

# Fusion and neutron transfer reactions with weakly bound nuclei within time-dependent and coupled channel approaches

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## Abstract

The time-dependent Schrödinger equation and the coupled channel approach based on the method of perturbed stationary two-center states are used to describe nucleon transfers and fusion in low-energy nuclear reactions. Results of the cross sections calculation for the formation of the  $^{198}\text{Au}$  and fusion in the  $^6\text{He}+^{197}\text{Au}$  reaction and for the formation of the  $^{65}\text{Zn}$  in  $^6\text{He}+^{64}\text{Zn}$  reaction agree satisfactorily with the experimental data near the barrier. The Feynman's continual integrals calculations for a few-body systems were used for the proposal of the new form of the shell model mean field for helium isotopes.

## 1 Time-dependent model for neutron transfer

It is a well-established fact that  $^6\text{He}$  nucleus consists of the  $^4\text{He}$  core and the two-neutron cluster, see *e.g.* Refs. [1–4]. The probability density for the ground state of the three-body systems  $^6\text{He}$  ( $\alpha + n + n$ ) was calculated in Ref. [5] using Feynman's continual integrals method [6, 7]. The ground state probability densities for few-body systems of  $^{4,5,6}\text{He}$  (see Fig. 1a for  $^6\text{He}$ ) were used for the proposal of the new forms  $V_1(r)$  for mean field in the shell model of helium isotopes (see Fig. 1c). For  $^4\text{He}$  properties of the mean field are similar to these determined in Ref. [8].

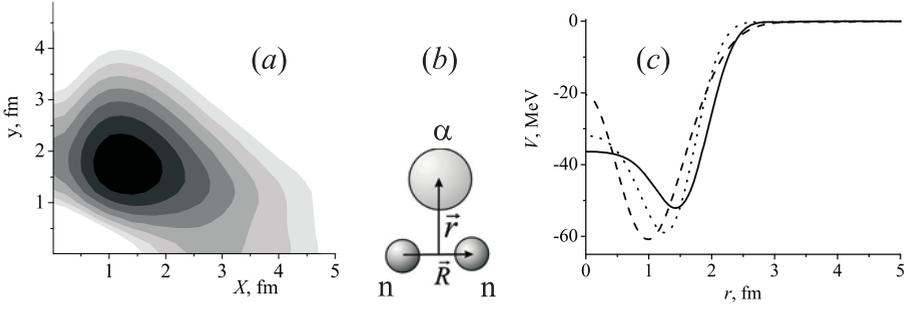


Figure 1: The probability density (a) for neutron-position symmetric configurations  $\alpha + n + n$  (b) of  ${}^6\text{He}$  calculated by Feynman's continual integrals method; (c) the mean field  $V_1(r)$  for neutrons in the shell model of  ${}^6\text{He}$  (solid line),  ${}^5\text{He}$  (dotted line) and  ${}^4\text{He}$  (dashed line).

The approximation of the independent external (valence) neutrons was used for the description of transfer reactions and the first (capture) stage of fusion reactions [9]. The two component spinor wave function of the neutron with the mass  $m$  and the radius vector  $\mathbf{r}$  changes according to the time-dependent Schrödinger equation (TDSE) with the spin-orbital interaction [10–12]

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \Delta + W(\mathbf{r}, t) + \hat{W}_{LS}(\mathbf{r}, t) \right\} \Psi(\mathbf{r}, t), \quad (1)$$

$$W(\mathbf{r}, t) = V_1(\mathbf{r} - \mathbf{R}_1(t)) + V_2(\mathbf{r} - \mathbf{R}_2(t)). \quad (2)$$

Here  $\mathbf{R}_{1,2}(t)$  are the radius vectors of the center of mass of the He projectile and the heavy target nuclei moving along classical trajectories. Potential  $V_2(r)$  is the energy of the neutron interaction with the heavy target nucleus in the Wood-Saxon form [13]. The operator of the spin-orbital interaction is

$$\hat{W}_{LS} = -\frac{b_1}{2\hbar} \sigma [(\nabla V_1) \hat{\mathbf{p}}] - \frac{b_2}{2\hbar} \sigma [(\nabla V_2) \hat{\mathbf{p}}]. \quad (3)$$

Here  $\hat{\mathbf{p}}$  is the momentum operator,  $\sigma$  are Pauli matrices, the factors  $b_{1,2}$  are

$$b_i = R_0^2 \lambda_i \frac{\hbar^2}{2m^2 R_0^2 c^2} = 0.022 R_0^2 \lambda_i, \quad (4)$$

$c$  is the speed of light,  $R_0 = 1$  fm and  $\lambda_i$  are phenomenological dimensionless constants. In this study value  $\lambda_2 \approx 35$  [13] for heavy nuclei and value  $\lambda_1 \approx 10$

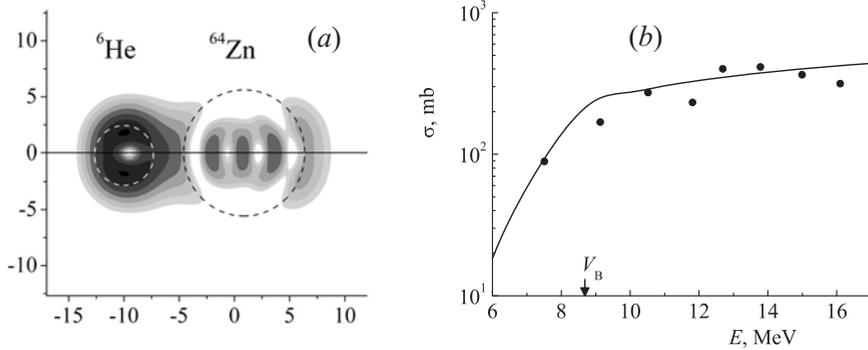


Figure 2: (a) The probability density of the valence neutrons of the  ${}^6\text{He}$  nucleus during a frontal collision with  ${}^{64}\text{Zn}$  at energy in the center of mass system  $E = 8$  MeV, a scale value is 1 fm, the radii of the circumferences equal to the radii of the nuclei. (b) The excitation function for the formation of the  ${}^{65}\text{Zn}$  isotope in the reaction  ${}^6\text{He} + {}^{64}\text{Zn}$ . Experimental data (solid circles) is from [15]. The theoretical curve is calculated within the TDSE approach;  $V_B$  is the Coulomb barrier.

for helium nuclei are used. The evolution of the probability density of the valence  $1p_{3/2}$  neutrons of the  ${}^6\text{He}$  nucleus during the collision with the  ${}^{197}\text{Au}$  nucleus was studied in Ref. [12]. The probability density during the frontal collision of  ${}^6\text{He} + {}^{64}\text{Zn}$  is shown in Fig. 2. The used mesh spacing in the TDSE method was not large then 0.2 fm, which is substantially smaller than 0.8 fm in a typical time-dependent Hartree-Fock (TDHF) calculation [14]. The colliding nuclei are enclosed in a box of typical dimensions  $50 \times 25 \times 25$  fm<sup>3</sup> for central collisions and  $50 \times 35 \times 25$  fm<sup>3</sup> for grazing collisions.

The presence of a stable structure in Fig. 2a indicates the formation of two-center (molecular) states of external neutrons. The neutron cloud in this case covers both nuclei, and an corresponding integral can serve as a measure of the probability  $w$  of the neutron transfer from one nucleus to another. Results of the cross section calculation using integration over the impact parameter  $b \geq b_0$

$$\sigma = 2\pi \int_{b_0}^{\infty} w(b) b db. \quad (5)$$

for the formation of the  ${}^{65}\text{Zn}$  isotope in the  ${}^6\text{He} + {}^{64}\text{Zn}$  reaction agree satisfactorily with the experimental data [15] near the barrier (Fig. 2b). The impact parameter  $b = b_0$  in Eq. (5) is corresponded to nuclei touching.

## 2 Coupled channel model for neutron transfer and fusion reactions

The semiclassical model with few independent quantum light particles (valence neutrons) and two classical heavy cores describes experimental neutron transfer cross sections satisfactory (see Fig. 3a and Ref. [12]). For simultaneous description of neutron transfer and fusion reactions we may use coupled channel (CC) approach (see *e.g.* Refs. [16,17]). The program CCFULL [16] includes pair-transfer coupling between the ground states only. It uses the macroscopic coupling form factor and the value of the coupling strength  $F_t = \text{const}$  is adjusted manually, which makes its choice unobvious. Microscopic approach proposed in Refs. [5,10,18] takes into account shell structure of colliding nuclei. In this approach TDSE and CC solutions are used simultaneously. For neutron rearrangement CC equations based on the method of perturbed stationary two-center states are

$$y''_{L,\mu} - \frac{L(L+1)}{R^2} y_{L,\mu} + \frac{2M}{\hbar^2} [E_\mu(R) - U(R)] y_{L,\mu} = F_t \sum_\nu T_{\mu\nu}(R) y_{L,\nu}. \quad (6)$$

Here  $M$  is the reduced mass,  $U(R)$  is the nucleus-nucleus potential,  $L$  is the relative orbital momentum,  $E_\mu = E_{\text{c.m.}} + Q_\mu(R)$ ,  $E_{\text{c.m.}} = \hbar^2 k_0^2 / 2M$ ,  $Q_\mu(R) = \varepsilon_0 - \varepsilon_\mu(R)$  is the distance-dependent Q-value,  $\varepsilon_0$  is the energy of the initial neutron state in the distant nucleus,  $\varepsilon_\mu(R)$  is the two-center (molecular) energy level and  $T_{\mu\nu}(R)$  is the reduced kinetic energy coupling matrix

$$T_{\mu\nu}(R) = \int \frac{\partial}{\partial R} \phi_\mu^*(\mathbf{r}; R) \frac{\partial}{\partial R} \phi_\nu(\mathbf{r}; R) d\mathbf{r}. \quad (7)$$

Here  $\phi_\nu(\mathbf{r}; R)$  are wave functions of the stationary two-center states calculated using series expansion of Bessel functions [10]. The effective coupling strength  $F_t$  in Eq. (6) is used to compensate the deletion of some complicated expressions in the exact equations of the perturbed stationary states method [5,10]. The value of the coupling strength  $F_t \approx 10$  was determined from the requirement of the correlation between the values of the channel functions  $|y_{L,\nu}|^2$  and the coefficients  $|a_\nu(R)|^2$  of the series expansion

$$|y_{0,\nu}(R)|^2 \sim |a_\nu(R(t))|^2 \quad (8)$$

$$a_\nu(R(t)) = \int \phi_\nu^*(\mathbf{r}; R(t)) \Psi(\mathbf{r}, t) d\mathbf{r} \quad (9)$$

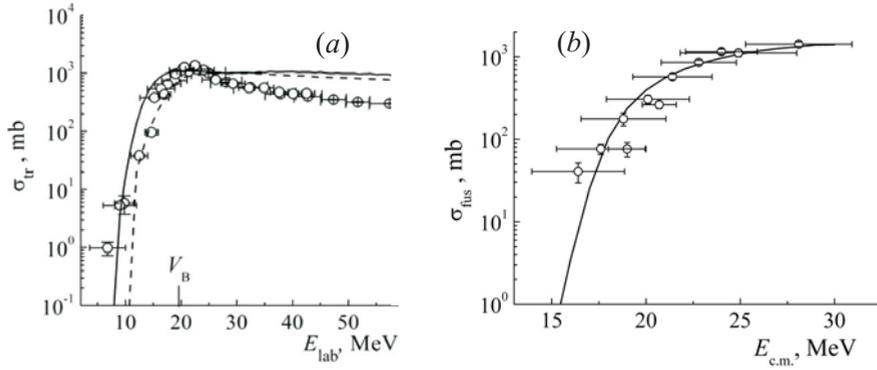


Figure 3: The excitation functions for the formation of the  $^{198}\text{Au}$  isotope (a) and fusion (b) in the reaction  $^6\text{He}+^{197}\text{Au}$ . Experimental data (circles) is from Refs. [19,20]. Theoretical curves were calculated within the coupled channel (solid lines) and the TDSE (dashed line) approaches in Ref. [21];  $V_B$  is the Coulomb barrier.

The transfer cross section  $\sigma_{\text{tr}}$  and the fusion cross section  $\sigma_{\text{fus}}$  are calculated by summation of probability density flux in the infinity and in the region of the nuclear surfaces contact  $R = R_c$

$$\sigma_{\text{tr}} = \frac{\pi k_0^2}{j_0} \sum_{L=0}^{\infty} (2L+1) \sum_{\nu} |j_{L,\nu}(R \rightarrow \infty)|. \quad (10)$$

$$\sigma_{\text{fus}} = \frac{\pi k_0^2}{j_0} \sum_{L=0}^{\infty} (2L+1) \sum_{\nu} |j_{L,\nu}(R \rightarrow R_c)|. \quad (11)$$

Here  $j_0 = \hbar k_0/M$  is the flux incoming from the infinity. In the simplest approximation coupling matrix elements have nonzero values only for the states localized in different nuclei in the limit  $R \rightarrow \infty$ . Results of calculations of the transfer and fusion cross section plotted in Fig. 3 agree satisfactorily with the experimental data near the barrier.

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