EPJ Web of Conferences **118**, 01017 (2016) DOI: 10.1051/epjconf/201611801017
FCCP2015

Perspectives on the hadronic contribution to the muon g-2

Marc Knecht[1,a]

[1] *Centre de Physique Théorique, CNRS/Aix-Marseille Univ./Univ. du Sud Toulon-Var (UMR 7332)*
CNRS-Luminy Case 907, 13288 Marseille Cedex 9, France

Abstract. The theoretical aspects of the various hadronic contributions to the anomalous magnetic moment of the muon are reviewed, as well as perspectives of their future evolution.

1 Introduction

The anomalous magnetic moment of the muon a_μ has been measured [1] with an unprecedented relative accuracy of 0.54ppm by the E821 experiment at the Brookhaven AGS. Its value reads [2]

$$a_\mu^{\rm exp} = 11\,659\,209.1(5.4)(3.3) \cdot 10^{-10}. \qquad (1)$$

At the theoretical level, the anomalous magnetic moment of the muon receives contributions from all interactions of the standard model. The by far dominant contribution comes from the electromagnetic sector, which accounts for more than 99.99% of its total value. The second most important contribution comes from the strong interactions, and finally the weak interaction provide, as expected, the smallest contribution. The values of these different contributions have been gathered in Table 1 below (in units of 10^{-10}). We will briefly discuss them in turn. Before that, let us consider the two following points.

• If one adds up the various contributions given in Table 1, one obtains a value which differs from the experimental one. The difference ranges from 2.8σ [5, 8]

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 23.9(8.6) \cdot 10^{-10}, \qquad (2)$$

to 3.5σ [4, 7]

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 27.6(8.0) \cdot 10^{-10}, \qquad (3)$$

depending on the input values used. Although this discrepancy is with us for quite some time now, it is nevertheless too small to provide conclusive evidence for the existence of physical degrees of freedom beyond those described by the standard model. For an overview of the possibilities concerning the latter, see e.g. Ref. [11], as well as Refs. [12–14] for more specific scenarios.

• Two new experiments are being planed, one at FNAL [15, 16] and the other one at J-PARC [17], with the aim of reducing the experimental uncertainty by a factor of 4 [18]. If all other values remain unchanged, i.e. those shown in Table 1 as well as the central value in Eq. (1), then this reduction of the uncertainties would by itself lead to roughly a 5σ discrepancy between theory and experiment. Things need, however, not develop as nicely as that, and in order to avoid ending up in a similar inconclusive situation as the present one regarding the existence of new physics, these experimental efforts need to be met by a comparable improvement of the theoretical accuracy.

Table 1. The contributions to a_μ (in units of 10^{-10}) using the latest published values

QED	+11 658 471.9	[3]
HVP-LO	+692.3(4.2)	[4]
	+694.9(4.3)	[5]
HVP-NLO	−9.84(7)	[5]
HVP-NNLO	+1.24(1)	[6]
HLxL	+10.5(2.6)	[7]
	+11.6(3.9)	[8]
EW 1 loop	+19.48(1)	[9]
EW 2 loops	−4.12(10)	[10]

2 QED and weak contributions

The contributions to a_μ coming from quantum electrodynamics and from the weak interactions belong to the realm of perturbation theory, which makes their evaluation straightforward, although quite tedious when it comes to including higher orders.

The value coming from pure (multi-flavour) QED includes contributions up to order $O(\alpha^5)$, i.e five loops. The one-loop contribution was computed by J. Schwinger [19] long time ago. Analytical results are also available for the two- and three-loop corrections (for a survey and references to the original publications, see [20]). Finally, the corrections at four and five loops were obtained more recently [3, 21], through numerical integration of Feynman-parameterised loop integrals. There are no uncertainties attached to the QED value at the level of precision shown here, which is sufficient to match the present experimental

[a] e-mail: marc.knecht@cpt.univ-mrs.fr

error on a_μ, but also the precision that forthcoming experiments expect to reach. The contributions at order $O(\alpha^4)$ that dominate the coefficient $C_\mu^{(8)}$ of the perturbative series

$$a_\mu^{\rm QED} = \sum_n C_\mu^{(2n)} \left(\frac{\alpha}{\pi}\right)^n, \tag{4}$$

come from Feynman graphs with internal electron loops, which are enhanced because they produce powers of $\ln(m_\mu/m_e)$. These contributions have been evaluated recently [22, 23], using asymptotic expansion techniques in the small quantity m_e/m_μ, thus providing an independent and welcome check (at a level of precision admittedly lower than the direct numerical evaluation [3, 21], but still higher than the one forthcoming experiments are aiming at). Looking more closely at the structure of the coefficients $C_\mu^{(2n)}$ of the perturbative series in Eq. (4), one notices that they increase drastically starting from the third order,

$$\begin{aligned} &C_\mu^{(2)} = 1/2 \quad C_\mu^{(4)} = 0.765\,857\,425(17) \\ &C_\mu^{(6)} = 24.050\,509\,96(32) \\ &C_\mu^{(8)} = 130.877\,3(61) \quad C_\mu^{(10)} = 751.92(93). \end{aligned} \tag{5}$$

This naturally raises the question whether a large coefficient $C_\mu^{(12)}$ could at least partly explain the $\sim 3\sigma$ discrepancy between theory and experiment mentioned above. Looking for diagrams that are enhanced by the presence of a maximal number of $\ln(m_\mu/m_e)$ factors leads to the estimate [3]

$$C_\mu^{(12)} \left(\frac{\alpha}{\pi}\right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 1 \cdot 10^{-12}. \tag{6}$$

This is way too small to explain even part of the discrepancy.

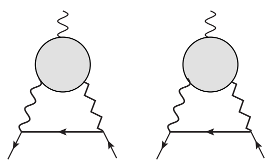

Figure 1. The hadronic contribution to the two-loop electroweak corrections to a_μ. The shaded blob represents the $\langle VVA \rangle$ QCD three-point function. It is connected to the muon line by a photon line and by the neutral Z^0, through the axial coupling of the latter to quarks and leptons.

The evaluation of the contributions at one loop coming from the electroweak sector [9] go back to the first loop calculations done within Weinberg's model for leptons [24]. The two-loop corrections were addressed somewhat later only [25–29]. Table 1 shows the value obtained by the recent reevaluation from Ref. [10]. The main change is a reduction of the uncertainty, due to the fact that the mass of the electroweak scalar boson is now known. These two-loop corrections also involve hadronic contributions, in particular those from the diagrams shown in Fig. 1, which are enhanced by a $\ln(M_Z/m_\mu)$ factor (which is eventually canceled by the corresponding contribution involving lepton loops instead of the blob). Although this contribution provides a substantial fraction of the uncertainty of the total two-loop electroweak correction to a_μ, there is no need to increase its accuracy for the time being. Notice that the discrepancy between theory and experiment represents about two times the total (one and two loops) correction from the electroweak sector. Moreover, the two-loop correction shown in Table 1 also contains an estimate of the leading-log three loop corrections [29]. There is therefore no reason to expect that higher-order electroweak contributions could account even for part of the discrepancy between theory and experiment.

Let us then leave the realm of perturbation theory and turn towards the remaining corrections, where the strong interactions are predominant.

3 Hadronic vacuum polarization

The contribution known as hadronic vacuum polarization (HVP) occurs first at order $O(\alpha^2)$, cf. Fig. 2. It provides to date the largest hadronic correction to a_μ, but also the largest contribution to the theoretical uncertainty (see Table 1). The leading order HVP correction can be expressed in the following way [30–32]

$$a_\ell^{\rm HVP-LO} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^\infty \frac{ds}{s} K(s) R^{\rm had}(s), \tag{7}$$

with

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}, \tag{8}$$

and $R^{\rm had}(s)$ represents the R-ratio of the cross section for $e^+e^- \to$ hadrons. This contribution can therefore be evaluated using available experimental input. The two most recent evaluations from Refs. [4] and [5] are shown in Table 1 (for more details concerning these two evaluations, see Ref. [33]). They are in good agreement (being based on the same data sets, this should not be a surprise) and give a relative precision of 0.6%, despite some tension between, for instance, the high-precision data collected by both BaBar and KLOE in the region of the ρ resonance, as shown in Table 2. Results from data collected in this region by the BESIII experiment have recently become

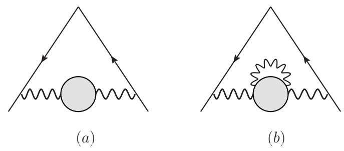

Figure 2. Diagrammatic representation of $a_\mu^{\rm HVP-LO}$, the leading hadronic vacuum polarization contribution to a_μ. In diagram (a), the shaded blob represents the $\langle VV \rangle$ QCD two-point function. Also shown is a next-to-leading contribution, diagram (b), actually included in $a_\mu^{\rm HVP-LO}$ (see text).

Table 2. The contribution to $a_\mu^{\mathrm{HVP-LO}}$ (in units of 10^{-10}) coming from the measurement of the $e^+e^- \to \pi^+\pi^-$ cross section in the region between 600 and 900 MeV. Values taken from Fig. 7 in Ref. [34].

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600-900\ \mathrm{MeV})$
BaBar	376.7(2.0)(1.9)
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	365.3(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)
BESIII	368.2(2.5)(3.3)

available [34] and are also shown in this Table. Ref. [35] provides an updated evaluation of $a_\mu^{\mathrm{HVP-LO}}$ including these and other data more recent than those available at the time of Refs. [4, 5]. One may notice that the discrepancy between theory and experiment given in Eqs. (2) and (3) represents about 3% or 4% of the value of $a_\mu^{\mathrm{HVL-LO}}$, which, as already mentioned, is itself known with a relative accuracy of about 0.6%. T he evaluation of $a_\mu^{\mathrm{HVP-LO}}$ would thus have to be wrong by several standard deviations in order to explain this discrepancy.

At present, the contribution from hadronic vacuum polarization is the main source of the total theoretical error. In order to match the experimental precision future experiments plan to achieve, the uncertainty on this quantity should be reduced below the 0.5% level. Forthcoming high-precision data from BaBar and KLOE-2, but also from BESIII, Belle II, the experiments at VEPP-2000 in Novosibirsk [36], or perhaps still other sources [37], will hopefully help to reach this goal.

In recent years, a lot of efforts have been devoted to explore the possibilities to obtain reliable determinations of $a_\mu^{\mathrm{HVP-LO}}$ from QCD simulations on the lattice. Several strategies are being considered in order to overcome the difficulties that this computation presents, and I refer to a dedicated presentation [38] at this workshop for more details. I will only make a remark at this stage. What is usually understood under $a_\mu^{\mathrm{HVP-LO}}$ is not only the contribution shown on the diagram on the left in Fig. 2, but also comprises the contribution shown on the diagram on the right of this same Figure. The reason for this situation is rather easy to understand. From an experimental point of view, it is not possible to correct the data for contributions arising from the exchanges of virtual photons between the final-state hadrons, or for the emission of soft photons with energies below the detection threshold of the experiment. Furthermore, some radiative modes, like $e^+e^- \to \pi^0\gamma$, are explicitly included in the evaluation of $a_\mu^{\mathrm{HVP-LO}}$. This last final state, for instance, measured in the energy range between 600 MeV and 1030 MeV, produces by itself a contribution of $4.4(1.9)\cdot 10^{-10}$ to $a_\mu^{\mathrm{HVP-LO}}$. Should the future lattice evaluations of HVP reach the sub-percent level, then comparison with the data-based determinations will be meaningful only if the second contribution in Fig. 2 and radiative corrections are evaluated separately, or if the numerical simulations include both QCD and QED field configurations, which might eventually turn out to be the most expedient way to address this issue.

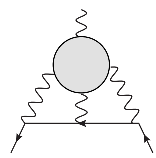

Figure 3. The hadronic light-by-light contribution to a_μ. The blob represents the hadronic four-point function (see text).

4 Hadronic light-by-light

Let us consider the second hadronic correction of importance as far as the contribution to the total theoretical uncertainty is concerned, namely hadronic light-by-light scattering (HLxL), represented on Fig. 3. The vertex function it depicts is expressed as follows, with p (resp. p') the four-momentum of the ingoing (resp. outgoing) muon, and $k = p' - p$,

$$\begin{aligned}
&\bar{\mathrm{u}}(p')\Gamma_\rho^{\mathrm{HLxL}}(p',p)\mathrm{u}(p) = \\
&= \bar{\mathrm{u}}(p')\Big[\gamma_\rho F_1^{\mathrm{HLxL}}(k^2) + \frac{i}{2m}\,\sigma_{\rho\tau}k^\tau F_2^{\mathrm{HLxL}}(k^2)\Big]\mathrm{u}(p) \\
&= -ie^6\int\frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\,\frac{1}{q_1^2\,q_2^2\,(q_1+q_2-k)^2} \\
&\times\frac{1}{(p'-q_1)^2-m^2}\,\frac{1}{(p'-q_1-q_2)^2-m^2} \\
&\times\bar{\mathrm{u}}(p')\gamma^\mu(\not{p}'-\not{q}_1+m)\gamma^\nu(\not{p}'-\not{q}_1-\not{q}_2+m)\gamma^\lambda\mathrm{u}(p) \\
&\times k^\sigma\frac{\partial}{\partial k^\rho}\,\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2,-k)
\end{aligned}\tag{9}$$

in terms of the QCD correlator $\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,q_3,q_4)$, $q_1+q_2+q_3+q_4=0$, the Fourier transform of the vacuum expectation value of the time ordered product of four light-quark electromagnetic currents. To extract a_μ^{HLxL}, the corresponding contribution to the anomalous magnetic moment, from this vertex function, one needs to project on the Pauli form factor $F_2^{\mathrm{HLxL}}(k^2)$, and then let the momentum k^μ go to $(0,0,0,0)$. In contrast to a_μ^{HVP}, there is no direct and simple connection of a_μ^{HLxL} with an experimental observable, i.e. something that would look like Eq. (7). In order to evaluate this contribution, one either has to rely on models that aim at reproducing the dynamics of the strong interactions in the whole ranges of integration over the loop

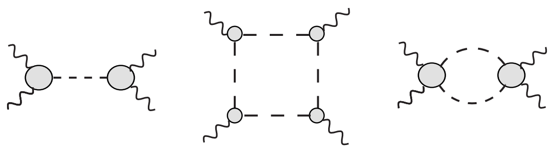

Figure 4. Various contributions to the four-point function $\Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4)$: poles due to single-meson intermediate states (left), charged meson loops (middle), two-meson intermediate states (right). The shaded blobs indicate the corresponding form factors.

momenta q_1 and q_2 in Eq. (9), or proceed through numerical simulation of QCD on a space-time lattice. Concerning the latter, see Ref. [39]. In order to implement the first approach, one needs some guiding principles. At our disposal, we have the chiral low-energy expansion and the $1/N_c$ expansion [40]. The second one turns out to be more useful from a practical point of view: the integration over the two momenta q_1 and q_2 in Eq. (9) does not only receive important contributions from the region where the chiral expansion stays relevant. From the point of view of the large-N_c expansion, one may identify individual contributions to the four-point function $\Pi_{\mu\nu\lambda\sigma}(q_1, q_2, q_3, q_4)$, like single-meson poles, or meson loops and two-particle intermediate states, and so on, as illustrated on Fig. 4,

$$\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) = \Pi_{\mu\nu\rho\sigma}^{\pi^0,\eta,\eta' \text{ poles}}(q_1, q_2, q_3, q_4) + \Pi_{\mu\nu\rho\sigma}^{\pi^\pm,K^\pm \text{ loops}}(q_1, q_2, q_3, q_4) + \Pi_{\mu\nu\rho\sigma}^{\pi\pi,K\bar{K}}(q_1, q_2, q_3, q_4) + \cdots . \quad (10)$$

Only two full calculations, Refs. [42, 43] and [44–46], have to date attempted to provide a description as complete as possible of the various contributions to the four-point function $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$. Although there are differences if specific contributions are compared on an individual basis, both groups obtained similar results for their final numbers for a_μ^{HLxL}, with later updates correcting for the sign of the pion-pole contribution, which was found to be wrong in the original publications, as first pointed out in Ref. [47]. In order to describe the QCD dynamics in the crucial region of intermediate energies, models like the extended Nambu–Jona-Lasinio model [48, 49] and/or the hidden local symmetry model [50, 51], which couples the lowest-lying vector resonance to the pseudoscalars, were used. The final result turns out to be numerically quite close to the contribution due to the exchange of a single neutral pseudoscalar between pairs of photons (see the first contribution on Fig. 4), the remaining contributions canceling each others. For a critical review, as well as updates, see Ref. [52]. Subsequent developments have focused on specific aspects, like: i) the reevaluation of individual contributions coming from the exchanges of single mesons, the pseudoscalars [8, 47, 53], the axial [35, 54], scalar or tensor mesons [54], ii) the large-N_c expansion, iii) short-distance constraints, and iv) dispersive approaches.

Concerning the large-N_c expansion, the general structure of a_μ^{HLxL} reads [7, 55, 56]

$$a_\mu^{\text{HLxL}} = N_c \left(\frac{\alpha}{\pi}\right)^3 \frac{N_c}{F_\pi^2} \frac{m_\mu^2}{48\pi^2} \left[\ln^2 \frac{M_\rho}{M_\pi} + c_\chi \ln \frac{M_\rho}{M_\pi} + \kappa\right] + O(N_c^0). \quad (11)$$

The coefficient of the $\ln^2$ constitutes an exact result [47, 55], whereas the constant c_χ is related to the amplitude for the decay $\pi^0 \to e^+e^-$ [55, 56] and can in principle be obtained from a sufficiently accurate measurement of the corresponding branching fraction. Nothing can be said from first principles on the constant κ and on the subleading $O(N_c^0)$ corrections, and here one is redirected to the existing model calculations, as provided by Refs. [42, 43] and [44–46].

An important aspect for the theoretical description of the four-point function $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$ are its properties at short-distances. Various possibilities have to be considered. First, since $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$ is not an order parameter, one has to match the low- and medium-energy part, that is described in terms of resonances, to the perturbative regime of QCD when all momenta reach high (euclidian) virtualities. Another quite interesting situation is provided by the limit, first considered in Ref. [57] (for q_1 in the euclidian region)

$$\lim_{\lambda\to\infty} \Pi_{\mu\nu\rho\sigma}(\lambda q_1, q_2 - \lambda q_1, q_3, q_4) = \frac{1}{\lambda}\frac{2i}{3}\frac{q_1^\gamma}{q_1^2}\, \epsilon_{\mu\nu\gamma}{}^{\tau} W_{\rho\sigma\tau}(q_3, q_4, q_2) + O(1/\lambda^2), \quad (12)$$

with $q_2 + q_3 + q_4 = 0$. In this expression, $W_{\mu\nu\rho}(q_1, q_2)$ denotes the Fourier transform of the the three-point QCD correlation function involving two hadronic electromagnetic currents and the flavour-neutral axial currents (including the flavour-singlet one). Finally, one might in principle also consider the behaviour in the limit

$$\lim_{\lambda\to\infty} \Pi_{\mu\nu\rho\sigma}(\lambda q_1, \lambda q_2, q_3 - \lambda q_1 - \lambda q_2, -q_3), \quad (13)$$

which doesn't seem to have been discussed so far. The limit where all virtualities become large (in the euclidian region) starts with a loop of massless quarks, supplemented with corrections from perturbative QCD,

$$\lim_{\lambda\to\infty} \Pi_{\mu\nu\rho\sigma}(\lambda q_1, \lambda q_2, \lambda q_3, \lambda q_4) = \Pi_{\mu\nu\rho\sigma}^{\text{pQCD}}(q_1, q_2, q_3, q_4) + \cdots, \quad (14)$$

with $q_1 + q_2 + q_3 + q_4 = 0$. In the approaches mentioned above, this condition is met through the presence of a constituent quark loop, which reproduces the correct limit at leading order, while the sum of the other contributions has then to vanish in this limit. In contrast to the previous situation, the constraint of Eq. (12) appears to be more "democratic", in the sense that all terms in the decomposition (10) can in principle contribute. This means that, in the limit considered in Eq. (12), they have to combine in order to build up, in the limit considered, the three-point correlation function on the right-hand side.

Recent (non lattice) developments concerning a_μ^{HLxL} have come from the side of dispersion relations. One approach [58–60] uses a decomposition of $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$ into a set of invariant functions, for which dispersion relations are written. In the corresponding absorptive parts one then isolates the one-particle (poles), two-particle, and so on, contributions, as illustrated on Fig. 4. The large-N_c expansion, together with the suppression of contributions involving higher invariant masses, provide the justification for considering contributions with only a low number of intermediate states. Notice, however, that, for instance, the two-pion states also contains resonant contributions (scalars or tensors) that, like the poles from the pseudoscalar mesons, occur already at leading order in the large-N_c limit. The idea is then to evaluate these dispersion relations in terms of experimental input for, for instance, various form factors they involve [61]. Whether experiment will be able to provide the necessary input data for all form factors involved at the required accuracy remains an open question [62]. A second approach [63, 64] writes a dispersion relation directly for the form factor $F_2(k^2)$ itself, using the representation in Eq. (9). The task consists then again to identify and evaluate the various discontinuities. So far, it has been shown [63] that one can recover the contribution from the pion pole through this method, using a vector-meson-dominance representation for the corresponding form factor. What is so far missing in these dispersive approaches, is how the short-distance properties will be implemented, that is how the various individual contributions will be made to talk to each others in the relevant asymptotic regimes.

The present discrepancy between theory and experiment represents about three times the values of $a_\mu^{\rm HLxL}$ reported in Table 1. Although the present determinations of a_μ^{HLxL} are still highly model dependent, they seem to incorporate the relevant physics in an appropriate manner. Other determinations relying on simple models, like, for instance, the constituent chiral quark model [65, 66] or holographic QCD [67, 68], do not show significant deviations from the results given in Table 1, although the uncertainties coming from the models themselves are of course difficult to estimate. Finally, higher-order corrections to $a_\mu^{\rm HLxL}$ have also been estimated [69], and found to be quite small.

5 Conclusion

There is a persistent discrepancy between the theory prediction of the anomalous magnetic moment of the muon within the standard model and its measured value, as shown in Eqs. (2) and (3). From the theoretical point of view, we see no hint of any need of higher order corrections at the level of the QED and weak contributions, even with the improvement in the experimental precision that future experiments plan to achieve. An independent cross-check of some of the tenth-order QED corrections enhanced by powers of $\ln(m_\mu/m_e)$, along the lines developed in Refs. [22, 23] for the eighth-order terms, if feasible, would be welcome, though. As far as the contributions of hadronic vacuum polarization is concerned, further improvement should be possible, since more precise data should become available in the future [36]. Actually, inclusion of more recent data on $e^+e^- \to$ hadrons in the determination of $a_\mu^{\rm HVP}$ even enhances the difference between theory and experiment [35]. Finally, there remains the contribution from hadronic light-by-light. There is no sign that important physics is missing in the existing evaluations. The issue here will rather consist in improving on the present precision and in obtaining more reliable estimate of the uncertainties.

Acknowledgements

I would like to thank the organizers of the Workshop “Flavour changing and conserving processes 2015” for the kind invitation to attend this very stimulating meeting. This work has been partially supported by the OCEVU Labex (ANR-11-LABX-0060) and the A*MIDEX project (ANR-11-IDEX-0001-02) funded by the "Investissements d’Avenir" French government program managed by the ANR.

References

[1] G. W. Bennett et al., Phys. Rev D **73**, 072003 (2006).

[2] K. A. Olive et al. [Particle Data Group], Chin. Phys. C **38**, 090001 (2014).

[3] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012) [arXiv:1205.5370 [hep-ph]].

[4] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **71**, 1515 (2011) [Eur. Phys. J. C **72**, 1874 (2012)] [arXiv:1010.4180 [hep-ph]].

[5] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G **38**, 085003 (2011) [arXiv:1105.3149 [hep-ph]].

[6] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, Phys. Lett. B **734**, 144 (2014) [arXiv:1403.6400 [hep-ph]].

[7] J. Prades, E. de Rafael and A. Vainshtein, chapter 9 in *Lepton Dipole Moments*, Advanced Series on Directions in High Energy Physics – Vol. 20, B. Lee Roberts and William J. Marciano Eds, World Scientific Co. Pte. Ltd. (2010).

[8] F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, 1 (2009) [arXiv:0902.3360 [hep-ph]].

[9] R. Jackiw, S. Weinberg, Phys. Rev. D **5**, 2396 (1972); I. Bars, M. Yoshimura, Phys. Rev. D **6**, 374 (1972); K. Fujikawa, B.W. Lee, A.I. Sanda, Phys. Rev. D **6**, 2923 (1972); G. Altarelli, N. Cabbibo, L. Maiani, Phys. Lett. B **40**, 415 (1972); W. A. Bardeen, R. Gastmans, B.E. Lautrup, Nucl. Phys. B **46**, 315 (1972).

[10] C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, Phys. Rev. D **88**, 053005 (2013) [arXiv:1306.5546 [hep-ph]].

[11] D. Stöckinger, chapter 12 in *Lepton Dipole Moments*, quoted in [7].

[12] See the contribution of D. Stöckinger to this workshop.

[13] See the contribution of E. J. Chun to this workshop.

[14] See the contribution of K. M. Patel to this workshop.

[15] R. M. Carey et al.. The New g-2 experiment - 2009. Fermilab. Proposal 0989.

[16] B. Lee Roberts, Chin. Phys. C **34**, 741 (2010).

[17] T. Mibe [J-PARC g-2 Collaboration], Chin. Phys. C **34**, 745 (2010).

[18] See the contribution of D. Hertzog to this workshop.

[19] J. Schwinger, Phys. Rev. **73**, 416L (1948).

[20] T. Kinoshita (chapter 3), S. Laporta and E. Remiddi (chapter 4) in *Lepton Dipole Moments*, quoted in [7].

[21] T. Kinoshita, M. Nio, Phys. Rev. D **73**, 053007 (2006). T. Aoyama et al., Phys. Rev. D **78**, 053005 (2008); Phys. Rev. D **78**, 113006 (2008); Phys. Rev. D **81**, 053009 (2010); Phys. Rev. D **82**, 113004 (2010); Phys. Rev. D **83**, 053002 (2011); Phys. Rev. D **83**,

053003 (2011); Phys. Rev. D **84**, 053003 (2011); Phys. Rev. D **85**, 033007 (2012).

[22] See the contribution of M. Steinhauser to this workshop.

[23] A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. D **92**, 073019 (2015) [arXiv:1508.00901 [hep-ph]].

[24] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[25] T. V. Kukhto, E. A. Kuraev, Z. K. Silagadze and A. Schiller, Nucl. Phys. B **371**, 567 (1992).

[26] S. Peris, M. Perrottet and E. de Rafael, Phys. Lett. B **355**, 523 (1995) [hep-ph/9505405].

[27] A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. D **52**, 2619 (1995) [hep-ph/9506256]; Phys. Rev. Lett. **76**, 3267 (1996) [hep-ph/9512369].

[28] M. Knecht, S. Peris, M. Perrottet and E. De Rafael, JHEP **0211**, 003 (2002) [hep-ph/0205102].

[29] A. Czarnecki, W. J. Marciano and A. Vainshtein, Phys. Rev. D **67**, 073006 (2003) [Phys. Rev. D **73**, 119901 (2006)] [hep-ph/0212229].

[30] C. Bouchiat and L. Michel, J. Phys. Radium 22, 121 (1961).

[31] L. Durand, Phys. Rev. **128**, 441 (1962); Err.-ibid. **129**, 2835 (1963).

[32] M. Gourdin and E. de Rafael, Nucl. Phys. B **10**, 667 (1969).

[33] See the contribution of Z. Zhang to this workshop.

[34] M. Ablikim *et al.* [BESIII Collaboration], arXiv:1507.08188 [hep-ex].

[35] See the contribution of F. Jegerlehner to this workshop.

[36] See the contribution of S. Eidelman to this workshop.

[37] See the contribution of L. Trentadue to this workshop.

[38] See the contribution of M. Petschlies to this workshop.

[39] See the contribution of C. Lehner to this workshop.

[40] E. de Rafael, Phys. Lett. B **322**, 239 (1994) [hep-ph/9311316].

[41] J. Bijnens, E. Pallante and J. Prades, Phys. Rev. Lett. **75**, 1447 (1995) [Phys. Rev. Lett. **75**, 3781 (1995)] [hep-ph/9505251].

[42] J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **474**, 379 (1996) [hep-ph/9511388].

[43] J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **626**, 410 (2002) [hep-ph/0112255].

[44] M. Hayakawa, T. Kinoshita and A. I. Sanda, Phys. Rev. Lett. **75**, 790 (1995) [hep-ph/9503463].

[45] M. Hayakawa, T. Kinoshita and A. I. Sanda, Phys. Rev. D **54**, 3137 (1996) [hep-ph/9601310].

[46] M. Hayakawa and T. Kinoshita, Phys. Rev. D **57**, 465 (1998) [Phys. Rev. D **66**, 019902 (2002)] [hep-ph/9708227].

[47] M. Knecht and A. Nyffeler, Phys. Rev. D **65**, 073034 (2002) [hep-ph/0111058].

[48] J. Bijnens, C. Bruno and E. de Rafael, Nucl. Phys. B **390**, 501 (1993) [hep-ph/9206236].

[49] J. Bijnens, Phys. Rept. **265**, 369 (1996) [hep-ph/9502335].

[50] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. B **259**, 493 (1985).

[51] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. **164**, 217 (1988).

[52] See the contribution of J. Bijnens to this workshop.

[53] A. Nyffeler, Phys. Rev. D **79**, 073012 (2009) [arXiv:0901.1172 [hep-ph]].

[54] V. Pauk and M. Vanderhaeghen, Eur. Phys. J. C **74**, 3008 (2014) [arXiv:1401.0832 [hep-ph]].

[55] M. Knecht, A. Nyffeler, M. Perrottet and E. de Rafael, Phys. Rev. Lett. **88**, 071802 (2002) [hep-ph/0111059].

[56] M. J. Ramsey-Musolf and M. B. Wise, Phys. Rev. Lett. **89**, 041601 (2002) [hep-ph/0201297].

[57] K. Melnikov and A. Vainshtein, Phys. Rev. D **70**, 113006 (2004)

[58] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP **1409**, 091 (2014) [arXiv:1402.7081 [hep-ph]].

[59] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP **1509**, 074 (2015) [arXiv:1506.01386 [hep-ph]].

[60] See the contribution of M. Procura to this workshop.

[61] G. Colangelo, M. Hoferichter, B. Kubis, M. Procura and P. Stoffer, Phys. Lett. B **738**, 6 (2014) [arXiv:1408.2517 [hep-ph]].

[62] See the contribution of A. Nyffeler to this workshop.

[63] V. Pauk and M. Vanderhaeghen, Phys. Rev. D **90**, 113012 (2014) [arXiv:1409.0819 [hep-ph]].

[64] See the contribution of M. Vanderhaeghen to this workshop.

[65] D. Greynat and E. de Rafael, JHEP **1207**, 020 (2012) [arXiv:1204.3029 [hep-ph]].

[66] See the contribution of D. Greynat to this workshop.

[67] L. Cappiello, O. Cata and G. D'Ambrosio, Phys. Rev. D **83**, 093006 (2011) [arXiv:1009.1161 [hep-ph]].

[68] See the contribution of L. Cappiello to this workshop.

[69] G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera and P. Stoffer, Phys. Lett. B **735**, 90 (2014) [arXiv:1403.7512 [hep-ph]].