Q with $E = \sqrt{S} - 2M - m$, while $\mu_{1,2}^{-1} = M^{-1} + (M + m)^{-1}$ and $\mu_3^{-1} = m^{-1} + (2M)^{-1}$ are the reduced masses, respectively. Because the inter-nucleon $\bar{k}_{1,2}$ is obtained from $k_{1,2}$ by absorbing energy m, therefore we can define

$$\bar{k}_{1,2}^2/2\mu_{1,2} \equiv k_{1,2}^2/2\mu_{1,2} + m. \tag{4}$$

Substituting Eq.(4) to Eq.(2), and compairing with Eq.(3), we obtain $\bar{z}_{1,2} = z_{1,2}$ which are the two-body sub-energies $\bar{z}_1$ or $\bar{z}_2$ in the virtual three-body system. Summing up k_i^2 with respect to $i = 1, 2, 3$,

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then we obtain $\overline{k}_3^2/2\mu_3 = k_3^2/2\mu_3 - m^2/2M$ where $\overline{k}_3$ is a virtual momentum for $\overline{k}_3^2 < 0$. Let us call the new MLS equations with the E2Q the *E2Q* equation [2].

In solving the E2Q and the Faddeev equations, the first difference occurs "at the NN′ threshold" where $E_{\rm cm} = 0$ and the integral variable $0 \leq \overline{k}_{1,2} < \infty$ in the E2Q equations, therefore $\{D_{\rm E2Q}\}^{-1}$ has a singular cut, while in the original Faddeev equations, $E = -m$ and $0 \leq k_i < \infty$, so that $\{D_{\rm Fadd}\}^{-1}$ is a regular function. The second appears in Eq.(4), $0 \leq \overline{k}_{1,2}^2 \leq 2m\mu_{1,2}$ gives $-2m\mu_{1,2} \leq k_{1,2}^2 \leq 0$, therefore the integral range of $\overline{k}_{1,2}$ is larger than $k_{1,2}$. In addition, a phenomenon at the three-body break up threshold $E = 0$ gives rise to the Efimov effect [3], with an infinite two-body scattering length, where $E = 0$ brings about a singular cut which is the same as our $E_{\rm cm} = 0$ case just mentioned above [2]. In a fourth difference, a phenomenon below the three-body threshold emerges as a long range NN′ interaction from an energy average of the E2Q [2]. We find that the calculated "NN′" and πD scattering lengths for the E2Q equations seem to be in better agreement with the experimental "NN" and πD data than the original three-body Faddeev calculations (Table 1).

Table 1. The "NN′" and πD scattering lengths are calculated using the original three-body Faddeev equation (Org-Fadd) and the E2Q calculation (E2Q). The potential-A is given by Thomas [4] and the potential-B is proposed by Fuda [5]. The E2Q results show good agreement with the experimental data both in the "NN" (^{3}S$_1$) and the πD cases [6, 7].

Method	Scattering length [fm]	System/State
Org-Fadd (potential-A[4])	0.280	NN′ ^{3}S$_1$
Org-Fadd (potential-B[5])	2.85	NN′ ^{3}S$_1$
E2Q (potential-B[5])	4.66	NN′ ^{3}S$_1$
EXP [6]	5.424±0.004	NN ^{3}S$_1$
Org-Fadd (potential-A[4])	0.033	πD
Org-Fadd (potential-B[5])	-0.019+0.019i	πD
E2Q (potential-B[5])	-0.023+0.019i	πD
EXP [7]	-0.038+0.009i	πD

Finally, it is stressed that the Hamiltonian *below* the three-body break up threshold is different from that *above* the threshold where the calculation should be carried out not with the "original" Faddeev equations but with the E2Q or modified Faddeev equations. This difference is emphasized in the NNπ system, because the two-body binding energy (*i.e.* the pion mass in the P_{11} state) is much larger than that in the nuclear three-body system, where such a discrepancy may be neglected.

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