Development of an Analytical Method for Predicting Flow in a Supersonic Air Ejector

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Abstract. The article deals with development of an analytical method for predicting flow in an ejector with twelve supersonic nozzles, which are located at the periphery of the mixing chamber of the ejector. Supersonic primary air stream makes the investigation more complex. The secondary air (atmospheric) is sucked in direction of the ejector axis. The shape of the mixing chamber is convergent – divergent and a throat is formed behind the primary nozzles. Each of the primary nozzles can be treated independently so there can be various number of nozzles under operation in the ejector. According to previous investigations, constant pressure mixing is assumed to occur inside a part of the mixing chamber. The method under investigation is considered for isentropic flow in the first approximation and after that the stagnation pressure corrections at the inlets are considered. Furthermore, the decrease in stagnation pressure in the mixing chamber is considered to take losses in the mixing chamber and diffuser into account. The numerical data of the stagnation pressure has been obtained from Ansys Fluent software. In addition, a comparison with previous experimental results is introduced.

1 Introduction

Nowadays, there are still higher and higher requirements for efficient operation of engineering devices and optimal operating regimes are required. The efficient working regimes are connected with losses and with properly working devices. Ejectors with their low efficiency, generally less than 30 per cent, require more attention and should be investigated in more details. The low efficiency is the most significant disadvantage of these devices. On the other hand, there are many advantages of ejectors. Among advantages especially belong absences of moving parts, ointment or seals. Other advantages are simply designs and relatively low manufacturing costs. They are usually used in places where sufficient amount of the working fluid is available or in industrial processes where some low-grade thermal energy is available, which is especially highly desirable in these days when new environment-friendly technologies are under development.

By the time that Keenan et.al. [1] performed the first comprehensive study of mixing, two cases of mixing were distinguished: the constant pressure mixing and the constant area mixing. From this follows usage of ejectors. Another distinction of ejectors is in terms of primary and secondary streams arrangement. The primary stream can either be supplied in the ejector axis and secondary stream is sucked at the periphery of the mixing chamber or primary stream enters the periphery of the mixing chamber and the secondary stream is sucked in the ejector axis. The latter design, which is usually used for wind tunnel propulsions, can be based on two arrangements. The first arrangement uses a slot primary nozzle. The second one uses several primary nozzles and is investigated in this paper, see figure 1. Many theoretical and experimental studies have been carried out to obtain the performance of ejectors so far. For more information, see [2-4].

The design of the air ejector under investigation was also investigated in more detail in works [5, 6 and 7]. The ejector is used for propulsion of a small experimental variable wind tunnel, see [8]. As can be seen in figure 1, the declination angle of the primary supersonic nozzles towards the ejector axis is 8.2°. The declination should prevent the occurrence of reversal flow in the ejector axis, as has been stated in work [9]. The method used in this paper for predicting ejector performance is based on assumptions which were first proposed by Christianovic and later simplified by Kiselev [10].

Four cases with two, four, eight and twelve activated primary nozzle have been selected for the analysis.
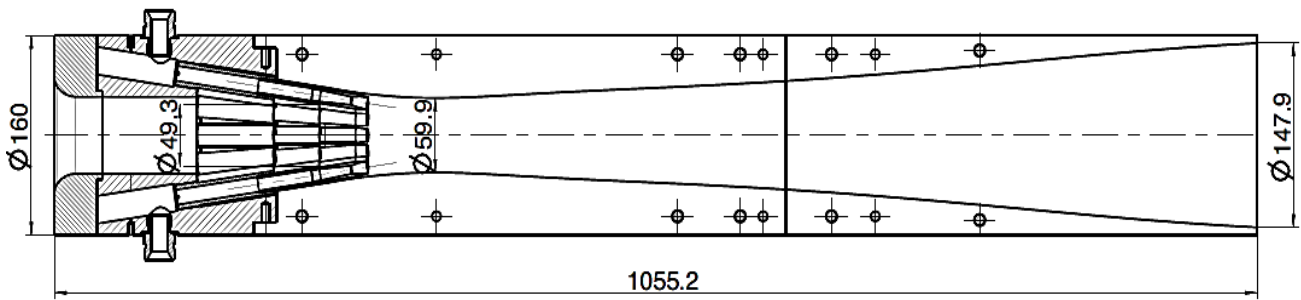
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Figure 1. Longitudinal cross-section of the supersonic ejector (All dimensions given in millimeters) [8].

2 Theoretical analysis

The aim of this paper is to obtain output values of secondary (or sucking) mass flow rate and entrainment ratio, which is defined as ratio of mass flow rates, i.e.

$$\Gamma = \frac{\dot{m}_2}{\dot{m}_1},\qquad(1)$$

where subscript 1 denotes primary and 2 secondary flow.

The entrainment ratio is considered as the most important indication of ejector performance in many supersonic ejector applications. Three operational regimes of supersonic ejectors are distinguished, see figure 2.
Due to availability of setting any number of the primary nozzles the situation inside the ejector is rather complex. Figure 3 shows the scheme of the investigated ejector. The section x-x, which is the ejector throat, is so called effective area where the flow is assumed to be choked.
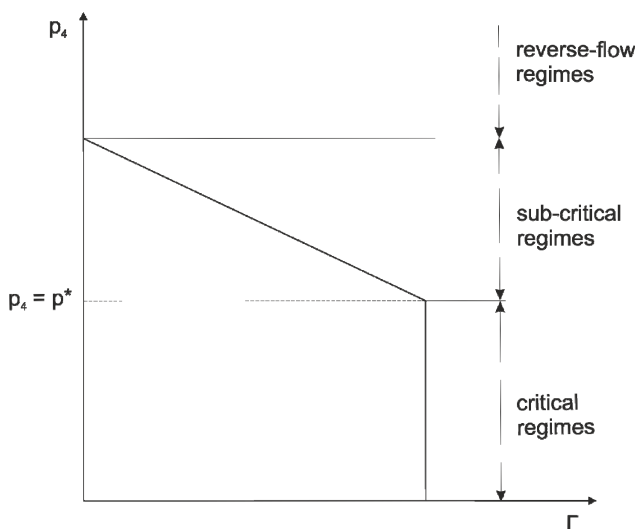

Figure 2. Ejector operational regimes.

The method used for the analysis is similar to the other methods that have been already investigated in previous works. These assumptions have been made:

1. The flow is considered to be steady and one-dimensional.
2. The primary and the secondary stream of air can be treated as ideal with constant κ and c_p.
3. The stagnation conditions are constant at the primary and secondary air inlets, as well as at the exit of the mixing chamber and diffuser.
4. The mixing chamber and the diffuser walls of the ejector are adiabatic.
5. Isentropic relations are used for calculations.
6. Shock waves are not considered.
7. Stagnation pressure coefficients are used for correction of the stagnation pressures at the boundaries (inlets) at section *12-12*.
8. Stagnation pressure correction has not been considered for the diffuser.
9. The mixed flow is considered as critical or sub-critical ($\lambda_3 \le 1$).

The used method is based on the following relations, which are the law of conservation of mass

$$c_1 \rho_1 A_1 + c_2 \rho_2 A_2 = c_3 \rho_3 A_3,\qquad(2)$$

where A_1 and A_2 are cross section areas at section *12-12* and A_3 is the exit cross section area of the mixing chamber, c denotes the stream velocities and ρ the stream densities. The other relations are the balance of momentum.

$$\dot{m}_1 c_1 + p_1 A_1 + \dot{m}_2 c_2 + p_2 A_2 = (\dot{m}_1 + \dot{m}_2) c_3 + p_3 (A_1 + A_2), \quad (3)$$

where p denotes stream static pressure and the balance of energy

$$\dot{m}_1 \left(c_{p1} T_1 + \frac{c_1^2}{2} \right) + \dot{m}_2 \left(c_{p2} T_2 + \frac{c_2^2}{2} \right) = (\dot{m}_1 + \dot{m}_2) \left(c_{p3} T_3 + \frac{c_3^2}{2} \right), \quad (4)$$

which has been derived for adiabatic flow in the mixing chamber and where T is static temperature. The relations mentioned above have been derived with the assumptions that isobaric specific heat capacities are constant, i.e.

$$c_{p1} = c_{p2} = c_{p3}, \quad (5)$$

And also ratio of specific heats is constant, i.e.

$$\kappa_1 = \kappa_2 = \kappa_3. \quad (6)$$

Stagnation pressure of the mixed flow after finished mixing is calculated by relation

$$p_{03} = p_{01} \frac{\sqrt{(1+\Gamma)(1+\Gamma\Theta_{21})}}{1 + \frac{p_{01}}{p_{02}} \Gamma \sqrt{\Theta_{21}} \frac{q(\lambda_1)}{q(\lambda_2)}} \frac{q(\lambda_1)}{q(\lambda_3)}, \quad (7)$$

where Θ_{21} is the ratio of the stagnation temperatures of both streams defined as

$$\Theta_{21} = \frac{T_{02}}{T_{01}} \quad (8)$$

and $q(\lambda)$ is the aerodynamic function of mass flow rate given by

$$q(\lambda) = \frac{\rho c}{(\rho c)^*} = \left(1 - \frac{\kappa-1}{\kappa+1} \lambda^2 \right)^{\frac{1}{\kappa-1}} \left(\frac{\kappa+1}{2} \right)^{\frac{1}{\kappa-1}} \lambda, \quad (9)$$

where λ is dimensionless speed obtained from ratio

$$\lambda = \frac{c}{c^*}, \quad (10)$$

where c is the velocity at given cross section and c^* is the critical velocity at the same cross section. Stagnation temperature after mixing is derived directly from the balance of energy

$$T_{03} = T_{01} \frac{1 + \Gamma\Theta_{21}}{1 + \Gamma}. \quad (11)$$

Dimensionless speed of the mixed flow is obtained from relation

$$z(\lambda_3) = \frac{z(\lambda_1) + \Gamma \sqrt{\Theta_{21}} z(\lambda_2)}{\sqrt{(1+\Gamma)(1+\Gamma\Theta_{21})}}, \quad (12)$$

where $z(\lambda)$ is aerodynamic function given by

$$z(\lambda) = \lambda + \frac{1}{\lambda}. \quad (13)$$

The following relations have been used also during calculations. Since there is no exact distinction between the mixing chamber and the diffuser due to the fluent shape of the inner channel, following relation has been used to determine the mixing chamber cross section A_3 and consequently the position of the end of the mixing chamber

$$A_{1et} + A_2 = A_3, \quad (14)$$

where A_{1et} is the total exit cross section area of all activated nozzles, A_2 is the cross section area remaining for the secondary flow at section 12-12 and A_3 is the exit cross section area of the mixing chamber or the inlet cross section area of the diffuser, respectively.

The most significant design parameter of the ejector is the ratio of cross section areas of both streams defined as

$$\mu(A_{1et}) = \frac{A_{1et}}{A_2}. \quad (15)$$

This approach allows calculation of ejector efficiency, which is defined as

$$\eta_E = \frac{\dot{m}_2}{\dot{m}_1} \frac{\left(\frac{p_4}{p_{02}} \right)^{\frac{\kappa-1}{\kappa}} - 1}{1 - \left(\frac{p_4}{p_{01}} \right)^{\frac{\kappa-1}{\kappa}}} \frac{T_{02}}{T_{01}}, \quad (16)$$

where p_4 is back-pressure, i.e. the pressure at the diffuser exit.

The relations mentioned above assume one-dimensional isentropic flow of ideal compressible fluid. These relations allow to calculate all important variables of the mixed flow for a given regime of the ejector.

The stagnation pressure correction has been treated by coefficients obtained from numerical data

$$p_0' = p_{12} \left\{ 1 - \eta_{nozzle} \left[1 - \left(\frac{p_{12}}{p_0} \right)^{\frac{\kappa-1}{\kappa}} \right] \right\}^{\frac{\kappa}{1-\kappa}}, \quad (17)$$
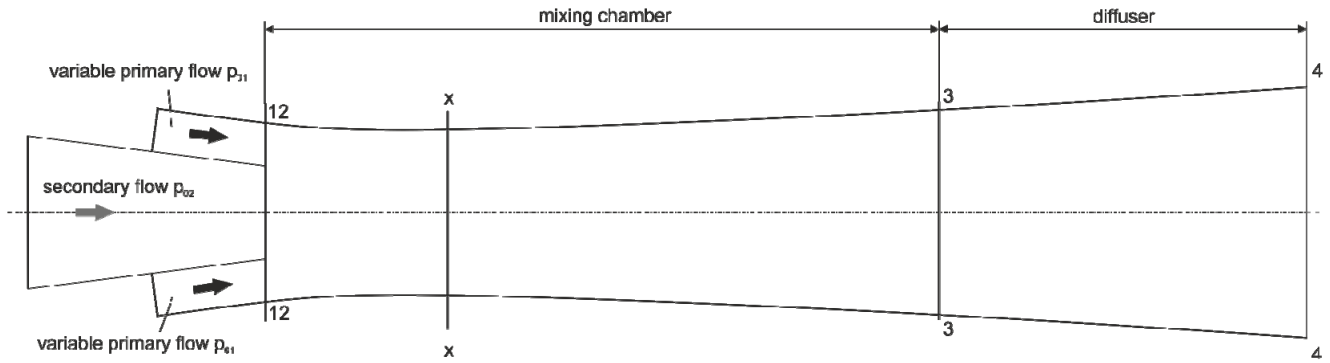
Figure 3. Analytical scheme of the ejector performance.

where η_{nozzle} is efficiency coefficient of the primary nozzle or the secondary inlet part, respectively, that has nozzle-like behavior as well. The efficiency has been calculated from resulta of numerical calculations using following equation

$$\eta_{nozzle} = \frac{1 - \left(\frac{p}{p_0'}\right)^{\frac{\kappa-1}{\kappa}}}{1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} = \varphi^2, \qquad (18)$$

where ϕ is a velocity coefficient, p is the static pressure obtained from numerical results as an area-weighted-average value, p_0 is the stagnation pressure in the beginning of expansion, i.e. in the nozzle inlet, obtained from numerical results as a mass-weighted-average value and similarly p_0' is stagnation pressure after expansion, i.e. at the nozzle exit, again as mass-weighted-average, see figure 4.

The stagnation conditions of the primary air have been set $p_{01} = 397$ kPa (absolute) and $T_{01} = 295$ K. The Stagnation pressure of the secondary air has been $p_{02} = 97$ kPa (absolute) and the stagnation temperature has been considered the same as for the primary air, i.e. $T_{02} = T_{01}$. Static pressure p_{12}, which varied from value 96.9 kPa down to 30.0 kPa, has been considered as the only one independent variable.
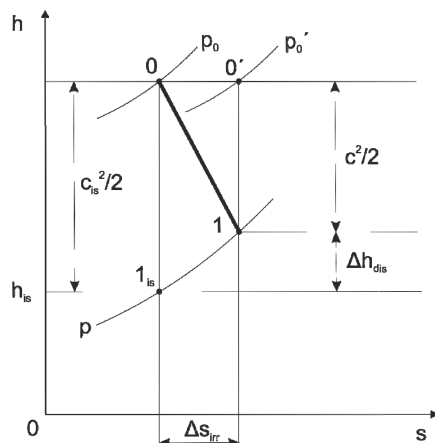

Figure 4. Enthalpy-entropy diagram.

3 Numerical analysis

A One fourth model of the ejector has been used to obtain numerical data of the stagnation and static pressures within the mixing chamber. The k-ω SST turbulence model was used to take for turbulence effects into account. This model is suitable for calculation of flows in supersonic ejectors as it was stated in previous works. For more information, see Ansys Fluent documentation [11].
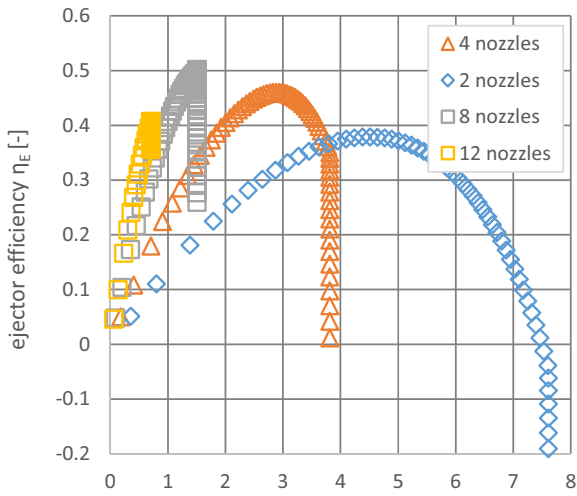
4 Results

In figures 5-8 courses of efficiency and relative back-pressure for different number of activated primary nozzles can be seen. In figure 5, the results for ejector efficiency obtained from the analytical method are shown. It can be seen that the whole curve, i.e. the position of maximal efficiency and position of choking, moves to lower ejection ratios with increasing number of activated nozzles. These observations are in agreement with the previous experimental data as shown in the next text. The courses of relative back-pressure obtained from the analytical method can be seen in figure 6. The highest value of entrainment ratio obtained from analytical method was approximately of 7.7 for two activated nozzles, while the lowest ejection ratios but the highest back pressure ratios are obtained for twelve nozzles.

In order to verify proposed analytical model, experimental data obtained by Kracik (2014) on a supersonic air ejector with a circular cross section were used, see figure 1.

Figure 7 and 8 show comparison of curves of efficiency and relative back-pressure obtained by presented analytical model and by experiments [8].

As mentioned earlier the maximum of efficiency moves to the left with increasing number of activated nozzles in both the presented model and the experiments. There are obvious relatively large disagreement between both the efficiency and the relative back-pressure curves. The analytical method underestimates reached ejection rations, but overestimates maximums of efficiency. One of the possible ways to reduce these discrepancies is to include friction and others losses that take place within the ejector mixing chamber and ejector diffuser. Further, it is necessary to analyse experimental and numerical

results more thoroughly to deeply understand flow processes inside the mixing chamber and include and describe all phenomena to predict the chocking conditions more precisely.

Figure 5. Comparison of ejector efficiency for various number of activated primary nozzles.
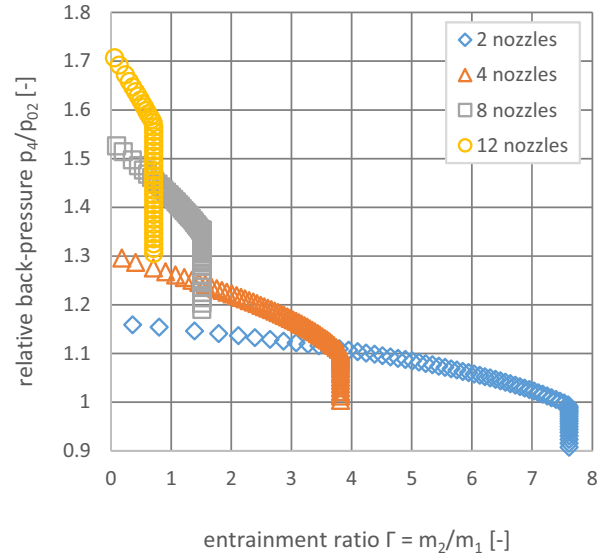

Figure 6. Comparison of relative back-pressure for various number of activated primary nozzles.
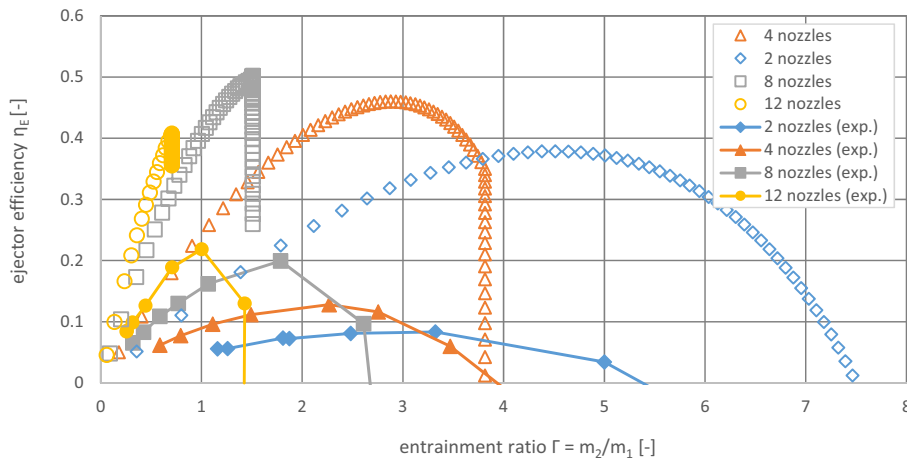

Figure 7. Ejector efficiency comparison of presented model and experimental data from [8] for various number of activated primary nozzles.
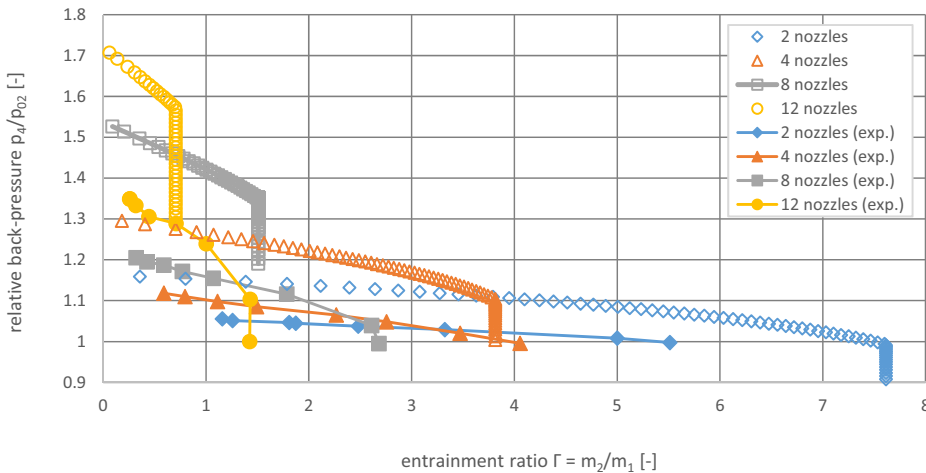

Figure 8. Relative back-pressure comparison of presented model and experimental data from [8] for various number of activated primary nozzles.

5 Summary and conclusions

The first attempt to predict ejector performance with twelve supersonic primary nozzles has been presented. Isentropic flow has been considered as the first approximation. Moreover, the correction coefficients have been taken into account for the primary and secondary inlet. It has been shown that the maximum of efficiency moves to lower ejection ratios with increasing number of activated nozzles, i.e. the primary mass flow rate is increased both in the presented model and experiments performed earlier. There are obviously significant discrepancies in both the efficiency and relative back-pressure curves between predicted and measured values.

It is obvious that friction and others losses that take place within the mixing chamber and diffuser must be considered in the method to improve agreement with experiments. Furthermore, a thorough analyse of experimental and numerical results is necessary to understand flow processes inside the mixing chamber and include and describe all phenomena and conditions to predict the chocking more precisely.

Acknowledgment

References

1. J.H. Keenan, E.P. Neumann and F. Lustwerk, *An Investigation of Ejector Design by Analysis and Experiment.* Journal of Applied Mechanics, Trans. ASME **17** (1950), pp 299 – 309.
2. J.H. Keenan, E.P. Neumann, *A simple air Ejector,* Journal of Applied Mechanics, Trans. ASME **64**, pp A75 – A81 (1942)
3. S. Aphornratana, I.W. Eames, *A small capacity steam-ejector refrigerator: experimental investigation of a system using ejector with movable primary nozzle,* International Journal Refrigeration, Trans. ASME **20**, pp 352 – 358 (1997)
4. B.J. Huang, J.M. Chang, C.P. Wang, V.A. Petrenko, *A 1-D analysis of ejector performance,* International Journal Refrigeration, Trans. ASME **22**, pp 354 – 364 (1999)
5. J. Kracik, V. Dvorak, *Experimental and numerical investigation of an air to air supersonic ejector for propulsion of a small supersonic wind tunnel,* EPJ Web of Conferences **92** (2015) 02038
6. J. Kracik, V. Dvorak, J. Kolar, *Development of air to air ejector for supersonic wind tunnel,* Web of Conferences **67** (2014) 02059
7. J. Kracik, V. Dvorak, J. Kolar, *Development of air to air ejector for supersonic wind tunnel,* Web of Conferences **67** (2014) 02059
8. J. Kracik, *Design of Supersonic Wind Tunnel* Diploma thesis, In Czech, (2014)
9. V. Dvorak, *Shape Optimization of Supersonic Ejector for Supersonic Wind Tunnel.* Applied and Computational Mechanics, Plzen, (2010)
10. G.N. Abramovich, *Applied Gas Dynamics,* Volume **2** Set edition (1973)
11. Ansys Fluent 15.0 *User´s Guide*, Ansys Inc.
12. J. Kolar, V. Dvorak, *Interaction of Shock Waves in Supersonic Ejector,* Proceedings of the 27th Meeting of Thermodynamics and Fluid Mechanics Departments, Plzen (2008)
13. J. Simak, *Computation of the Flow and Interaction of Shock Waves in a 2D Supersonic Ejector,* Colloquium Fluid Dynamics 2009, Prague (2009)
14. Y. Bartosiewicz, Z. Aidoun, P. Desevaux, Y. Mercadier, *Numerical and Experimental Investigations on Supersonic Ejectors,* International Journal of Heat and Fluid Flow **26** (2005)
15. J. Kolar, *Design and Optimization of Ejector for Supersonic Wind Tunnel*, Diploma thesis, Liberec, In Czech (2007)
16. B.J. Huang, J.M. Chang, C.P. Wang, V.A. Petrenko, *A 1-D analysis of ejector performance,* International Journal Refrigeration, Trans. ASME **22**, pp 354 – 364 (1999)
17. B.M. Kiselev, *Rasčet odnorazmernych gazovych tečenij,* Prikladnaja matem. i mech, No **1** (1949)
18. W. Cheng, M. Liu, D. Chong, J. Yan, A.B. Little, Y. Bartosiewicz, *A 1D model to predict ejector performance at critical and sub-critical operational regimes,* International Journal of Refrigeration **36**, pp 1750 – 1761. (2013)