Λ polarization in an exact fluid dynamical model for heavy-ion collisions

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Abstract

Λ polarization is calculated in an exact analytical, rotating model based on parameters from a high resolution (3+1)D Particle-in-Cell Relativistic hydrodynamics calculation. The polarization is attributed to effects from thermal vorticity and for the first time the effects of the radial and axial acceleration are also studied separately.

At finite impact parameters, the initial state (IS) has non-vanishing angular momentum. Early studies neglected effects arising from the non-vanishing angular momentum, but interest increased recently. With the development of hydrodynamic modeling, rotation and its consequences were studied as well.

Thermal vorticity arises from the inverse temperature field in heavy ion collisions, and due to the non-vanishing angular momentum and shear in the initial stages. We look at polarization in effects arising from thermal vorticity in an exact rotating model [1], corresponding to an appropriate time-period of the collision based on a (3+1)D fluid dynamical model

Conventionally, $[x, z]$-plane is the reaction plane, with y being the axis of rotation. Then the initial angular momentum points into the negative y-direction, with an absolute value of approximately $1.45 \cdot 10^4 \hbar$.

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Following [2], the polarization arises from the thermal velocity field, $\beta^\mu(x) = u^\mu(x)/T(x)$, due to equipartition between vorticity and spin as

$$\Pi_\mu(p) = \hbar\epsilon_{\mu\sigma\rho\tau}\frac{p^\tau}{8m}\frac{\int d\Sigma_\lambda p^\lambda n_F(x,p)(1-n_F(x,p))\partial^\rho\beta^\sigma}{\int d\Sigma_\lambda p^\lambda n_F(x,p)} \ . \tag{1}$$

where $\epsilon_{\mu\rho\sigma\tau}$ is the completely antisymmetric Levi-Civita symbol, n_F the Fermi-Jüttner distribution for spin-1/2 particles, $(1-n_F)$ is the Pauli blocking factor and p is the four-momentum of the Λ.

The Λ polarization is determined by measuring the angular distribution of the decay protons in the Λ's rest frame. By Lorentz boosting the polarization vector, $\mathbf{\Pi}(\boldsymbol{p})$, in the participant frame, one can obtain the polarization vector $\mathbf{\Pi}_0(\boldsymbol{p})$ in Λ's rest frame:

$$\mathbf{\Pi}_0(\boldsymbol{p}) = \mathbf{\Pi}(p) - \frac{\boldsymbol{p}}{p^0(p^0+m)}\mathbf{\Pi}(p)\cdot\boldsymbol{p} \ , \tag{2}$$

where $(p^0, \boldsymbol{p})$ is the Λ four-momentum and m its mass.

As the Λ is transversely polarized, $\Pi^\mu p_\mu = 0$, one can confine himself to the spatial part of Π^μ. The simplified spatial part of polarization vector is:

$$\begin{aligned}
\mathbf{\Pi}(p) &= \mathbf{\Pi_1}(p) + \mathbf{\Pi_2}(p) \\
&= \frac{\hbar\epsilon}{8m}\frac{\int dV n_F(x,p)(\nabla\times\boldsymbol{\beta})}{\int dV n_F(x,p)} \\
&\quad + \frac{\hbar\boldsymbol{p}}{8m}\times\frac{\int dV n_F(x,p)(\partial_t\boldsymbol{\beta}+\nabla\beta^0)}{\int dV n_F(x,p)} \ .
\end{aligned} \tag{3}$$

Using the vorticity evaluated in [3], for the non-relativistic Exact model, we deduced the analytical solution for the Λ polarization:

$$\begin{aligned}
\mathbf{\Pi}(p) = \frac{\hbar}{8mT}\Bigg[&\frac{p_y c_9}{c_3\sqrt{c_4}}\frac{M_{-1,\frac{1}{2}}}{M_{-\frac{1}{2},0}}\boldsymbol{e}_x \quad + \quad \Big(2\epsilon\omega - \frac{|p_x|c_9}{c_3\sqrt{c_4}}\times\frac{M_{-1,\frac{1}{2}}}{M_{-\frac{1}{2},0}}\Big)\boldsymbol{e}_y \\
&+ \quad \Big(\frac{|p_x|c_7 c_1}{2c_2} - \frac{p_y c_8}{c_3\sqrt{c_4}}\frac{M_{-1,\frac{1}{2}}}{M_{-\frac{1}{2},0}}\Big)\boldsymbol{e}_z \Bigg] \ , \tag{4}
\end{aligned}$$

where c_i $(i = 1-9)$ are parameters in terms of scaling variables and $M_{\mu,\nu}(z)$, is the Whittaker function, the confluent hypergeometric function [1].

As seen in eq. (4), the polarization consists of two terms, $\mathbf{\Pi_1}(p)$ and $\mathbf{\Pi_2}(p)$, which arise from local vorticity $(\nabla\times\boldsymbol{\beta})$ and expansion $(\partial_t\boldsymbol{\beta})$. One can see from Fig. 1, that the second term in the polarization is of comparable

magnitude to the term arising from local vorticity. In a previous calculation [4], the p dependence of n_F, was considered negligible in the integral, and the time derivative and gradient terms were also assumed to be smaller. The present calculation shows that in general these terms are not negligible.
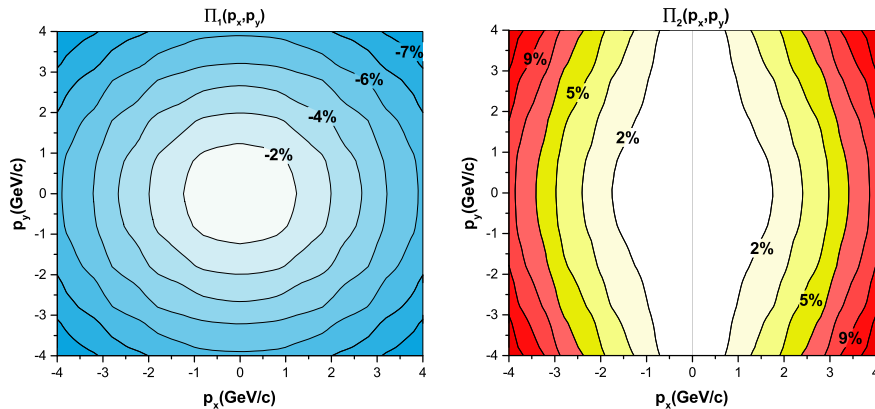

Figure 1: (color online) The two terms of Λ polarization $\boldsymbol{\Pi}_1(\boldsymbol{p})$ (left panel), $\boldsymbol{\Pi}_2(\boldsymbol{p})$ (right panel) in the participant frame at time $t = 0.5$ fm/c after the equilibration of the rotation, in the Exact model. Based on Ref. [1].

The Λ polarization is measured via the angular distribution of the decay protons in the Λ's rest frame, as shown in Eq. (2). The resulting distribution is shown in Fig. 2. The structure of $\Pi_{0y}(\boldsymbol{p})$ is similar to the one obtained in [4], but it reaches 12% at high p_x values, greater than 9% in Ref. [4], due to the contribution of the "second", $\partial_t \boldsymbol{\beta}$ term. These new studies indicate that the dynamics of the expansion may also lead to non-negligible contribution to the observable polarization.

Recently the vorticity and polarization were also studied in another fluid dynamical model [5], where the initial shear flow is neglected. This results in negligible thermal vorticity (Figs. 3 and 13 of Ref. [5]), and consequently a negligible polarization from the vorticity, i.e. from the "first term" discussed here. On the other hand, there is qualitative agreement between Fig. 12 of Ref. [5] and this work in the sense that only the y-directed (i.e. $[x, z]$ or $[x, \eta]$) component of the vorticity leads to an overall average net polarization. This arises in both models from the initial angular momentum and points into the $-y$-direction. In Ref. [5] this arises as a consequence of viscous evolution of the initial, vorticity-less flow, while in our Exact model it is present in the initial state.
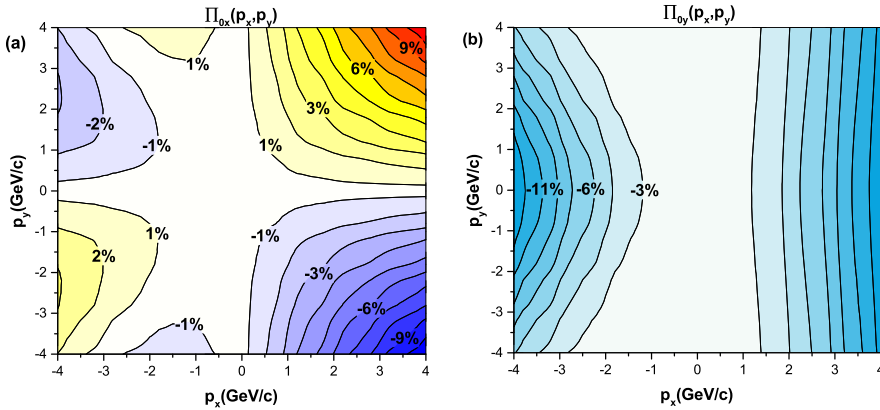
Figure 2: (color online) The radial, x, and axial, y, components of Λ-polarization, $\mathbf{\Pi}_0(\boldsymbol{p})$, in the Λ's rest frame. Both plots are asymmetric due to the Lorentz boost to the Λ rest frame. From [1].

In this work we analyzed and compared the two terms, and the Exact model. Including both rotation, expansion, and vorticity arising from both effects. This study indicates that the assumptions regarding the initial state are influencing the predictions on the observed vorticity.

Acknowledgements

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