

Analysis of reaction size by method of scattering radius

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Abstract

We formulate the method of a scattering radius, which characterizes a spatial size of the scattering area, from partial wave decompositions in the coupled-channel problem. The method of the scattering radius is applied to the $p + {}^{12}\text{C}$ scattering in the range of $E_p = 29.95$ MeV to 65 MeV, and the scattering radii for the elastic scattering and the various inelastic channels are derived. We have found that the scattering radii for the inelastic channels with a well developed 3α structure are strongly enhanced in comparison to the non- 3α channels.

1 Introduction

Clustering phenomena, in which a nucleus is decomposed into some subunits, are well known to appear in the excited states of the lighter mass system [1]. In ${}^{12}\text{C}$, for instance, three α particles are strongly overlapped to each other in its ground 0_1^+ state, while they are weakly coupled in the excited 0_2^+ state, called Hoyle state, at $E_x = 7.65$ MeV [2]. One of characteristic features in cluster states is an extension of a matter radius. Previous theoretical calculations suggest that a radius of a cluster state is larger by about 50 % than a radius of a ground state [1,2]. Unfortunately, a size of a cluster state is difficult to measure directly by experiments because a cluster state has a

very short life-time. In a naive expectation, however, such a spatial extension may be reflected in observables of scattering phenomena. In previous studies, some enhancement factors originated from a spatial extension of a cluster structure are studied especially in the 3α state in ^{12}C [3, 4].

Recently, we have proposed the method of the gscattering radius on the basis of a partial wave decomposition of a scattering cross section [5]. In this report, the method of the scattering radius is applied to the multi-channel problem in the $p + ^{12}\text{C}$ scattering, going to the various inelastic channels, and the results of the scattering radii are discussed in connection to the intrinsic structures of ^{12}C .

2 Framework

We solve a set of the coupled-channel equations for the proton plus ^{12}C system, which is given in a symbolic form

$$(T_f(\mathbf{R}) - V_{f,f}(\mathbf{R}) - E_f) \chi_f(\mathbf{R}) = - \sum_{i \neq f} V_{f,i}(\mathbf{R}) \chi_i(\mathbf{R}) . \quad (1)$$

Here the subscripts of f and i design a channel. $T_f(\mathbf{R})$ represents the kinetic energy of the relative motion of the $p + ^{12}\text{C}$ system with a relative coordinate \mathbf{R} , while $V_{f,i}(\mathbf{R})$ denotes the coupling potential for the transition from the channel i to the channel f . The total energy in the channel f , E_f , is given by the relation of $E_f = E - \epsilon_f$ with a proton incident energy E and an internal energy of ^{12}C , ϵ_f . $\chi_f(R)$ is the proton- ^{12}C relative wave function for the channel f , which should be solved in the coupled-channel equations. In the present calculation, we include all the discrete states of ^{12}C in addition to the ground 0_1^+ state; the rotational state of 2_1^+ , the vibrational 3_1^- state, the 3α cluster states of 0_2^+ and 2_2^+ .

The coupling potential ($V_{f,i}(\mathbf{R})$) has a component of the real and the phenomenological imaginary potential. The real potential is calculated by the folding model [6]. We employ the DDM3Y (Density-Dependent Michigan 3-range Yukawa) [7] effective nucleon-nucleon force and the ^{12}C transition density calculated by the microscopic cluster model [2]. The imaginary potential, which has a Woods-Saxon form factor, is introduced in the diagonal transition ($f = i$) [5, 8], in order to reproduce the observed differential cross sections as much as possible. In the present calculation, the proton is treated as the spin-less particle.

In order to characterize a size of scattering area, we simply define the effective orbital spin \bar{L} [5] from the partial cross section $\sigma(L)$ for the incident

orbital spin L like

$$\bar{L} = \sqrt{\frac{\sum_L \left\{ \sqrt{L(L+1)} \right\}^4 \sigma(L)}{\sum_L \left\{ \sqrt{L(L+1)} \right\}^2 \sigma(L)}}. \quad (2)$$

The scattering radius R_{SC} corresponding to \bar{L} can be simply obtained from the relation of $\bar{L} = kR_{SC}$, where k denotes the wave number of the incident channel, measured in the laboratory system.

3 Results

We calculate the differential cross sections of the elastic and inelastic scattering in the energy range of $E_p = 29.95$ MeV to 65 MeV. The coupled-channels calculation with the 3α RGM + DDM3Y nicely reproduces all of the differential cross sections in the angular range of $\theta_{c.m.} = 30^\circ$ to 120° [8]. From the partial cross sections obtained by the coupled-channels, the effective orbital spins \bar{L} and the scattering radii, R_{SC} are calculated for all the channels. The \bar{L} and R_{SC} derived for $E_p = 65$ MeV are summarized in table 1. In this table, the matter radii of all the states (\bar{r}) [2] are also shown in the bottom row for comparison.

Table 1: The effective orbital spin \bar{L} and the scattering radius R_{SC} at $E_p = 65$ MeV. The respective matter radii \bar{r} are also shown in the bottom column [2]. R_{SC} and \bar{r} are shown in units of fm.

	0_1^+	0_2^+	2_1^+	2_2^+	3_1^-
\bar{L}	4.6	6.4	5.7	6.8	6.0
R_{SC} (fm)	2.6	3.6	3.2	3.9	3.4
\bar{r} (fm)	2.40	3.5	2.4	4.0	2.8

From table 1, we can see the enhancement of R_{SC} for the the 0_2^+ and 2_2^+ channels, which are the resonant states just above the 3α threshold, in comparison to the scattering radii of the 0_1^+ and 2_1^+ channels. The enhancement of the scattering radii in the 0_2^+ and 2_2^+ channels can be attributed to the spatial extension of the 3α structures of these states [2]. There is a considerable enhancement of the scattering radius in the 3_1^- channel. This enhancement may be due to the extended feature of the resonant 3_1^- states.

4 Summary and discussion

In summary, we have introduced a scattering radius from the partial wave decomposition in the coupled-channels problem. The method of the scattering radius has been applied to the $p + {}^{12}\text{C}$ scattering. We have found a clear enhancement of the scattering radius for the final 3α channels in comparison to the elastic and non- 3α channels. This enhanced scattering radius for the cluster channels is attributed to the spatial extension of the 3α structure.

The scattering radius does not necessarily mean the matter radius of a target nucleus itself but our scattering radius is consistent to the magnitude correlation of the matter radius, which is predicted by the structure calculation [2]. Thus, the scattering radius can be used for the measure of the spatial size of the individual reaction channels. The systematic studies are very important to establish the validity of the method of the scattering radius. In particular, a recent study in Ref. [9] suggests that the 0_6^+ state in ${}^{16}\text{O}$ corresponds to the 4α dilute state, and the state is expected to have the extended matter radius. Systematic studies are now underway.

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