Coupled-rearrangement-channels calculation of the three-body system under the absorbing boundary condition

M. Iwasaki[1], R. Otani[1], M. Ito[1] and M. Kamimura[2]

[1]Department of Pure and Applied Physics, Kansai University, 3-3-35 Yamatecho Suita, Osaka, Japan
[2]RIKEN Nishina Center for Accelerator-based Science, RIKEN, Wako, Saitama, Japan

Abstract

We formulate the method of the absorbing boundary condition (ABC) in the coupled-rearrangement-channels variational method (CRCMV) for the three-body problem. In the present study, we handle the simple three-boson system, and the absorbing potential is introduced in the Jacobi coordinate in the individual rearrangement channels. The resonance parameters and the strength of the monopole breakup are compared with the complex scaling method (CSM). We have found that the CRCVM + ABC method nicely works in the three-body problem with the rearrangement channels.

1　Introduction

One of the current issues in nuclear physics is the structure of the borromean three-body systems, which mainly appear around Neutron- or Proton-drip lines and its dynamics of continuum states above the particle decay threshold [1]. The borromean systems around the drip line are weak binding systems and hence, they are easily excited to unbound continuum states. Therefore, it is very important to describe the structure of the borromean systems as well as its reaction dynamics in a consistent manner.

In a naive treatment of the unbound states, a scattering boundary condition should be imposed among the interacting particles, but imposing the scattering boundary condition for three particles is complicated in general. In order to avoid the complexity of the three-body scattering boundary condition, non-Hermitian transformation is often introduced. There are two representative methods of the non-Hermitian transformations: the complex scaling method (CSM) [2] and the method of the absorbing boundary condition (ABC) [3, 4].

In CSM, the coordinates and momenta in the Hamiltonian of the total system is simply rotated in the complex plane, while, in ABC, the negative imaginary potential is placed at the outer region of the system. These two procedures commonly give rise to the damping of the wave function at the asymptotic region. Therefore, the three-body scattering problem can be reduced to the bound-state-like problem, in which the variational calculation using the basis-expansion technique is available.

The CSM calculations are greatly successful in describing the Coulomb breakup into the three-body continuum [2]. In the CSM calculations, the rearrangement of the Jacobi coordinates among the three particles, is completely taken into account. On the contrary, in the ABC method, no coordinate rearrangements are considered in the three-body calculations [5, 6].

The variational calculations with the rearrangement channels is called the coupled-rearrangement channels variational method (CRCVM), and the CRCVM treatment is essential to obtain the rapid convergence of the binding energy of the three-body system [7]. The CSM + CRCVM calculation in the three-body system has already been established, but the formulation of the CRCVM + ABC still remains unclear. Since the application of CSM is difficult in some special problems [4], it is important to develop ABC in the CRCVM treatment. In the present report, we apply ABC to CRCVM in the three-body problem, and the formulation of CRCVM + ABC is discussed.

2 Framework

In the present calculation, we handle the identical three-boson system, which has the following total Hamiltonian,

$$\hat{H} = -\frac{\hbar^2}{2\mu_{\mathbf{r}}}\nabla_{\mathbf{r}}^2 \ - \frac{\hbar^2}{2\mu_{\mathbf{R}}}\nabla_{\mathbf{R}}^2 \ + \sum_{i>j} V(s_{ij}), \tag{1}$$

where, the first two terms represent the kinetic energies with the Jacobi coordinate of $\mathbf{r}$ and $\mathbf{R}$, which denote the relative coordinate of the first and

second particles (1–2) and the coordinate between the center of mass of two particle and the third particle ((12)–3), respectively. $\mu_\mathbf{r}$ and $\mu_\mathbf{R}$ denote the reduced masses respective to each of the Jacobi coordinates. In the present calculations, a kinematic factor is set to be $\hbar^2/m = 1/2$ (MeV·fm^2) for simplicity.

The last term denotes the two body potential between the ith particle and the jth particle with relative distance s_{ij}, which is assumed to be the Gaussian potential [5], such as

$$V(s) = -2e^{-0.16s^2} + e^{-0.04s^2}. \tag{2}$$

In the ABC method, total Hamiltonian is modified to include the negative imaginary potential $-i\hat{W}$ with a strength η, such as

$$\hat{H} \rightarrow \hat{H}^\eta = \hat{H} - i\eta\hat{W}, \quad \hat{W} = \sum_{c=1,2,3} (r_c^4 + R_c^4). \tag{3}$$

Here, the absorbing potential $\hat{W}$, which is given by the polynomial function with the fourth power [3,4], are introduced along the Jacobi coordinate in the c-th rearrangement channels. We solve the Schrödinger equation for $\hat{H}^\eta$

$$\hat{H}^\eta \Psi_n^\eta(r, R) = E_n(\eta)\Psi_n^\eta(r, R). \tag{4}$$

The wave function $\Psi_n^\eta(r, R)$ and energy eigenvalue $E_n(\eta)$ for the n-th eigenstate depend on the strength of the absorber η. The S-wave relative motion is assumed in $\Psi(r, R)$ and hence, it depends on the magnitudes of the Jacobi coordinates, r and R. Eq. (4) is solved by the basis-expansion method, in which the oscillating Gaussian basis is employed [7]. The boson symmetry is exactly considered by introducing the rearrangement channels.

We also calculate the strength function of the three-body breakup into the continuum. A general definition of the strength function is given by

$$\mathcal{S}(E) = \sum_\nu \left| \left\langle \Psi_\nu | \hat{O} | \Psi_i \right\rangle \right|^2 \delta(E - E_\nu), \tag{5}$$

where Ψ_ν is the ν-th final state having the energy eigenvalue E_ν in continuum. In Eq. (5), $\hat{O}$ denotes the operator of an external field, and the monopole field is considered, which is the squared particle coordinate measured from the center of mass for the total system.

The calculation of the strength function in Eq. (5) can be achieved by implementing the extended completeness relation (ECR) [8]. The detailed explanation of ECR for the ABC solutions is shown in Ref. [8]. We have also applied the complex scaling method (CSM) on the calculation of the energy eigenvalues and the strength function. The ABC calculation is checked by comparing with the CSM calculation.

3 Results

Figure 1 shows the distribution of the energy eigenvalues obtained by solving Eq. (4) for the identical three-bosons system under the absorbing boundary condition. If the absorbing potential is switched off, the eigenvalues are located along the real (horizontal) axis. The energy eigenvalues are distributed in the complex energy plane, when the absorbing potential is switched on. In the negative real energy region, there is a bound state (open square), which is almost invariant under the absorbing boundary. In the positive energy region, the two resonant poles (solid circle and solid square) are separated from the continuum spectra (open circles).
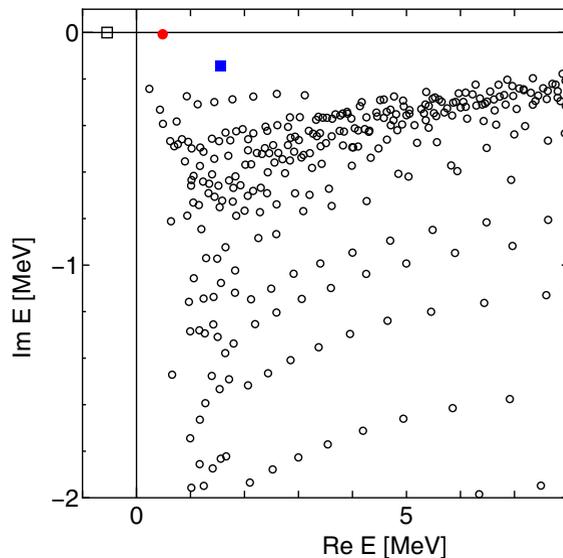

Figure 1: Distribution of the energy eigenvalues in the complex energy plane. The open square and the open circles show the bound state and the non-resonant continuum states, respectively. The solid circle and the solid square show the first and second resonances, respectively.

When the absorber strength η increases, the imaginary energies of the continuum states monotonically increase. The energies of the resonant states $E_{res}(\eta)$ are almost invariant with respect to the variation of η, but it slightly depends on η. The strength η is fixed so as to minimize the expectation value of the absorber, which corresponds to the numerical error, arising from the existence of the absorber and the incomplete basis-expansion [3, 4].

The resonance parameters of E_R and Γ_R are calculated at the optimal strength of η_o according to the relation of $E_{res}(\eta_o) = E_R - i\Gamma_R/2$. The resonance parameters for the first resonance are summarized in table 1. The optimal strength η_o is quite weak ($\sim 10^{-6}$) and hence, the expectation value of $< W(\eta_o) >$ is also small ($\sim 10^{-5}$). The parameters of E_R and Γ_R in ABC (the second row) is almost the same as the CSM results (the bottom row). We have confirmed that the ABC results on the second resonance also

Table 1: Resonance parameters for the first resonance calculated at the optimal strength η_o. The resonance energy (E_R) and the resonance width (Γ_R) are shown in unit of MeV. $< W(\eta_o) >$ represent the expectation value of the absorber at η_o. In the second row, the result of the ABC method is shown, while the results of CSM is shown in the lowest row.

Method	E_R	$\Gamma_R/2$	$< W(\eta_o) >$	η_o
ABC	0.424	$5.402{\times}10^{-3}$	$5.717{\times}10^{-5}$	$1.22{\times}10^{-6}$
CSM	0.424	$5.260{\times}10^{-3}$	X	X

reproduce the CSM result.

Finally, we discuss the application of the ABC method to the strength function of the three-body breakup. Figure 2 shows the strength function from the ground state to the three-body continua, which is induced by the monopole external field. The solid curve shows the result of the ABC method, while the dashed curve shows the CSM result. The ABC calculation follows the curve obtained by the CSM calculation over all energy region although a small oscillation arise in the higher energy region of $E \geq$ 3 MeV, where the magnitude of the strength is smaller by about 10^{-6} than the peak magnitude at $E \sim 0.5$ MeV.
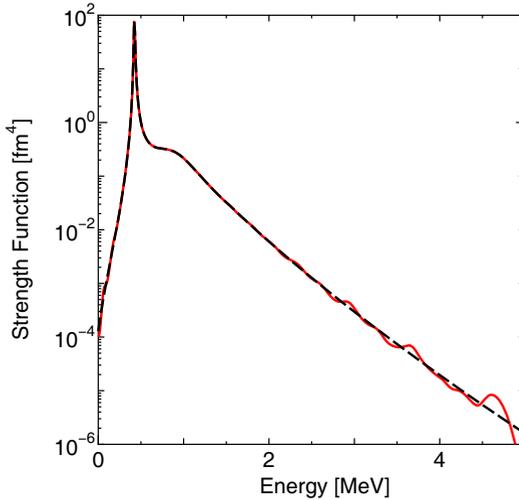

Figure 2: Monopole strength functions calculated from the ABC and CSM results. The solid and dashed curves show the ABC and CSM results, respectively.

The sharp peak at $E \sim 0.5$ MeV is generated by the contribution from the first resonance, while the broad shoulder at $E \sim 1$ MeV is due to the existence of the second resonance. The individual contributions, such as the resonance and the non-resonant continuum, in the strength function is common between ABC and CSM. Therefore, the strength function in the ABC calculation is consistent to the CSM treatment.

4   Summary

In the present study, we have formulated the absorbing boundary condition (ABC) method in the three-body problem with the coupled-rearrangement channel variational method (CRCVM). The CRCVM + ABC has been applied to the schematic three-boson system, and the results are compared to the CRCVM with the complex scaling method (CSM). The resonance parameters and the strength function calculated in ABC is completely consistent to the CSM calculation. Therefore, we can conclude that the formulation of the CRCVM + ABC method is successful in the calculation of the three-body system with the rearrangement channels.

In the future subject, we should apply the ABC method to the more realistic system such as the triple α system, in which its reaction rate is often discussed [9]. The realistic application of CRCVM + ABC is now under progress.

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