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Theoretical studies of possible toroidal high-spin isomers in the light-mass region

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Abstract

We review our theoretical knowledge of possible toroidal high-spin isomers in the light mass region in $28 \leq A \leq 52$ obtained previously in cranked Skyrme-Hartree-Fock calculations. We report additional toroidal high-spin isomers in ^{56}Ni with $I{=}114\hbar$ and $140\hbar$, which follow the same (multi-particle)–(multi-hole) systematics as other toroidal high-spin isomers. We examine the production of these exotic nuclei by fusion of various projectiles on ^{20}Ne or ^{28}Si as an active target in time-projection-chamber (TPC) experiments.

1 Introduction

Wheeler suggested that under appropriate conditions the nuclear fluid may assume a toroidal shape [1]. If toroidal nuclei could be made, there would be a new family tree for the investigation of the nuclear species.

The rotating liquid-drop model is useful as a qualitative guide to point out essential energy balances leading to possible toroidal figures of equilibrium [2]. A quantitative assessment relies on microscopic descriptions that include both the bulk properties of the nucleus and the single-particle shell effects in self-consistent mean-field theories, such as the Skyrme-Hartree-Fock (SHF) approach [3]. The non-collective rotation with an angular momentum about the symmetry axis is permissible quantum mechanically for an axially symmetric toroid by making particle-hole excitations and aligning the angular momenta of the constituents along the symmetry axis [4]. As a consequence, only a discrete, quantized set of total angular momentum $I{=}I_z$ states is allowed.

In our recent works [5], we showed by using a cranked SHF approach that toroidal high-spin isomeric states have general occurrences in $28 \leq A \leq 52$ for even-even $N{=}Z$ and $N{\neq}Z$ nuclei. Toroidal high-spin isomers have also been found theoretically in similar HF calculations in this mass region [6].

We would like to review the systematics of these nuclei and suggest ways how these nuclei may be produced.

2 Light-mass toroidal high-spin isomers

We have located the toroidal high-spin isomers at their energy minima using the cranked SHF approach [5]. We find that the ratio of the torus major radius R to the torus minor radius d, R/d, increases with angular momentum and approximately linearly with the mass number while the minor radius d remains essentially unchanged (see Ref. [5]). It is useful to classify these nuclei according to the n-particle n-hole nature of the isomer, relative to the toroidal nucleus at $I = 0$. One finds that all np-nh families follow a regular well-behaved pattern as shown in Fig. 1, where we plot the total energy $E^{\rm tot}(I)$ of the isomer ${}^AZ^t(I)$ as a function of R/d for different toroidal isomers with various aligned angular momenta I.

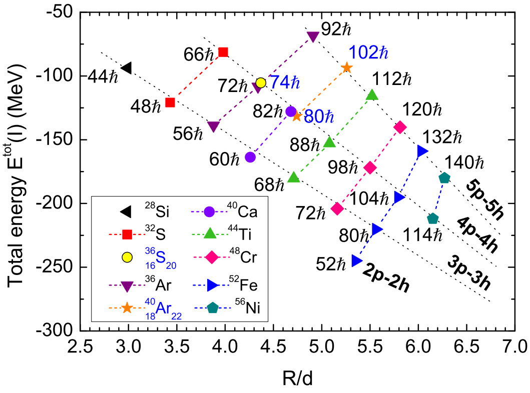

Figure 1: (Color online.) The total energies of the isomeric toroidal states of $^{28}_{14}$Si, $^{32}_{16}$S, $^{36}_{16}$S, $^{36}_{18}$Ar, $^{40}_{18}$Ar, $^{40}_{20}$Ca, $^{44}_{22}$Ti, $^{48}_{24}$Cr, $^{52}_{26}$Fe, and $^{56}_{28}$Ni and their associated angular momenta $I{=}I_z$ values along the symmetry axis, as a function of R/d. The np-nh configurations relative to the $I{=}0$ states are also indicated.

We collect the properties of all known 21 toroidal high-spin isomers up to ^{52}Fe obtained previously in [5] in Fig. 1. These systematics predict the possible presence of np-nh toroidal high-spin isomers for ^{56}Ni at $R/d \sim 6.0$. Indeed, we found energy minima for ^{56}Ni with I=114$\hbar$ and 140$\hbar$ at R/d=6.15 and 6.27, respectively, in subsequent cranked SHF calculations.

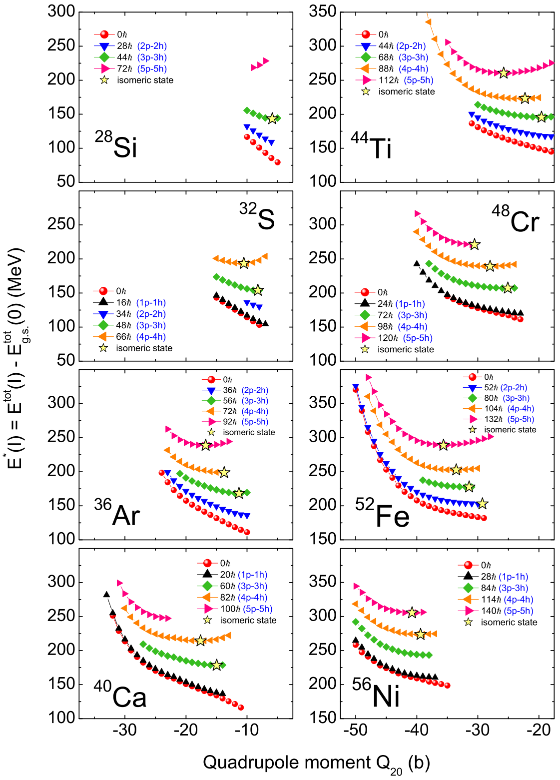

Figure 2: (Color online.) The excitation energies $E^*(I)$ of high-spin toroidal states of ^{28}Si, ^{32}S, ^{36}Ar, ^{40}Ca, ^{44}Ti, ^{48}Cr, ^{52}Fe, and ^{56}Ni as a function of the quadrupole moment Q_{20} for different angular momenta along the symmetry axis, I=I_z.

In Fig. 2 the excitation energies $E^*(I)$ of even-even N=Z toroidal states relative to the energy of the ground states are presented as a function of the quadrupole moment Q_{20} for different I values along the symmetry axis. They include those from our earlier work in [5] and the newly found ^{56}Ni toroidal isomers. The locations of energy minima $E^*(^AZ^t(I))$ representing toroidal high-spin isomeric states are indicated by star symbols.

3 Production of light-mass toroidal isomers

As the question of the mean half-life of these isomers remains unresolved, one can design experiments that could detect the existence of the exotic toroidal high-spin nuclei of Figs. 1 and 2 at appropriate energies, if the mean half-lives are longer than $\hbar$/MeV or 200 fm/c. One way is to search for these isomers as resonances or metastable nuclei by bombarding projectile nucleus ${}^{A_p}Z_p$ on an active-target nucleus ${}^{A_T}Z_T$ for the production of the toroidal high-spin isomer ${}^{A}Z^t(I)$ with angular momentum I=I_z,

$$ {}^{A_p}Z_p + {}^{A_T}Z_T \to {}^{A}Z^t(I). \tag{1}$$

The active target can be, for example, ^{20}Ne, ^{36,38,40}Ar, or ^{28}Si. We shall consider the cases of ^{20}Ne and ^{28}Si as active-targets.

In recent years, TPC chambers have been used to study the nuclear spectroscopy of metastable nuclei [7, 8]. The idea is to use a chamber of noble gas under a high voltage so that the gas itself or an embedded solid layer serves as the target, and the nuclear trajectories show up as tracks. The production of a composite nucleus with a long half-life would show up as a single track with the mass and charge arising from the fusion of the projectile and target nuclei. The production of binary products indicates a two→two reaction from which one can examine the elastic and inelastic channels and study the excitation function and angular distribution to search for various meta-stable states. Previously, many metastable states formed by colliding various projectile nuclei with an active He target have been found by such a technique [7].

The cross section for producing a toroidal isomer at the correct energy and angular momentum is [9] (p. 517)

$$\sigma_{\rm res}(E, {}^{A}Z^t(I)) = \frac{4\pi}{k^2}(2I+1)\frac{\Gamma^2/4}{[E-E_{\rm res}({}^{A}Z^t(I))]^2+\Gamma^2/4}B_{\rm in}B_{\rm out}, \tag{2}$$

where E is the c.m. energy, $E_{\rm res}({}^{A}Z^t(I))$ is the c.m. resonance energy for the toroidal high-spin isomer with spin I, k is the c.m. momentum in the initial state, and Γ is the full width at half maximum height of the resonance. The quantities $B_{\rm in}$ and $B_{\rm out}$ are the branching fractions for the resonance into the initial-state and finial-state channel, respectively. Here, the width Γ and the branching fractions $B_{\rm in}$ and $B_{\rm out}$ may need to be determined experimentally. The resonance energy $E_{\rm res}$ (in the c.m. system) is given by

$$E_{\rm res}({}^{A}Z^t(I)) = M({}^{A}Z^t(I)) - M({}^{A_T}Z_T) - M({}^{A_p}Z_p), \tag{3}$$

where $M({}^{A}Z^t(I))$, $M({}^{A_T}Z_T)$, and $M({}^{A_p}Z_p)$ are nuclear masses of ${}^{A}Z^t(I)$, ${}^{A_T}Z_T$, and ${}^{A_p}Z_p$, respectively. In terms of the binding energies $B({}^{A}Z)$, $B({}^{A_T}Z_T)$, and $B({}^{A_p}Z_p)$, and excitation energy $E^*({}^{A}Z^t(I))$, we have

$$E_{\rm res}({}^{A}Z^t(I)) = E^*({}^{A}Z^t(I)) - B({}^{A}Z) + B({}^{A_T}Z_T) + B({}^{A_p}Z_p). \tag{4}$$

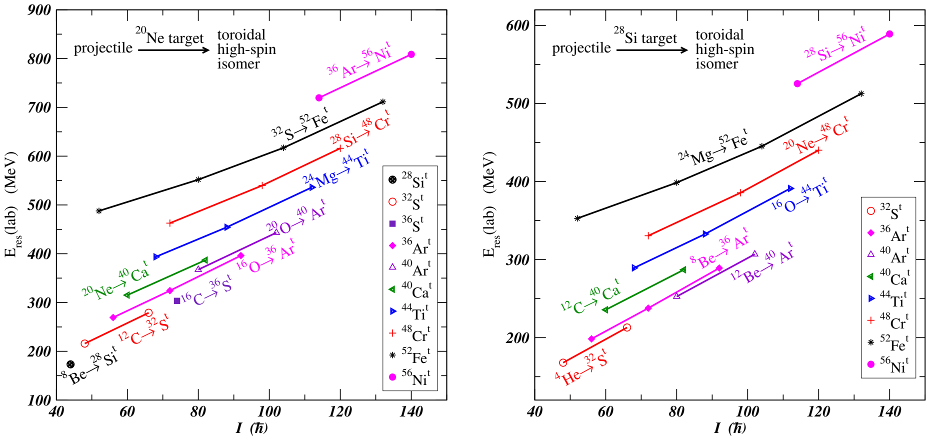

Figure 3: (Color online) The resonance energies $E_{\rm res}(^AZ^t(I))$(lab) in the laboratory system in the bombardment of various projectiles $^{A_p}Z_p$ on ^{20}Ne (left panel) or ^{28}Si (right panel) as an active target for the production of toroidal isomers $^AZ^t(I)$ with different angular momenta I.

The resonance energy $E_{\rm res}(^AZ^t(I))$(lab) in the laboratory system is given by $E_{\rm res}(^AZ^t(I)) \times (A_p + A_T)/A_T$. For the production of toroidal high-spin isomers by collision on ^{20}Ne or ^{28}Si as an active target, the resonance energies are shown in Fig. 3. The knowledge of the predicted values of $E_{\rm res}(^AZ^t(I))$(lab) will facilitate the search of toroidal high-spin isomers.

4 Conclusions and discussion

Under (multi-particle)–(multi-hole) excitation involving large orbital angular momentum orbitals, non-collective rotations of many light nuclei lead to equilibrium configurations whose densities may assume the shape of a torus. The np-nh systematics of toroidal high-spin isomers fit a regular pattern which can be used to predict possible presence of toroidal high-spin isomer in ^{56}Ni. We found additional equilibrium energy minima for ^{56}Ni with I=114$\hbar$ and 140$\hbar$ in subsequent cranked SHF calculations.

We examine the production of light-mass toroidal high-spin isomers by fusion of various projectile nuclei with an active target, in which the trajectories of the reaction products can be examined as a function of the collision energies. Resonance energies for the production of toroidal high-spin isomers have been calculated for ^{20}Ne or ^{28}Si as an active target, based on the Skyrme-Hartree-Fock energies obtained for the isomers.

The technology of building a TPC using 90% of ^{20}Ne as its dominant ingredient has been developed by the ALICE collaboration [10]. The utilization of a similar TPC detector with ^{20}Ne or ^{28}Si as an active target for nuclear spectroscopy may prove useful for the search of toroidal isomers.

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