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Numerical simulation heat transfer by natural convection in liquid metal with a sinusoidal temperature

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Abstract. This study focuses on the numerical simulation of heat transfer by natural convection in a rectangular enclosure, filled with a liquid metal (low Prandtl number) partially heated from below with a sinusoidal temperature. The value of the study lies in its involvement in the crystal growth for the manufacture of semiconductors and electronics cooling. Indeed, the occurrence of convection during crystal growth can lead to in homogeneities that lead to striations and defects that affect the quality of the crystals obtained by the Bridgman techniques or Chochrawlski. Temperature of the oscillations, due to the instabilities of the convective flow in the liquid metal, also induces non-uniform cooling in the solidification front. Convection is then studied in order to reduce it. A modelling of the problem in two dimensions was conducted using Comsol computer code that is based on the finite element method, by varying the configuration of the control parameters, namely, the Rayleigh number, the nature of fluid (Prandtl number) and amplitude of temperature on heat transfer rate (Nusselt number) on convective structures that appear.

1 Introduction

Natural convection in liquid metals is the subject of studies by metallurgists and engineers of nuclear reactors. Indeed, liquid metals have an excellent heat transfer medium and thus make it possible to cool the nuclear reactor, the electronic components and especially during the crystal growth process [1].

Many studies have been conducted in recent years and have allowed to better understand the behavior of liquid metals when stressed thermally. Stewarl and Weinberg [2] were among the first to study the natural convection in a two-dimensional cavity fluid containing low Prandtl number with insulated horizontal walls. Their results show that the behavior of the flow in the liquid metal is different from conventional fluids. This study has given researchers a starting point for research in liquid metals. [3], [4] studied this phenomenon in several conditions with comparison of the experimental results by digital. Their results confirm that the liquid metals are an excellent heat transfer medium.

Interest in the study of this problem is also its involvement in the growth of crystals for the manufacture of semiconductors. Indeed, the occurrence of convection during crystal growth can lead to in homogeneities that lead to striations and defects that affect the quality of the crystals obtained by the Bridgman techniques and Czochralski. Temperature of the oscillations due to flow instabilities in the liquid metal also induce non-uniform cooling in the forehead solidification. In this case, convection is studied in order to avoid or reduce it.

In addition to this practical aspect, the study of convective flows in liquid metals is of interest of fundamental research perspective. Indeed this type of opaque fluids at low melting temperatures (eg 302.8 K for gallium) are fluids at low Prandtl number (Pr) and behavior when subjected to temperature gradients are quite different from those observed in fluids high number of Pr (such as water and air).

Our work is to study the numerical heat transfer by natural convection in a rectangular enclosure filled with a liquid metal (low Prandlt), partially heated from below with a sinusoidal temperature. The results obtained by the Comsol software using the numerical method of finite elements.

2 Problem and mathematical model

2.1 Problem definition

A schematic of the two dimensional system, and the coordinates and boundary conditions are shown in Figure 1. The square cavity is partially heated by sinusoidal temperature and the top wall is maintained cold temperature. The vertical wall and the no-heated portions of bottom wall are adiabatic. The height of the cavity is H. The finite-sized heater is length L_1 , and here temperature strength is T_C = 1+A*sin(w*t). The

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dimension length the portions no-heated and of the heater are same at a value L_1=H/3. All boundaries satisfy the no-slip velocity conditions.

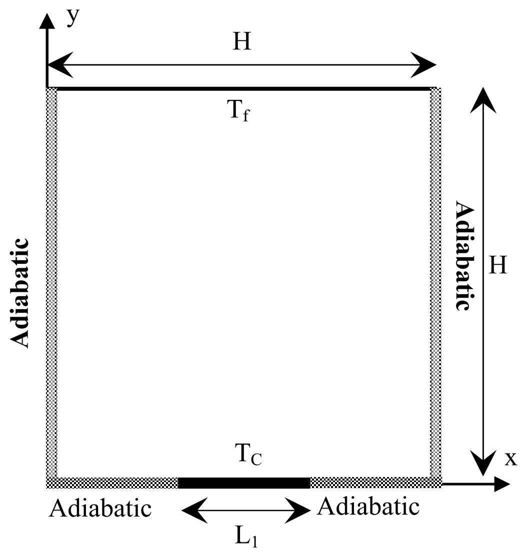

Figure 1. Schematic of the physical system.

2.2 Mathematical model

Following assumptions are made: the flow is laminar of an incompressible Newtonian fluid and two-dimensional, there is no viscous dissipation, the gravity acts in the vertical direction, fluid properties are constant and fluid density variations are neglected except in the buoyancy term (the Boussinesq approximation), and radiation heat exchange is negligible.

The unsteady state, two dimensional continuity, momentum and energy equations governing the flow and heat transfer are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+ g.\beta.(T-T_C) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The continuity, momentum and energy equations are written. The non-dimensional governing equations are obtained as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{5}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{6}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra.Pr.\theta \tag{7}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{8}$$

Here, the x and y coordinates are normalized as: $X=\frac{x}{H}$; $Y=\frac{y}{H}$. U and V are the dimensionless velocity components in the X and Y directions, non-dimensionalized as: $U=\frac{u.H}{\alpha}$; $V=\frac{v.H}{\alpha}$. P is the dimensionless effective pressure, scaled as: $P=\frac{p.H^2}{\rho.\alpha^2}$. θ is the dimensionless temperature, non- dimensionalized as: $\theta=\frac{T-T_f}{T_C-T_f}$. a and W are the dimensionless respectively amplitude and period, non- dimensionalized as: $a=\frac{A}{T_C-T_f}$ and $W=\frac{w.\alpha}{H^2}$.

The steady-state solutions are obtained from unsteady state equations, Eqs (5)-(8). The local and average Nusselt numbers are calculated respectively as:

$$\left.\begin{aligned} Nu_L &= -\frac{\partial \theta}{\partial Y} \\ Nu_m &= \int_0^1 Nu_L dX \end{aligned}\right\} \tag{9}$$

The stream function is calculated from its definition as:

$$U=-\frac{\partial \psi}{\partial Y},\ V=\frac{\partial \psi}{\partial X} \tag{10}$$

2.3 Boundary conditions

The boundary conditions of the system are shown in Figure. 1. Velocities are zero on all solid surfaces, normal pressure gradient are zero on all solid surfaces at the outside boundaries. On the adiabatic boundaries, temperature gradient is zero. On the bottom centre portion horizontal boundary, a sinusoidal temperature is applied. Thus,

On all solid boundaries: $U=V=0$

$$Y=0,\quad \frac{H}{3}<X<\frac{2H}{3}\ :\ \theta=1+a.\sin(W.\tau)$$

$$Y=0,\quad 0<X<\frac{H}{3}\ and\ \frac{2H}{3}<X<1:\ \frac{\partial \theta}{\partial Y}=0$$

$$Y=H,\quad 0<X<H\ :\ \theta=0$$

$$X=0\ and\ X=H, 0<Y<H:\ \frac{\partial \theta}{\partial X}=0$$

3 Numerical techniques

Numerical results are obtained by solving the system of unsteady differential equations (1)-(4), with appropriate boundary and initial conditions, using Comsol Multiphysics based for Galerkin finite elements method. The computational domain consists of bi-quadratic elements which correspond to 590 elements and a Lagrange quadratic interpolation has been chosen.

4 Results and discussion

The square cavity with height and length H=1 here studied at Rayleigh number from 10^3 to 10^6. The partially centre portion is hot by the sinusoidal temperature with the amplitude and period are varied between respectively a=0 to 0.8 and W=0 to 1. The working fluid is chosen as liquid metal (Gallium) Pr = 0.025.

4.1 Stream functions and isotherms

The figure 2 shows the streamlines and isotherms for different Rayleigh numbers with an amplitude and a period is respectively 0.8 and 0.5 for the liquid metal. The results show that the flow of fluid within the square cavity becomes large by increasing the number of Rayleigh. Can be observed at low flow intensities (Ra = 10^3) the formation of two counter-rotating cells and subsequently turns into a single main drive unit of the fluid mass in a direction of counterclockwise rotation with the appearance of recirculation zones have low flow intensities located in the four corners of the cavity. For Ra = 10^3, the isotherms are parallel to the horizontal wall of the heating characterizes the conductive heat transfer. Nevertheless, from Ra = 10^4 the onset of transfer with convection , beyond this value of Ra the transfer is purely convective. Indeed, the intensity of the natural convection entrained the elevation large quantities of hot fluid to be subsequently transported to the opposite side (cold).

For comparison, the structure of the flow and isotherms for the air (Figure 3) are different from the liquid metal. As noted, the presence of two counter-rotating cells to Ra = 10^3 and 10^5 and a single vortex to Ra = 10^4 and 10^6, it is also noted that the intensity of the flow increases by increasing Ra. Heat transfer by convection is triggered from as Ra = 10^4. Thus, there is a symmetry of the isotherms for Ra = 10^5 and the transfer becomes fully convective for Ra = 10^6. Note that the effect found on the stream lines, affects the structure of the isotherms, This changing of the flow structure explained by the density of air is lighter than the liquid metal.

4.2 Heat transfer

Local Nusselt number profile on the cold wall for amplitude 0.8 and period 1 is shown in Figure 4. In this figure, the profile is traced for X to 0 from 1. As discussed earlier, we can see clearly that the local Nusselt number profile along X is line in Ra=10^3, axisymmetric for Ra=10^4, 10^5 and symmetric (Gaussian form) for Ra=10^6. We clearly notice that the value of the local Nusselt number believed along the horizontal axis of the cavity then it starts decreased from the center of the cavity. This is due to the increase in the central portion along the horizontal axis X of the temperature gradient between the fluids and parietal layers cooled wall. Indeed, the fluid in contact with the hot wall is heated and transported to the top of the cavity under the effect of the decrease in density of the fluid. We note the maximum values for the local Nusselt number, this can be explained by the high values of the Rayleigh number that represents the predominant effect of convective fluid movement.

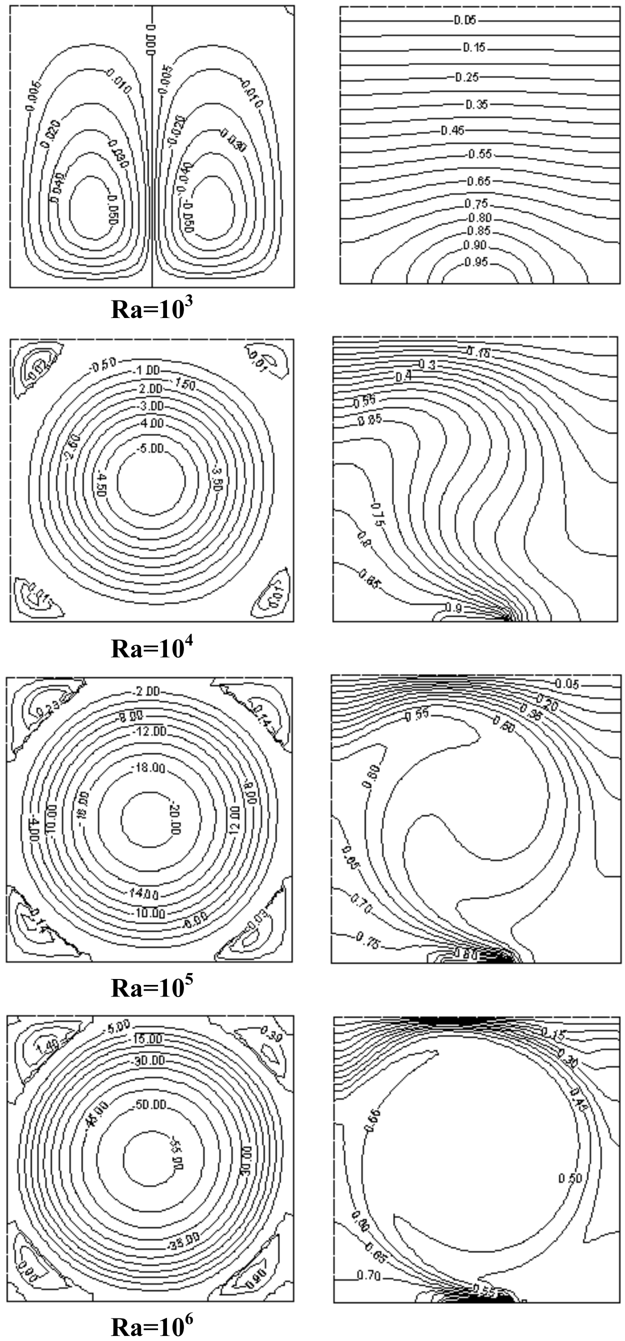

Figure 2. Stream functions (in the left) and isotherms (in the right) for a=0.8, W=0.5 with different values of Rayleigh for metal liquid.

Average Nusselt number on the cold wall is calculated by Eq. (5) and presented in the figure 5 and 6 respectively with differentes values the amplitude for period is fixed at 0.5 and differentes values the period for amplitude is fixed at 0.2. In the case of liquid metal the average Nusselt number increases with the number of Rayleigh, therefore the heat transfer becomes more intense with the increase of Rayleigh number (Figure 4) and it decreases with increase in the value of the amplitude of the heating temperature. However, for the case of air, the average transfer rate remains almost constant at low amplitude values, and then decreases with the decrease of the amplitude value of the heating temperature. The transfer rate increases with the increase of Rayleigh number and the period of the heating temperature (Figure 6) for the case of fluid.

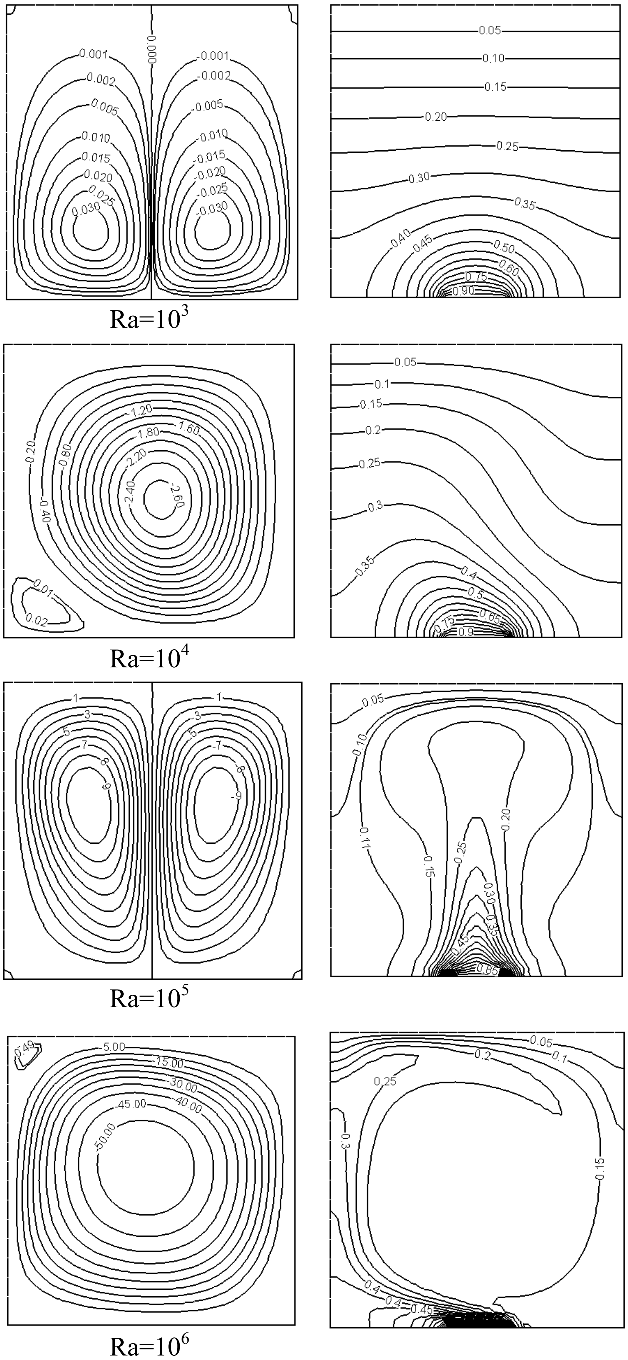

Figure 3. Stream functions (in the left) and isotherms (in the right) for a=0.8, W=0.5 with different values of Rayleigh for air.

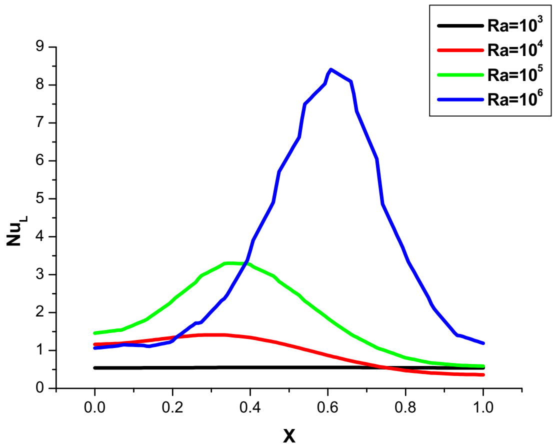

Figure 4. Local Nusselt number at (Y=1, X) along cold wall of the enclosed for a=0,8 and W=1 for various Rayleigh number

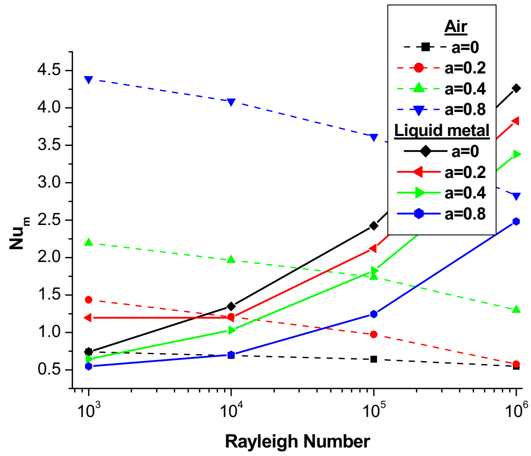

Figure 5. Average Nusselt number the liquid metal and air as a function of Rayleigh number with the amplitude value as a emperature for period fixed at 0.5

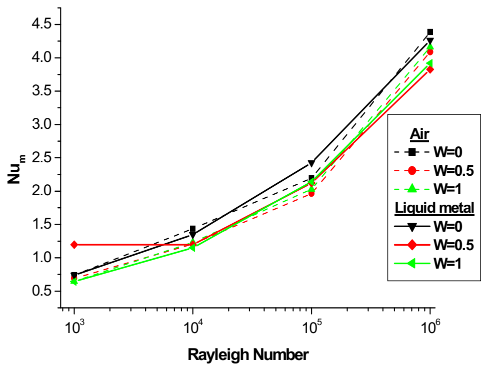

Figure 6. Average Nusselt number of the liquid metal and air as a function of Rayleigh number with the period value as a temperature for amplitude fixed at 0.2

4.3 Maximum streamt function

Figures 7 and 8 respectively show the variations of the maximum current in absolute value function as a function of time for different values of the amplitude and the period. We read on these curves the instant the heating temperature was imposed. Naturally, at this moment we can see that the steady state is well established. A transition period characterizing the fluid response is observed at the start of heating by the sinusoidal temperature.

After the transition period, the solution becomes periodically characterized by sinusoidal oscillation having the same period as the heating temperature. The oscillations of the maximum current in absolute value function are around a mean value substantially identical to that of the steady state (a = 0), which increases an amplitude of a. Nevertheless, increasing the amplitude, it is noted that the maximum value of the amplitude above the steady state (a = 0) is less than that below.

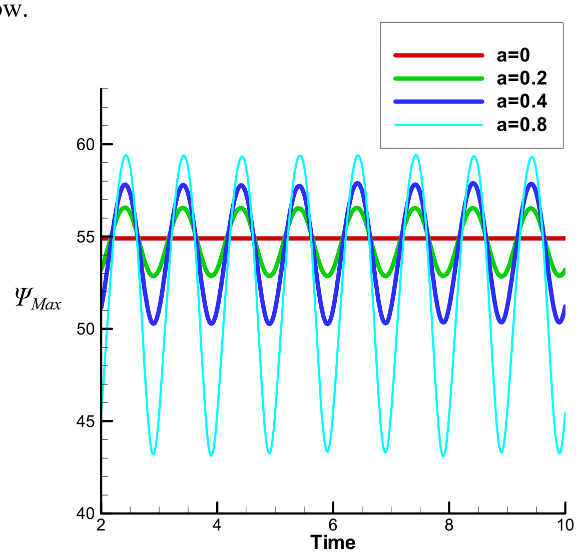

Figure 7. Variation of maximal stream function for time for different value of amplitude with period at 1

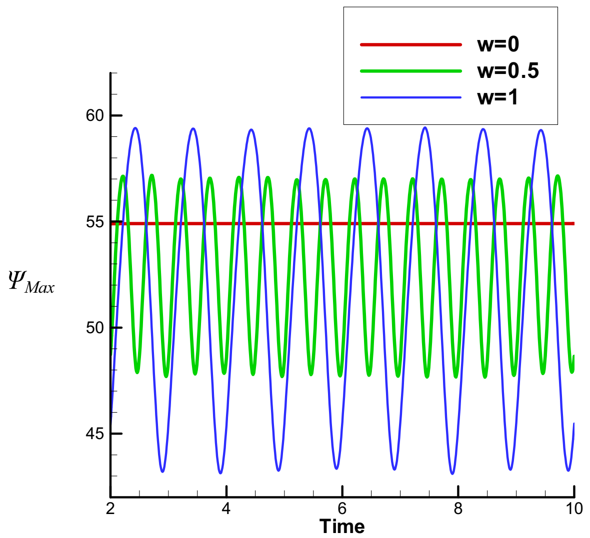

Figure 8. Variation of maximal stream function for time for different value of period with amplitude at 0.8

5 Conclusion

A numerical study has been carried out in square enclosures is partially heated by sinusoidal temperature in bottom and the top wall is maintained cold temperature. The vertical wall and the no-heated portions of bottom wall are insulated. The influence of Rayleigh number and of the amplitude, the period of the heated part on the heat transfer characteristics is examined. In view of the results and discussion presented, the following main conclusions have been drawn.

- Whatever the nature of the fluid, increasing the Rayleigh number causes a remarkable change in the structure of the flow field and the temperature field, and stabilizes the convective flow.

- The curves of the stream function and the temperature profile of a changing sinusoidally around the mean values. Comparing amplitudes of the oscillation parameters, it may be noted that these amplitudes are of different sizes with respect to that of the heating temperature, this leads to the conclusion that the oscillation amplitudes depend on the positions of the particles in the cavity, while the oscillation frequencies are the same everywhere in the fluid.

- The effect of the variation of the amplitudes of oscillation parameters has an effect on the transfer rate.

- The rate of transfer of liquid metals is important with respect to large values of the number of Prandlt such as air, it can be concluded that the liquid metal have a significant calorific value is for cooling as for heating.

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