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Spatial properties of pairing and quarteting correlations in nuclei

V. V. BARAN[1,4] and D.S. DELION[1,2,3]

[1] "Horia Hulubei" National Institute of Physics and Nuclear Engineering,
Bucharest-Măgurele, România
[2] Academy of Romanian Scientists, Bucharest, Romania
[3] Bioterra University, Bucharest, Romania
[4] Department of Physics, University of Bucharest, Romania

Abstract

Interest in the study of the spatial properties of pairing correlations in nuclei has recently been revived. The main interesting physical quantity is the coherence length (CL). We investigate the CL as obtained from schematic pairing potentials. We then generalize to quarteting correlations by considering the CL build out of proton and neutron pairs. We point out the main differences between the pairing and quarteting CLs. Finally we comment on the important role of proton-neutron correlations as they strongly modify the behaviour of the CL when taken into account.

The coherence properties of the spatial distribution of the two-particle density are known to give information about nuclear correlations [1, 2]. The main physical quantity that describes the coherence property is the CL, defined as the root mean square relative distance averaged over the pairing density, in the case of superfluid nuclei. It has been found [3, 4] that this quantity is of the order of the nuclear radius inside the nucleus and smaller around the nuclear surface. In contrast, the correlations between proton and neutron pairs by the analogous quarteting CL.

1 Theoretical background

The pairing interaction generates the most important two-body correlations beyond the mean field in even-even nuclei. We describe such systems within the standard HF+BCS approach, where the averaged particle number is conserved, separately for protons and neutrons. We consider in our basis bound single particle (sp) states with negative energy, as well as relatively narrow sp resonances with positive energy up to $e_{max} = 10$ MeV with a decay width $\Gamma \leq 1$ MeV (the background contribution is not relevant [6]), given by a Woods Saxon central field with universal parameterization [8].

The information about the spatial properties of pairing correlations is contained in the anomalous density $\kappa(\mathbf{r_1}, \mathbf{s_1}; \mathbf{r_2}, \mathbf{s_2}) = \langle BCS|\mathcal{A}\{\hat{\psi}(\mathbf{r_1}, \mathbf{s_1})\hat{\psi}(\mathbf{r_2}, \mathbf{s_2})\}|BCS\rangle$. After passing to the spherical basis of states and recoupling from the j-j to the L-S scheme, we retain only the largest singlet component, $\kappa_1(\mathbf{r_1}, \mathbf{r_2})$. By changing to the relative $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ and center of mass coordinate $\mathbf{R} = (\mathbf{r_1} + \mathbf{r_2})/2$ and averaging over their relative angle θ one obtains the averaged pairing tensor $\bar{\kappa}_1^2(r, R)$. The CL is then defined to be the relative distance averaged over this pairing tensor:

$$\xi(R) = \sqrt{\frac{\int \mathrm{d}r\ r^4\ \bar{\kappa}_1^2(r, R)}{\int \mathrm{d}r\ r^2\ \bar{\kappa}_1^2(r, R)}} \equiv \sqrt{\int \mathrm{d}r\ r^2\ w(r, R)}\ . \qquad (1)$$

Intuitively it gives the size of the Cooper pair at a certain point inside the nucleus.

In order to investigate medium to heavy α-decaying nuclei having the valence protons and neutron in different major shells it is customary to construct a quartet by two separate proton and neutron pairs [10, 11]. By considering the overlaps of the proton-proton and respectively neutron-neutron wave functions (given by the corresponding pairing tensors) with the two-proton and respectively two-neutron part of the α-particle wave-function, we construct the quartet tensor $\kappa_q(\mathbf{R}_\pi, \mathbf{R}_\nu) = \langle \kappa_\pi(\mathbf{r_1}, \mathbf{r_2})|\phi_{00}^{(\beta_\alpha/2)}(r_\pi)\rangle \cdot \langle \kappa_\nu(\mathbf{r_3}, \mathbf{r_4})|\phi_{00}^{(\beta_\alpha/2)}(r_\nu)\rangle$, where $\mathbf{r}_{\pi,\nu} = \mathbf{r}_{1,3} - \mathbf{r}_{2,4}$, $\mathbf{R}_{\pi,\nu} = (\mathbf{r}_{1,3} + \mathbf{r}_{2,4})/2$, $\phi_{nl}^{(\beta)}$ is the standard radial spherical Harmonic Oscillator function and $\beta_\alpha \approx 0.5 fm^{-2}$. This quantity plays the role of the pairing tensor in the quarteting case. The Quartet CL ξ_q is now defined by proceeding similarly with the pairing case (see eq. 1). It is a function of the center of mass of the two pairs $\mathbf{R}_\alpha = (\mathbf{R}_\pi + \mathbf{R}_\nu)/2$.

By taking into account proton-neutron correlations, described by the part in the α-particle wave function dependent on the relative distance $\mathbf{r}_\alpha = \mathbf{R}_\pi - \mathbf{R}_\nu$, we may also define the so-called alpha CL. The corresponding correlation tensor is $\kappa_\alpha(r_\alpha, R_\alpha) = \kappa_q(r_\alpha, R_\alpha)\phi_{00}^{(\beta_\alpha)}(r_\alpha)$, and the CL is $\xi_\alpha(R_\alpha)$. The alpha correlation tensor is directly related to the amplitude to find a quartet in the alpha state, namely the formation amplitude [10]: $\mathcal{F}(R_\alpha) = \int_0^\infty \mathrm{d}r_\alpha \; r_\alpha^2 \; \kappa_\alpha(r_\alpha, R_\alpha)$.

2 Numerical application and results

We analyzed all even-even nuclei with $20 < Z < 100$ and known experimental pairing gaps, determined by the binding energies of neighboring nuclei [7]. We present here the results obtained by solving the BCS equations with the Gaussian interaction defined by $v(r_{12}) = -v_0 e^{-[r_{12}/r_0]^2}$ depending on the relative radius r_{12}. There is no a priori reason to consider that the parameters of the in-medium effective interaction are equal to those of the bare interaction. Thus we compared the case r_0=2 fm, corresponding to the singlet "bare" value in the free space, with the larger $r_0 = 4.5$ and even $r_0 = R_N = 1.2A^{1/3}$, corresponding to the nuclear radius. The strength v_0 is always chosen as for the Fermi level gap to reproduce the experimental gap. We note that the corresponding CLs are all similar in shape for all considered interactions, and in agreement with Ref. [3].

For the bare Gaussian, the ratio of the mean CL to the nuclear radius decreases from 1.4 for light nuclei down to around unity for heavy nuclei. The systematic study also revealed that the proton CL approximately scales as $2.4A^{1/5}$ fm, while for the neutrons the scaling goes as $2.0A^{1/4}$ fm. As a general trend, the CL is larger for neutrons, but shell effects are stronger for protons. We also investigated the temperature dependence of the CL by solving the finite temperature BCS equations. One would normally expect a strong variation induced by thermal pair breaking. By going up to the critical temperature, the CL saw almost no variation. It was only slightly affected in the internal region by the variable mixing between the odd and even parts of the pairing tensor [4].

Moving on to the quarteting case, we first checked that our treatment correctly evidences the surface nature of α condensation. In fact, $\mathcal{F}(R_\alpha)$ defined above has a pronounced maximum on the nuclear surface. Given the way we constructed the quartet by putting together proton and neutron pairs, we find that the formation amplitude is strongly dependent on the strength of the pairing interaction. We find that the quartet CL is larger in the internal region, similarly to the pairing CL. After taking into account proton-neutron correlations the shape is significantly altered: the α-CL oscillates about the geometrical dimension of the α-particle of 1.9 fm.

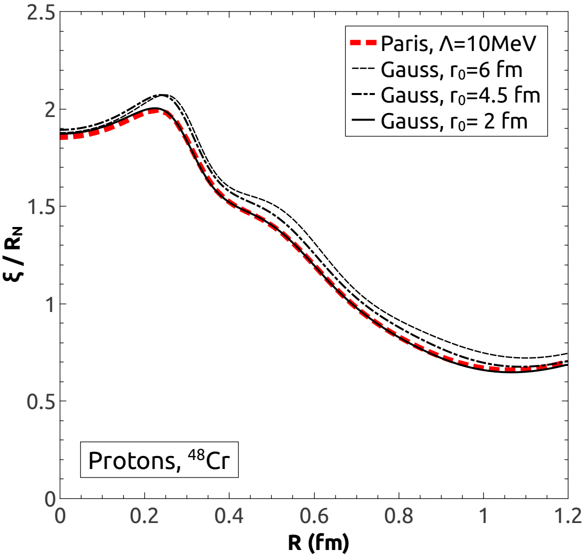

Figure 1: Proton CL in units of the nuclear radius in ^{48}Cr for Gaussian potentials with various width parameters and for the realistic Paris potential renormalized down to the cutoff energy $\Lambda = 10$ MeV (see [9]).

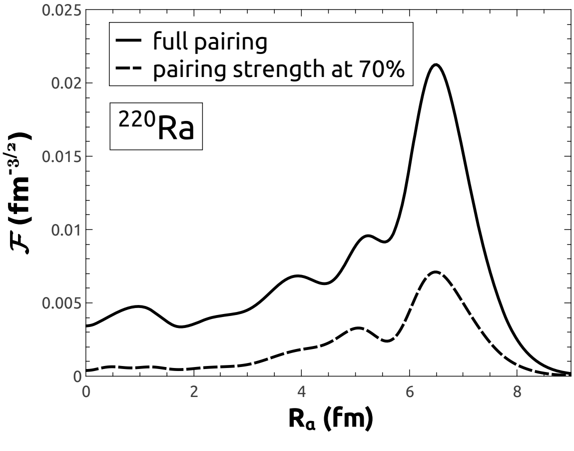

Figure 2: Formation amplitude $\mathcal{F}$ vs the quartet center of mass R_α for the nucleus ^{220}Ra.

3 Conclusions

The pairing CL has similar characteristics for all considered interactions. It is larger than the geometrical radius for light nuclei and approaches this value for heavy nuclei. Our analysis provides evidence of strong shell effects. The pairing CL also is insensitive to the gradual decrease in the intensity of the pairing correlation due to incresing temperature. The quarteting CL describes correlations between proton and neutron pairs and resembles in shape the pairing CL. We evidenced the important role played by proton-neutron correlations, as they change completely the behavior of the quarteting CL. Namely, the new alpha CL has oscillating values around the α-particle geometrical radius. Its mean value weakly depends on the nuclear mass.

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