EPJ Web of Conferences **113**, 08021 (2016)
DOI: 10.1051/epjconf/201611308021

Higgs particles interacting via a scalar Dark Matter field

Yajnavalkya Bhattacharya[1,2,a] and Jurij Darewych[2,b]

[1] *New Jersey Institute of Technology, Newark, NJ, USA*
[2] *York University, Toronto, Canada*

Abstract. We study a system of two Higgs particles, interacting via a scalar Dark Matter mediating field. The variational method in the Hamiltonian formalism of QFT is used to derive relativistic wave equations for the two-Higgs system, using a truncated Fock-space trial state. Approximate solutions of the two-body equations are used to examine the existence of Higgs bound states.

Dark matter particles (DM) of mass m are described by a spinless, massive scalar feld ϕ, – interacting with the self-coupled Standard Model Higgs field χ, with mass μ. The Lagrangian density of this model is ($\hbar = c = 1$)

$$\mathcal{L} = \frac{1}{2}\partial^\nu\phi\,\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - \kappa\,\phi^4 + \frac{1}{2}\partial^\nu\chi\,\partial_\nu\chi - \frac{1}{2}\mu^2\chi^2 - \lambda\,\upsilon\,\chi^3 - \frac{1}{4}\lambda\,\chi^4$$
$$-g_1\,\chi\,\phi^2 - \eta_1\chi^2\phi^2 - \eta_2\chi\phi^3 - \eta_3\chi^3\phi - g_2\,\chi^2\,\phi \tag{1}$$

where κ, λ, g_1, g_2, υ and η_j ($j = 1, 2, 3$) are coupling constants; λ, κ, η_j being dimensionless, and υ, g_i, (i=1,2), having dimensions of mass. In canonical quantization the classical fields ϕ, χ are promoted to operators.

In the Hamiltonian formalism of QFT, the equations to be solved are $\hat{P}^\beta|\Psi\rangle = Q^\beta|\Psi\rangle$, $\hat{P}^\beta = (\hat{H}, \hat{\mathbf{P}})$ and $Q^\beta = (E, \mathbf{Q})$ are the energy-momentum operator and corresponding eigenvalues. The $\beta = 0$ (energy) component of the equation is generally impossible to solve. Approximate solutions can be obtained using the variational principle $\langle\delta\Psi_{trial}|\hat{H} - E|\Psi_{trial}\rangle_{t=0} = 0$ where $\hat{H}$ is normal ordered, and $|\Psi_{trial}\rangle$ is a suitable trial state. Trial states are taken to be superpositions of channel Fock states. The simplest trial states that yield non-trivial results are

$$|\psi_{trial}\rangle = \int d\mathbf{p_1}d\mathbf{p_2}F_1(\mathbf{p_1}, \mathbf{p_2})\,h^\dagger(\mathbf{p_1})h^\dagger(\mathbf{p_2})\,|0\rangle + \int d\mathbf{p_1}\,d\mathbf{p_2}\,d\mathbf{p_3}\,F_2(\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3})h^\dagger(\mathbf{p_1})h^\dagger(\mathbf{p_2})d^\dagger(\mathbf{p_3})|0\rangle \tag{2}$$

where h denotes Higgs, d Dark Matter operators that satisfy the usual commutation rules. F_i, ($i = 1, 2$) are variational channel wave functions.

For the case where g_1=η_j=0, the equations of motion, in the rest frame $\overrightarrow{Q}$=0, that follow from the variational principle are:

$$F_1(\mathbf{q_1}, -\mathbf{q_1})\,[2\omega(\mathbf{q_1}, \mu) - E] = -g_2\int d\mathbf{p}\frac{F_2(-\mathbf{q_1}, \mathbf{q_1} + \mathbf{p}, \mathbf{p})}{(2\pi)^{3/2}\sqrt{2\omega(\mathbf{q_1}, \mu)}\sqrt{2\omega(\mathbf{p}, m)}\sqrt{2\omega(\mathbf{q_1} + \mathbf{p}, \mu)}}, \tag{3}$$

[a] e-mail: yajnaval@gmail.com
[b] e-mail: darewych@yorku.ca

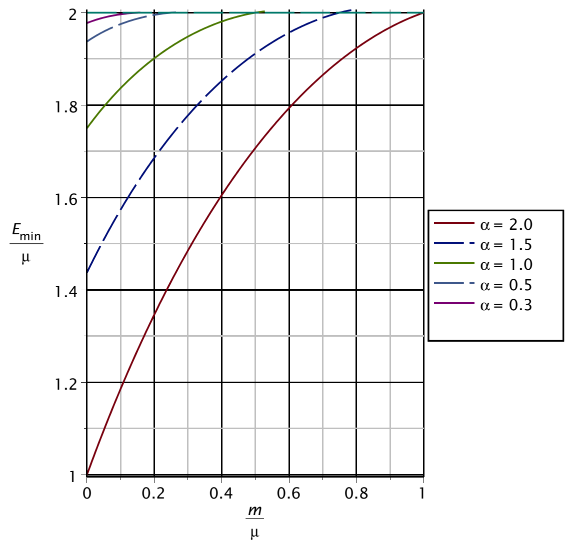

Figure 1. E_{min}/μ as a function of m/μ for various values of α. For a given value of α, the two-Higgs ground state binding energy decreases with increasing m/μ from the Coulombic value, $\frac{1}{4}\mu\alpha^2$ at $m = 0$, to zero at a critical value of m. The critical values of m/μ, beyond which no two-Higgs bound states are possible correspond to points where the curves cross the line $E_{min} = 2\mu$. These critical points occur where $m/\mu = \alpha/(2Z)$, where $Z \lesssim 1$. Accurate numerical solutions of Equation (6) yield $Z = 0.839908$.

$$F_2(\mathbf{q_1},\mathbf{q_2},\mathbf{q_1}+\mathbf{q_2})\left[\omega(\mathbf{q_1},\mu)+\omega(\mathbf{q_2},\mu)+\omega(\mathbf{q_1}+\mathbf{q_2},m)-E\right]$$
$$= -g_2\frac{F_1(-\mathbf{q_2},\mathbf{q_2})}{(2\pi)^{3/2}\sqrt{2\omega(\mathbf{q_1},\mu)}\sqrt{2\omega(\mathbf{q_1}+\mathbf{q_2},m)}\sqrt{2\omega(-\mathbf{q_2},\mu)}}. \quad (4)$$

Exact, analytic solutions of the coupled, relativistic equations are not possible, so approximate variational-perturbative solutions will be considered. In the lowest order approximation, we set $\omega(\mathbf{q_1},\mu)+\omega(\mathbf{q_2},\mu) \simeq E$ in (4) whereupon equation (4) simplifies to

$$F_2(\mathbf{q_1},\mathbf{q_2},\mathbf{q_1}+\mathbf{q_2})\left[\omega(\mathbf{q_1}+\mathbf{q_2},m)\right] = -g_2\frac{F_1(-\mathbf{q_2},\mathbf{q_2})}{(2\pi)^{3/2})\sqrt{2\omega(\mathbf{q_1},\mu)}\sqrt{2\omega(\mathbf{q_1}+\mathbf{q_2},m)}\sqrt{2\omega(-\mathbf{q_2},\mu)}}. \quad (5)$$

Thus in the rest frame, equation (3) becomes a single relativistic equation

$$f(\mathbf{q})\left[2\omega(\mathbf{q},\mu)-E\right] = \alpha\mu^2\int d^3\mathbf{p}\,\frac{f(\mathbf{p})}{\omega(\mathbf{q},\mu)\,\omega^2(\mathbf{p}-\mathbf{q},m)\,\omega(\mathbf{p},\mu)}. \quad (6)$$

where $f(\mathbf{q}) = F_1(-\mathbf{q},\mathbf{q})$, and $\alpha = g_2^2/(16\pi^2\mu^2)$ is a dimensionless coupling constant.

Approximate variational solutions of (6) in the non-relativistic limit, for the ground state, are obtained using the trial state $f(\mathbf{p}) = \dfrac{\omega(\mathbf{p},\mu)}{(p^2+b^2)^2}$, where b is an adjustable parameter obtained by minimizing E. The results are given in **Figure 1**.