Effect of breakup and transfer on complete and incomplete fusion in $^6\text{Li}+^{209}\text{Bi}$ reaction in multi-body classical molecular dynamics calculation

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Abstract

The effect of breakup and transfer in $^6\text{Li}+^{209}\text{Bi}$ reaction is studied in a multi-body classical molecular dynamics approach in which the weakly-bound projectile $^6\text{Li}$ is constructed as a 2-body cluster of $^4\text{He}$ and $^2\text{H}$ in a configuration corresponding to the observed breakup energy. This 3-body system with their individual nucleon configuration in their ground state is dynamically evolved with given initial conditions using Classical Rigid Body Dynamics (CRBD) approach up to distances close to the barrier when the rigid-body constraint on the target, inter-fragment distance, and $^2\text{H}$ itself are relaxed, allowing for possible breakup of $^2\text{H}$ which may result in incomplete fusion following the transfer of the $n$ or $p$. Relative probabilities of the possible events such as scattering with and without breakup, DCF, SCF, ICF($x$) where $x$ may be $^4\text{He}$, $^2\text{H}$, $^4\text{He}+n$, $^4\text{He}+p$, $n$, $p$ are calculated. Comparison of the calculated event-probabilities, complete, and incomplete fusion cross sections with the calculation in which $^2\text{H}$ is kept rigid demonstrates the effect of the transfer reactions on complete and incomplete fusion in the 4-body reaction. Events ICF($^4\text{He}+n$) corresponding to $n$-stripping followed by breakup of the resultant $^5\text{Li}$ to $^4\text{He}+p$ are found to contribute significantly in the fusion process in agreement with a recent experimental observation of direct reaction processes in breakup of weakly-bound projectiles.
1 Introduction

Fusion reactions involving weakly-bound projectiles are complicated by the breakup of the projectile into two fragments. Complete fusion (CF) involves capture of both the fragments directly (DCF) or sequentially (SCF); incomplete fusion (ICF) involves capture of only a part of the projectile [1]. If one of the fragment itself is a weakly-bound nucleus like $^2$H, then its own breakup in the approach phase is also possible, resulting in nucleon transfer processes and breakup of the remaining unstable projectile residue. In a recent experiment it is observed that breakup of projectiles like $^6$Li is predominantly triggered by nucleon transfer such as $n$-stripping [2].

Such reactions are usually studied in CDCC formalism [1], a semi-classical coupled channel approximation [3] and, a classical trajectory model [4]. However, none of these approaches account for CF, ICF, and ICF following nucleon transfer, within the same model calculation. The multi-body, 3-Stage Classical Molecular Dynamics (3S-CMD) model [5], apart from demonstrating CF and ICF events is also able to account for a process equivalent to a direct reaction leading to ICF in the same model calculation.

The details of the present model calculation are given in section-2. Using this model, the effect of breakup and transfer on CF and ICF in $^6$Li+$^{209}$Bi collisions is studied by calculating various event-probabilities which are discussed in section-3. Calculated CF, ICF and total fusion (TF) cross section are presented in section-4. Finally conclusions are given in section-5.

2 Calculation details

The $^6$Li+$^{209}$Bi collision is studied in a multi-body, 3S-CMD model [5] in which $^6$Li is constructed as a cluster of $^4$He($\alpha$) and $^2$H($d$) nuclei held together in a configuration corresponding to the observed breakup energy equal to 1.467 MeV. The projectile fragments and the target are first generated with a variational potential minimization code with a soft-core Gaussian NN-potential and approximately reproduced ground state properties of the nuclei as in ref [5]. Collision simulation is carried out in the three stages: (1) The projectile and the target nuclei are initially brought along their Rutherford trajectories, (2) This system is then dynamically evolved using Classical Rigid Body Dynamics (CRBD) up to distances close to the barrier, followed by (3) CMD evolution of the entire many-body system. If one or both the projectile fragments are further constrained to be rigid then these nuclei are dynamically evolved as in the CRBD calculation even in the stage-3.
The rigid-body constraint on the bond between $^2$H and $^4$He in $^6$Li, as well as on the target $^{209}$Bi are relaxed in the stage-3 for $R_{cm} < 13$ fm. One of the projectile fragments ($^4$He) is always kept rigid. By allowing $^2$H in the projectile to be non-rigid, thereby allowing the possibility of its own breakup, and comparing the results with the calculation in which it is kept rigid even in the stage-3 near or inside the barrier, demonstrates the effect of direct reaction process in this reaction.

3 Event probabilities

Dynamical simulations of $^6$Li+$^{209}$Bi collisions with different impact parameters, energies and initial orientations may result in events such as scattering with (NCBU) and without breakup (NBUS), DCF, SCF, ICF($x$) where $x$ is either $^4$He or $^2$H which is captured. When $^2$H is also allowed to breakup near the target, $x$ may also be $^4$He+$n$, $^4$He+$p$, $n$ or $p$.

Events fractions defined as $F(b)=\left(\frac{N_{\text{events}}}{N_{\text{total}}}\right)$ are calculated, where $N_{\text{total}}$ is total no of initially random orientations for given $E_{cm}$ and $b$, and $N_{\text{events}}$ is no of trajectories analyzed as DCF, SCF, ICF etc. events. Trajectories for $b=0$ fm to $b_{\text{max}}=8.6, 7.0, 5.0$ fm for $E_{cm}=50, 36, 29$ MeV, respectively, in steps of 0.2 fm are analyzed. For each value of $b$, $N_{\text{total}}=500$ for $E_{cm}=50$ and 36 MeV, and 2000 for 29 MeV. Calculated $F(b)$ when $^2$H is kept rigid are shown in figure-1(a) for $E_{cm}=50$ MeV. Events following breakup (ICF+NCBU) increases with $b$ but decreases again for larger values of $b$. DCF is the major component of CF with a few SCF events also. Events ICF($^4$He) which are negligible at low $b$, rises to a peak value at higher $b$.

Figure-1(b) shows the results when $^2$H is kept non-rigid in stage-3 for $E_{cm}=50$ MeV. This figure shows events ICF($^4$He+$n$) which are equivalent to $n$-stripping followed by breakup of the resultant unstable $^5$Li $\rightarrow$ $^4$He+$p$. Distribution of ICF($^4$He+$n$) is much broader and larger compared to ICF($^4$He) in figure-1(a). This is in qualitative agreement with the experiment [2] which shows the importance of direct reaction processes. Similar calculations for $E_{cm}=36$ and 29 MeV shows reduction in events following breakup.

Integration of $F(b)$ over $b \leq b_{\text{max}}$ for a given $E_{cm}$ gives event-probabilities, shown as staked bar-charts in figure-2. It can be noted from figure-2(a), for $^2$H-rigid case, that DCF, SCF and CF events increase with $E_{cm}$. Figure-2(b) shows ICF($^4$He+$n$) events which are seen even at the lowest energy. Its relative importance increases as $E_{cm}$ increases, contributing significantly in the fusion process. The relative strength of ICF($^4$He+$p$) events corresponding to $p$-stripping is much smaller compared to ICF($^4$He+$n$) events.
Figure 1: Event fractions for (a) $^2$H(d)-rigid(R), (b) $^2$H(d)- non-rigid(NR)

Figure 2: Event probabilities for (a) $^2$H(d)-rigid(R), (b) $^2$H(d)- non-rigid(NR)

4 Fusion cross sections

CF is defined as an event in which $^2$H and $^4$He both are captured by $^{209}$Bi for a long interval of time. ICF is defined as an event in which only one of the projectile fragments or a part of the projectile is captured by $^{209}$Bi after their break-up. In a sharp cut-off approximation, it is assumed that all the trajectories with $b<b_{cr-CF}$, critical impact parameter for CF, are fused and those for $b>b_{cr-CF}$ are scattered or lead to ICF events up to $b<b_{cr-TF}$. Scattering with or without breakup occur for $b>b_{cr-TF}$.

For given $E_{cm}$, a large number of Monte-Carlo sampled initial orientations (as in section-3) are considered and $b_{cr-CF}$ and $b_{cr-TF}$ are determined for every orientation. Ion-ion potential is obtained as a function of the separation between the centre of masses of the target and all the projectile
fragments for $b = b_{cr-CF}$ or the center of masses of the target and the projectile-fragment that is captured for $b = b_{cr-TF}$.

Barrier parameters obtained from the ion-ion potential corresponding to $b_{cr-CF}$ or $b_{cr-TF}$ are used in the Wong’s formula [6] to calculate $\sigma_{CF}$ or $\sigma_{TF}$ respectively for a given orientation. An average over different orientations gives $\sigma_{CF}$ and $\sigma_{TF}$ for a given collision energy. ICF cross section are calculated from $\sigma_{TF} = \sigma_{CF} + \sigma_{ICF}$.

Conventionally, one uses the Wong’s formula with $b=0$ approximation. Calculated $\sigma_{CF}$ and $\sigma_{TF}$ for $^6\text{Li} + ^{209}\text{Bi}$ reaction for $b=0$ are shown in figure-3(a) and compared with the experimental $\sigma_{CF}$ and $\sigma_{TF}$ [7]. Experimental $\sigma_{TF}$ shown in figure-3 are obtained as a sum of the $\sigma_{CF}$ and $\sigma_{ICF}$ of ref [7]. The calculated cross sections with $b=0$ seems to match well with the experimental $\sigma_{CF}$ and $\sigma_{TF}$ respectively, except at very low energies.

The $b=0$ approximation in the Wong formula is, however, justified at low energies only. Higher partial wave may contribute significantly at higher energies [8]. We still use the Wong formula but, with the barrier parameters corresponding to the critical impact parameter as in ref [5, 9]. Calculated $\sigma_{CF}$ at $b_{cr-CF}$ from ref [5] are shown in figure-3(b). Moreover, since the contribution of ICF increases at higher values of $b$, reaches a maximum and diminishes again at grazing impact parameters. Therefore, to account for the effect of increased number of ICF events, the cross sections must include the contributions from trajectories with higher $b$ also. Therefore, $\sigma_{TF}$ is calculated for $b = b_{cr-TF}$ and the results for CF and TF for central and non-central collisions are compared in figure-3.
The $\sigma_{CF}$ and $\sigma_{TF}$ calculated using the Wong formula with barrier parameters corresponding to $b_{cr-CF}$ or $b_{cr-TF}$ are shown in figure-3(b) and compared with the corresponding experimental data. The differences in the $\sigma_{CF}$ and $\sigma_{TF}$ correspond to $\sigma_{ICF}$ which are also shown in figure-3. Calculated $\sigma_{ICF}$ in figure 3(b) corresponding to $b=b_{cr-TF}$ is much larger at higher energies compared to the calculated $\sigma_{ICF}$ in figure-3(a) for $b=0$ case. This large difference arises because of the increased number of ICF events at $b>0$ when $^2$H is non-rigid. The $\sigma_{CF}$ and $\sigma_{ICF}$ in figure-3(b) are of the same order of magnitude, although $\sigma_{CF}$ in figure-3(b) are enhanced compared to those in figure-3(a). The $\sigma_{TF}$ in figure-3(b) are also enhanced due to the enhanced values of $\sigma_{ICF}$ and are overestimated compared to the experimental values.

5 Conclusions

The effect of breakup and transfer in $^6$Li+$^{209}$Bi reaction is studied in a multi-body classical molecular dynamics approach. Comparison of the calculated event probabilities, $\sigma_{CF}$ and $\sigma_{TF}$ between, a 4-body simulation in which $^2$H is allowed to breakup near the barrier, and the 3-body calculation in which $^2$H is kept rigid, clearly demonstrates the effect of the transfer reactions on complete and incomplete fusion. It is found that ICF($^4$He+$n$) corresponding to $n$-stripping contributes significantly in the fusion process. This observation is in agreement with the experimental observation of the importance of direct reaction processes in breakup of weakly-bound projectiles. The present approach is able to account for DCF, SCF, ICF and a process equivalent to a direct reaction leading to ICF process in the same model calculation.

References