EPJ Web of Conferences **117**, 07024 (2016) DOI: 10.1051/epjconf/201611707024
NN2015

High-spin molecular resonances in $^{12}\mathrm{C} + {}^{12}\mathrm{C}$

E. Uegaki[1] and Y. Abe[2]

[1]Dept. of Mathematical Science, Akita University, Akita 010-8502, Japan
[2]RCNP, Osaka University, Ibaraki 567-0047, Japan

Abstract

Resonances observed in the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ collisions are studied with a molecular model. At high spins $J = 10$–18, a stable dinuclear configuration is found to be an equator-equator touching one. Firstly, normal modes have been solved around the equilibrium, with spin J and K-quantum number being specified for rotation of the whole system. Secondly, with respect to large centrifugal energy, Coriolis coupling has been diagonalized among low-lying 11 states of normal-mode excitations, which brings K-mixing. The analyses of decay widths and excitation functions have been done. The molecular ground state exhibits alignments of the orbital angular momentum and the $^{12}\mathrm{C}$ spins, while some of the molecular excited states exhibit disalignments with small widths. Those results are surprisingly in good agreement with the experiments, which will light up a new physical picture of the high-spin $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ resonances.

1 Introduction

Well above the Coulomb barrier of the $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ system, series of resonances have been found with high spins over $10\hbar$, which exhibit prominent peaks in the elastic and inelastic 2^+ channels [1]. Band Crossing Model (BCM), based on the double resonance mechanism, has successfully explained resonance mechanism with the aligned configurations of the orbital angular momentum and the spins of the excited states of $^{12}\mathrm{C}$ nuclei [2].

On the other hand, for high-spin resonances observed in $^{24}\mathrm{Mg} + {}^{24}\mathrm{Mg}$ and $^{28}\mathrm{Si} + {}^{28}\mathrm{Si}$, we have developed a new molecular model [3, 4]. The model has an advantage to describe various geometrical configurations of two constituent deformed nuclei with the method of normal mode around the stable energy minimum under strong nucleus-nucleus interaction, which gives intuitive understanding on nuclear structure of the molecular resonances. For the oblate-oblate systems such as $^{28}\mathrm{Si} - {}^{28}\mathrm{Si}$ and $^{12}\mathrm{C} - {}^{12}\mathrm{C}$, the stable configurations with axial asymmetry exhibit a series of low-lying excited states with $K = 2$ or 4, in which spins of the constituent nuclei incline to the molecular z'-axis. Thus, those states are expected to have an interesting property of spin disalignments with the orbital angular momentum in contrast to the alignments in BCM. How to resolve an apparent contradiction?

In rotating systems, Coriolis coupling plays an important role in optimization of the centrifugal energy. Actually, a diagonalization of the coupling has brought a result that the molecular ground state with the $K = 1$ mixed configuration has aligned components similar to those of BCM [5]. In addition, a chain coupling of Coriolis is expected to connect the $K = 2$ and $K = 4$ states with the elastic channel, which gives rise to intermediate structures comparable to the observation.

2 Dynamical aspect and normal modes

Assuming a constant deformation and axial symmetry of the constituent nuclei, we have seven degrees of freedom $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2)$, as illustrated in fig. 1. The relative vector (R, θ_2, θ_1) of two $^{12}\mathrm{C}$ nuclei defines the molecular z'-axis. Orientations of the symmetry axes of $^{12}\mathrm{C}$ as internal degrees of freedom are described with Euler angles (α_i, β_i) referring to the

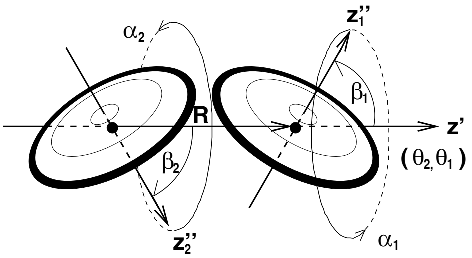

Figure 1: Dinuclear molecular coordinates; seven degrees of freedom of $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ (oblate-oblate system) are described with $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2)$.

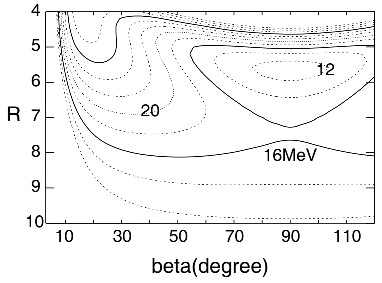

Figure 2: Contour map of the effective potential energy V_{JK} with $J = 14$ and $K = 0$ for the $R - \beta(\beta_1 = \beta_2)$ degrees at $\alpha = \pi/2$. Contours are in 2 MeV.

molecular axes. With α_1 and α_2, $\theta_3 = (\alpha_1 + \alpha_2)/2$ of the total K-rotation around z'-axis and $\alpha = (\alpha_1 - \alpha_2)/2$ of twisting motion are defined.

The kinetic energy is given classically in terms of angular velocities at first, and then the quantization is done to describe the whole rotating system. The resultant kinetic energy is classified into three parts, as $\hat{T} = \hat{T}_{\rm rot} + \hat{T}_{\rm int} + \hat{T}_{\rm C}$, where $\hat{T}_{\rm rot}$ denotes the rotational energy with the total angular momentum operator $\hat{\boldsymbol{J}}'$, $\hat{T}_{\rm int}$ the energy of internal motions, and finally $\hat{T}_{\rm C}$ the Coriolis coupling. The nucleus-nucleus interaction is given by a folding potential with nucleon-nucleon interaction DDM3Y. Expecting a stable configuration, we describe the interaction potential by the internal degrees of freedom $(R, \alpha, \beta_1, \beta_2)$, which specify geometrical configurations. Consistently with the coordinate system, at first we introduce a rotation-vibration type wave function specified with K-quantum number as basis one, $\Psi_\lambda \sim D^J_{MK}(\theta_i)\chi_K(R, \alpha, \beta_1, \beta_2)$, where χ_K describes the internal motions.

In order to know dynamical aspects of multidimensional internal motions, we calculate the effective potential for given J and K,

$$V_{JK}(R, \alpha, \beta_1, \beta_2) = V_{\rm interaction}(R, \alpha, \beta_1, \beta_2) + T'_{\rm rot}(J, K; R, \alpha, \beta_1, \beta_2), \quad (1)$$

where $T'_{\rm rot}$ denotes a multidimensional centrifugal potential with specified J and K, reduced from the rotational kinetic energy operator $\hat{T}_{\rm rot}$. In fig. 2, a multidimensional energy surface is displayed for $J = 14$. A local minimum is obtained around $\beta = 90^\circ$ and $R = 5.7$ fm, which is called as *equator-equator* (E-E) configuration. The barrier height is 4.8 MeV located at $R = 7.5$ fm.

Couplings among various molecular configurations are taken into account by the method of normal mode around the equilibrium at $\beta_i = 90^\circ$, which gives rise to the molecular modes of excitation. We expand the effective potential $V_{JK}(R, \alpha, \beta_1, \beta_2)$ into a quadratic form to describe normal modes. Combining the kinetic energy operator $\hat{T}_{\rm int}$, we obtain hamiltonian H_0, which is almost separated into each internal degree of freedom, as $H_0 = H(R) + H(\alpha) + H(\Delta\beta_1) + H(\Delta\beta_2) + H_{\rm coupl}(\alpha, \beta_1, \beta_2)$, where $\Delta\beta_i$ denote $\beta_i - \pi/2$. This means the internal degrees of freedom $(R, \alpha, \Delta\beta_1, \Delta\beta_2)$ are the normal modes themselves. Thus we use basis wave functions, $\Psi_{JMK}(\theta_i; R, \alpha, \beta_1, \beta_2) \sim D^J_{MK}(\theta_i) f_n(R)\phi_\nu(\alpha)\varphi_{n_1}(\Delta\beta_1)\varphi_{n_2}(\Delta\beta_2)$, where f_n, φ_{n_1} and φ_{n_2} are H.O. functions, while ϕ_ν is a rotational function $e^{i\nu\alpha}$ as the confinement is very weak in α-degree. The energy of the basis state is given as $E_{JK}(n, \nu, n_1, n_2) = E_0(R_{\rm e}) + \{J(J+1) - K^2 - 1\}\hbar^2/2\mu R_{\rm e}^2 + \{K^2 - 2\}\hbar^2/4I + \nu^2\hbar^2/4I + (n + 1/2)\hbar\omega_R + (n_1 + n_2 + 1)\hbar\omega_\beta$, where $\hbar\omega_\beta \sim 5$ MeV. Additional $H_{\rm coupl}$ includes coupling among β_i and α such as $\Delta\beta_1\Delta\beta_2 \cos 2\alpha$ as well as corrections for weak K-dependence of confinements on β_i-motions.

$H_{\rm coupl}$ and $H(\alpha)$ for the twisting motion have been diagonalized. In fig. 3, the spectrum of the normal modes with $J = 12$ is displayed, classified with quantum numbers $K = 0$–4. The axially asymmetric shape of the whole composite system allows states with $K = 2m$, m being integer, in low excitation. With $1\hbar\omega_\beta$ excitation around the equilibrium, the system has states with $K = 2m + 1$. The molecular quantum numbers also indicate rotational motions of two constituent ^{12}C nuclei by $\{n_1, \mu_1 = (K + \nu)/2\}$ and $\{n_2, \mu_2 = (K - \nu)/2\}$, respectively. A set of numbers $I_i = n_i + |\mu_i|$ is related to the set of spins of ^{12}C nuclei, like a channel assignment. In fig. 3, single 2^+ and mutual 2^+ assignments are depicted with connected lines.

3 Role of Coriolis coupling

With the normal-mode hamiltonian, the total hamiltonian is written as

$$H = H_0 + \hat{T}_{\rm C} + (higher\ order). \quad (2)$$

Generally in the rotating system, we have Coriolis coupling $\hat{T}_{\rm C}$, which can be described with a simple product $-\boldsymbol{J'} \cdot (\boldsymbol{S'_1} + \boldsymbol{S'_2})/\mu R^2$ of the total angular momentum operator $\boldsymbol{J'}$ and spins of ^{12}C $\boldsymbol{S'_i}$. Note that they refer to the molecular axes as designated with prime. In the model, it is given with many terms; $\boldsymbol{J'}$ is consisted with differential operators of (θ_i), which are combined into lowering and raising operators of K, $J'_\pm = J'_x \pm iJ'_y$, respectively. As for spins $\boldsymbol{S'_i}$, they are described usually with differential operators of Euler angles β_i, *i.e.*, given with those of the internal variables $(\alpha, \beta_1, \beta_2)$.

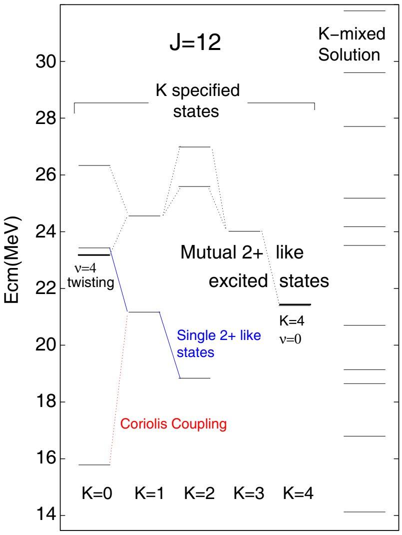

Figure 3: Energy spectra of molecular normal modes of ^{12}C+^{12}C for $J = 12$, specified with $K = 0$ to 4 from the *l.h.s.* Diagonalization of the Coriolis coupling gives a new spectrum shown in the rightmost column.

We consider the Coriolis terms as residual coupling, which causes K-mixing with $\Delta K = 1$. The mixing induces aligned components. Actually, in the molecular ground state, a $(K = 1, \nu = 1)$ mixed component appears to bring strong alignments in the single 2^+ channel [5]. In the rightmost column of fig. 3, levels of the K-mixed solutions caused by the Coriolis coupling are displayed for $J = 12$.

4 Molecular spectrum and excitation functions

We have found the stable E-E configuration for high spins from $J = 10$ to $J = 18$. For lower spins than $J = 10$, stable configurations are not found in the E-E one, but belly-belly one, *etc.* in closer contact of ^{12}C nuclei. In fig. 4, the molecular bands with the E-E structure are shown. Of course, there are many possible molecular bands for lower spins, which may correspond to those observed around the Coulomb barrier region.

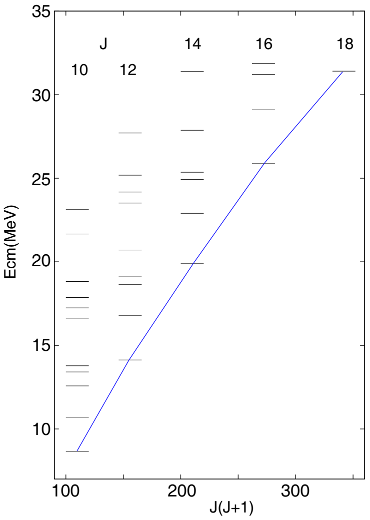

Figure 4: Molecular band structure for $J = 10$–18. The ground band is connected for eye-guide. The levels with widths larger than 4 MeV are omitted.

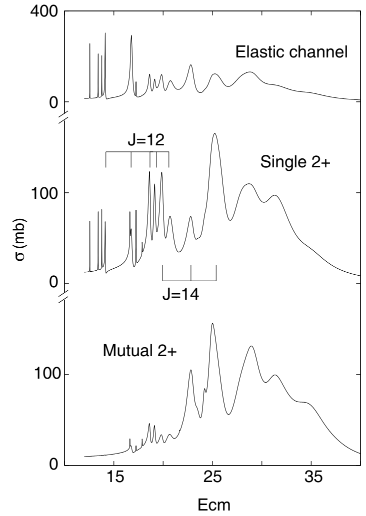

Figure 5: Theoretical excitation functions for the elastic, single and mutual 2^+ channels. The levels of $J = 10$–18 are included in. Spins are indicated for some resonance peaks (see text).

We have obtained many molecular excited states with small widths for the same spin, the number of which are comparable with the experimental observations. To obtain such a wide variety of the excited states, we introduced 11 bases of the molecular-mode states with low excitation energies, corresponding to single 2^+ and mutual 2^+ excitations. Important are low-lying states with high μ_i excitations, such as $\nu = 4$ or $K = 4$, which contains components of spin disalignments due to rotations of constituent ^{12}C nuclei around the molecular z'-axis. Surprisingly, widths of those states are not large due to very small amounts of the elastic components. The elastic components are almost exhausted in the molecular ground state and the first excited state.

In fig. 5, calculated excitation functions are displayed. Ledoux et al. assigned spins of three resonance states around $E_{\mathrm{cm}} = 20$ MeV as $J = 12$ for lower two states (18.4, 19.3 MeV) and $J = 14$ for the 20.3 MeV state [6]. This spin assignment is very important, because spins of all three resonances are suggested to be $J = 12$ by Cormier [1]. In the present results, the molecular ground state with $J = 14$ appears at 19.9 MeV in good agreement with the Ledoux's data. As for $J = 12$, the ground state is obtained at 14 MeV, while the second and third excited states appear around 19 MeV (see fig. 5). Thus three close-by resonances obtained around $E_{\rm cm} = 19$ MeV are in good correspondence with three intermediate peaks on the gross peak observed in the single 2^+ channel. It is noted that the excitation functions of both single and mutual 2^+ excitations are in good agreement with experiment [1].

5 Conclusions

Dinuclear molecular model has been applied to high-spin resonances of the $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ system. The model is capable to describe various angular momentum couplings, by the role of the Coriolis coupling. The resultant molecular ground state has its own characteristics as a token of aligned configuration, consistent with BCM, while in the excited states high K-rotational states appear relatively low in energy as non-aligned configurations. The latter multi-excited states are expected to weakly couple to the elastic channel and to be observed as resonances with small intermediate widths. As a result, characteristic features of excitation functions in the single 2^+ and mutual 2^+ channels, *i.e.*, the prominent gross peaks topped with the intermediate structures have been very well reproduced over a wide energy range. This gives rise to a novel visage to the known molecular resonances. Experiments to explore nature of those resonance states are strongly called for.

References

[1] Cormier T. M. et al., *Phys. Rev. Lett.* **40** (1978) 924.

[2] Abe Y., Kondō Y. and. Matsuse T., *Prog. Theor. Phys. Suppl.* No. **68** (1980) 303, and *references therein.*

[3] Uegaki E. and Abe Y., *Prog. Theor. Phys.* **90** (1993) 615.

[4] Uegaki E. and Abe Y., *Prog. Theor. Phys.* **127** (2012) 831, *ibid.* 877.

[5] Uegaki E. and Abe Y., *J. Phys.: Conf. Series* **569** (2014) 012091.

[6] Ledoux R. J. et al., *Phys. Rev. C* **27** (1983) 1103.