On the interrelationship among leptonic $g - 2$, EDMs and lepton flavor violation

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Abstract. We present a concise review of the status of charged lepton flavor violation (cLFV) in scenarios beyond the SM. We emphasize that the current experimental resolutions on cLFV processes are already testing territories of new physics (NP) models well beyond the LHC reach. On the other hand, with the expected sensitivities of next-generation experiments, cLFV will become the most powerful probe of NP signals at our disposal. Finally, the interrelationship among leptonic $g - 2$, EDMs and cLFV will turn out to be of outmost importance to disentangle among different NP scenarios.

1 Introduction

The origin of flavor remains, to a large extent, an open problem. However, significant progress has been achieved in the phenomenological investigation of the sources of flavour symmetry breaking which are accessible at low energies, ruling out models with significant misalignments from the SM Yukawa couplings at the TeV scale.

The search for LFV in charged leptons is probably the most interesting goal of flavour physics in the next years. The observation of neutrino oscillations has clearly demonstrated that lepton flavour is not conserved. The question is whether LFV effects can be visible also in other sectors of the theory, or if we can observe LFV in processes that conserve total lepton number. The most promising LFV low-energy channels are probably $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$, $\mu \rightarrow e$ conversion in Nuclei as well as $\tau$ LFV processes which will be further investigated at the Super-Belle machine. The future sensitivities of next-generation experiments are collected in table 1.

Moreover, the flavour-conserving component of the same diagrams generating $\mu \rightarrow e\gamma$ induces non-vanishing contributions to the anomalous magnetic moment of leptons as well as to the lepton EDMs. In this context, the current anomaly for the muon ($g - 2$), reinforces the expectation of detecting $\mu \rightarrow e\gamma$ within the reach of the MEG experiment. Once some clear deviation from the SM is established, the next most important step is to identify correlations among different non-standard effects that can reveal the flavour-breaking pattern of the new degrees of freedom providing, at the same time, a powerful tool to disentangle among different New Physics scenarios. The above program represents one of the most exciting proofs of the synergy and interplay existing between the LHC, i.e.

<table>
<thead>
<tr>
<th>LFV Process</th>
<th>Present Bound</th>
<th>Future Sensitivity</th>
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<tbody>
<tr>
<td>$\mu \rightarrow e\gamma$</td>
<td>$5.7 \times 10^{-11}$ [1]</td>
<td>$\approx 6 \times 10^{-14}$ [2]</td>
</tr>
<tr>
<td>$\mu \rightarrow 3e$</td>
<td>$1.0 \times 10^{-12}$ [3]</td>
<td>$\approx 10^{-16}$ [4]</td>
</tr>
<tr>
<td>$\mu^+ Au \rightarrow e^- Au$</td>
<td>$7.0 \times 10^{-13}$ [5]</td>
<td>?</td>
</tr>
<tr>
<td>$\mu^- Ti \rightarrow e^- Ti$</td>
<td>$4.3 \times 10^{-12}$ [6]</td>
<td>?</td>
</tr>
<tr>
<td>$\mu^- Al \rightarrow e^- Al$</td>
<td>?</td>
<td>$\approx 10^{-16}$ [7, 8]</td>
</tr>
<tr>
<td>$\tau \rightarrow e\gamma$</td>
<td>$3.3 \times 10^{-8}$ [9]</td>
<td>$\sim 10^{-8} - 10^{-9}$ [10]</td>
</tr>
<tr>
<td>$\tau \rightarrow \mu\gamma$</td>
<td>$4.4 \times 10^{-8}$ [9]</td>
<td>$\sim 10^{-8} - 10^{-9}$ [10]</td>
</tr>
<tr>
<td>$\tau \rightarrow 3e$</td>
<td>$2.7 \times 10^{-8}$ [11]</td>
<td>$\sim 10^{-9} - 10^{-10}$ [10]</td>
</tr>
<tr>
<td>$\tau \rightarrow 3\mu$</td>
<td>$2.1 \times 10^{-8}$ [11]</td>
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<table>
<thead>
<tr>
<th>Lepton EDM</th>
<th>Present Bound</th>
<th>Future Sensitivity</th>
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<tbody>
<tr>
<td>$d_e$ (e cm)</td>
<td>$8.7 \times 10^{-29}$ [12]</td>
<td>?</td>
</tr>
<tr>
<td>$d_\mu$ (e cm)</td>
<td>$1.9 \times 10^{-19}$ [13]</td>
<td>?</td>
</tr>
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</table>

Table 1. Present and future experimental sensitivities for relevant low-energy observables.

the high-energy frontier, and high-precision low-energy experiments, i.e. the high-intensity frontier.

2 Leptonic $g - 2$, EDMs and LFV: a model-independent analysis

The physics responsible for neutrino masses and mixing might or might not be related to the physics related to cLFV. On general grounds, we can say that:

- neutrino masses might be naturally explained within seesaw scenarios which introduce heavy right-handed Majorana neutrinos typically at the grand-unification (GUT) scale. These scenarios can also explain the baryon-antibaryon asymmetry in the universe through the leptogenesis mechanism. The new interactions of the model generally violate lepton-number $L = L_e + L_\mu + L_\tau$ (LNV),

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• In the Standard Model (SM) with massive neutrinos, where the only source of LFV is coming from the operators responsible for the neutrino masses, the LFV effects are loop suppressed and proportional to the GIM factor $(m_\nu/M_W)^2$, therefore, completely negligible. For instance, it turns out that $\text{BR}(\mu \to e\gamma) \sim 10^{-15}$.

• On the other hand, generic models for new physics (NP) at the TeV scale contain new sources for LFV (but not necessarily for LNV), leading to decay rates accessible with future experiments.

From the low-energy point of view, these observations can be accounted for by considering the SM as an effective theory and extending its Lagrangian,

$$L_{\text{eff}} = L_{\text{SM}} + \frac{1}{\Lambda_{\text{LNV}}} \mathcal{O}^{\text{dim-5}} + \frac{1}{\Lambda_{\text{LFV}}} \mathcal{O}^{\text{dim-6}} + \ldots .$$

Here, the dimension-5 operator responsible for the neutrino masses is uniquely given in terms of the lepton doublets $L$ and the Higgs doublet $H$ in the SM,

$$\mathcal{O}^{\text{dim-5}} = (g_\nu)^j_i (\bar{L} H y (H^\dagger L)^c) + h.c.$$ (2)

and the misalignment between the Yukawa matrices $g_\nu$ and the Yukawa coupling matrix $Y_\nu$ in the charged-lepton sector leads to a non-trivial mixing matrix $U_{\text{PMNS}}$ for neutrino oscillations.

For instance, within scenarios with right-handed Majorana neutrinos (type-I see saw), one can identify $g_\nu/\Lambda_{\text{LNV}} = Y_\nu M^{-1} Y^c_\nu$, where $Y_\nu$ is the Yukawa matrix in the neutrino sector, and $M$ the Majorana mass matrix.

Examples for a dimension-6 operator are

$$\bar{\mu}_R \sigma^{\mu\nu} H e_L F_{\mu\nu}, \quad (\bar{\mu}_L Y^c e_L) \left( \tilde{f}_i \tilde{f}_j \right), \quad (\bar{\mu}_R e_L) \left( \tilde{f}_i \tilde{f}_j \right),$$ (3)

where $f = e, u, d$ and the first dipole-operator leads to LFV decays like $\mu \to e\gamma$ while the second and third ones generate, at the leading order, only processes like $\mu \to eee$ and $\mu \to e$ conversion in Nuclei. Obviously, the underlying dipole-transition $\mu \to e\gamma$ with a virtual $\gamma$ also contributes to $\mu \to eee$ and $\mu \leftrightarrow e$ conversion in Nuclei.

In particular, within NP theories where the dominant LFV effects are captured by the dipole-operator, the following model-independent relations hold

$$\text{BR}(\ell_i \to \ell_j \gamma) = \frac{(g_\ell)^j_i}{3\Lambda} \left( \log \frac{m^2_{\ell_j}}{m^2_{\ell_i}} - 3 \right) \text{BR}(\ell_i \to \ell_j \gamma),$$

$$\text{CR}(\mu \to e \text{ in N}) = \frac{c_{\text{SM}}}{c_{\text{EXP}}} \times \text{BR}(\mu \to e\gamma),$$ (4)

and therefore, the current MEG bound $\text{BR}(\mu \to e\gamma) \sim 5 \times 10^{-13}$ already implies that $\text{BR}(\mu \to eee) \lesssim 3 \times 10^{-15}$ and $\text{CR}(\mu \to e \text{ in N}) \lesssim 3 \times 10^{-15}$.

However, it is worth stressing that in many NP scenarios non-dipole operators, such as those shown in eq. 3, provide the dominant sources of LFV effects. Therefore, in such cases, $\mu \to eee$ and $\mu \leftrightarrow e$ conversion in Nuclei represent the best probes of LFV.

Dipole transitions $\ell \to \ell'\gamma$ in the leptonic sector are accounted for by means of the effective Lagrangian

$$\mathcal{L} = e \frac{m^2_\ell}{2} \left( \bar{\ell}_R \sigma_{\mu\nu} A_{\ell R} \ell_L^c + \bar{\ell}_L^c \sigma_{\mu\nu} A_{\ell R} \ell_R \right) F^{\mu\nu},$$ (5)

where $\ell, \ell' = e, \mu, \tau$. Starting from eq. (5), we can evaluate LFV processes, such as $\mu \to e\gamma$,

$$\frac{\text{BR}(\ell \to \ell'\gamma)}{\text{BR}(\ell \to \ell'\gamma)} \sim \frac{48\pi^2 a}{G_F^2} \left( |A_{\ell R}|^2 + |A_{\ell R}|^2 \right).$$ (6)

The underlying $\ell \to \ell'\gamma$ transition can generate, in addition to LFV processes, also lepton flavor conserving processes like the anomalous magnetic moments $\Delta \mu$ as well as leptonic electric dipole moments (EDMs, $d_e$). In terms of the effective Lagrangian of eq. (5) we can write $\Delta \mu$ and $d_e$ as

$$\Delta \mu = 2 \frac{m^2_e}{\Lambda^2} \text{Re}(A_{\ell R}), \quad d_e = m_f \text{Im}(A_{\ell R}).$$ (7)

On general grounds, one would expect that, in concrete NP scenarios, $\Delta \mu, d_e$ and $\text{BR}(\ell \to \ell'\gamma)$, are correlated. In practice, their correlations depend on the unknown flavor and CP structure of the NP couplings and thus we cannot draw any firm conclusion.

Parametrizing the amplitude $A_{\ell R}$ as $A_{\ell R} = c_{\ell R}/\Lambda^2$, where $\Lambda$ refers to the NP scale, we can evaluate which are the values of $\Lambda$ probed by $\mu \to e\gamma$. We find that

$$\text{BR}(\mu \to e\gamma) \approx 10^{-12} \left( \frac{500 \text{ TeV}}{\Lambda} \right)^4 \left( \frac{|c_{\mu\ell}|^2 + |c_{e\mu}|^2}{\Lambda^2} \right),$$ (8)

and therefore, for $c_{\mu\ell} \sim 1$ and/or $c_{e\mu} \sim 1$, we are left with $\Lambda > 500$ TeV.

Since the anomalous magnetic moment of the muon $a_\mu = (g - 2)_\mu/2$ exhibits a $\sim 3.5\sigma$ discrepancy between the SM prediction and the experimental value $[14] \Delta a_\mu = a^\text{EXP}_\mu - a^\text{SM}_\mu = 2.90(90) \times 10^{-9}$, it is interesting to monitor the implications for $\text{BR}(\ell \to \ell'\gamma)$ assuming that such a discrepancy is due to NP. In particular, we find that

$$\text{BR}(\mu \to e\gamma) \approx 10^{-12} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{\mu\ell}}{2 \times 10^{-3}} \right)^2,$$

$$\text{BR}(\tau \to \ell\gamma) \approx 10^{-8} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{e\tau}}{5 \times 10^{-3}} \right)^2.$$ (9)

where $\theta_{\gamma\ell} = \sqrt{|c_{\ell R}|^2 + |c_{e\mu}|^2/c_{\mu\ell}|^2}$.

Therefore, we learn that the $a_\mu$ anomaly can be accommodated while satisfying the $\text{BR}(\mu \to e\gamma)$ bound only for extremely small flavor mixing angles $\theta_{\gamma\ell}$.

Similarly, from eq. 7, we find that $d_e \approx 2 \times 10^{-24} \times [\text{Im}(c_{\mu\ell})/\text{Re}(c_{\mu\ell})] e cm$ whenever $\Delta a_\mu \approx 3 \times 10^{-8}$. Therefore, also the electron EDM exceeds the current experimental bound by many orders of magnitudes unless there exists a dynamical mechanism suppressing the relevant CP violating phases.

### 3 Specific NP Models

The phenomenology of cLFV observables has been worked out in a number of well motivated NP scenarios. Among the most important questions are (i) which are the best probes among cLFV processes for any given NP model, (ii) how the predictions compare with the
present/foreseen experimental bounds, (iii) what the constraints are on new sources of LFV and new-particle masses, (iv) what are the correlations among different LFV observables.

Concerning the latter point, it should be stressed that 1) ratios for branching ratios of processes such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$ would provide a direct access to the flavor structure of the NP model while 2) a comparative analysis of processes with the same underlying flavor transition (such as $\mu \to e\gamma$ and $\mu \to eee$) would provide information about the operators which are generating potential LFV signals.

In the following, we briefly discuss two classes of NP models: supersymmetric (SUSY) extensions of the SM and strongly interacting models based on the partial-componenesis paradigm.

3.1 SUSY models

In SUSY models, new sources for LFV stem from the soft SUSY-breaking sector since the lepton and slepton mass matrices are generally misaligned. The leading effects for cLFV processes arise from sneutrino-chargino and slepton-neutralino loops. In the generic MSSM, it is useful to stick to the mass-insertion approximation, assuming small off-diagonal entries in the slepton mass matrices $(\delta_{ij}^{a})_{i} = (m_{\tilde{L}_{i}}^{2} - m_{\tilde{L}_{j}}^{2})/m_{\tilde{L}_{i}}^{2}$, where $A, B = L, R$ and $m_{\tilde{L}}$ is an average slepton mass.

A scenario which has received particular attention after the discovery of the Higgs-like boson at the LHC is the so-called “disoriented A-terms” scenario [15]. The assumption of disoriented A-terms is that flavor violation is restricted to the trilinear terms

$$(\delta_{LR}^{ij})_{ij} \sim \frac{A_{f} \theta_{ij}^{f} m_{f}}{m_{f}^{2}}, \quad f = u, d, \ell,$$  

(10)

where $\theta_{ij}^{f}$ are generic mixing angles. This pattern can be obtained when the trilinear terms have the same hierarchical pattern as the corresponding Yukawa matrices but they do not respect exact proportionality. A natural realization of this ansatz arises in scenarios with partial compositeness [16], where also the SM flavor puzzle can be accounted for. Interestingly, the structure of eq. (10) allows us to naturally satisfy the very stringent flavor bounds of the down-sector thanks to the smallness of down-type quark masses. On the other hand, sizable A-terms help to account for a Higgs boson with mass around 125 GeV while keeping the SUSY scale not too far from the TeV.

The bounds from the lepton sector can be satisfied under the (natural) assumption that the unknown leptonic flavor mixing angles are of the form $\theta_{ij}^{\ell} \sim \sqrt{m_{L}/m_{f}}$ [16]. In particular, we get the following predictions [17]

$$BR(\mu \to e\gamma) \approx 6 \times 10^{-13} \left| \frac{A_{e}}{\text{TeV}} \right| \left( \frac{\theta_{12}^{e}}{\text{TeV}} \right)^{2} \left( \frac{m_{\tilde{L}}}{\text{TeV}} \right)^{2} e \text{ cm},$$

$$d_{e} \approx 4 \times 10^{-28} \text{ cm}^{2} \left( \frac{A_{e}}{\text{TeV}} \right) \left( \frac{\theta_{12}^{e}}{\text{TeV}} \right)^{2} \left( \frac{m_{\tilde{L}}}{\text{TeV}} \right)^{2} e \text{ cm},$$

$$\Delta_{a_{\mu}} \approx 1 \times 10^{-9} \left( \frac{m_{\tilde{L}}}{\text{TeV}} \right)^{2} \left( \frac{\tan \beta}{30} \right)^{2},$$

(11)

where we have assumed that the only possible sources of CP violation arise from A terms, as well. These estimates are fully confirmed by the numerical analysis shown in fig. 1 which has been obtained by means of the following scan: $0.5 \leq |A_{i}|/m \leq 2$ with $\sin \phi_{A_{i}} = 1$, $m \leq 2$ TeV, $(M_{2, \mu, M_{1}}) \leq 1$ TeV and $10 \leq \tan \beta \leq 50$ [17].

It is interesting that disoriented A-terms can account for $(g-2)_{\mu}$, satisfy the bounds on $\mu \to e\gamma$ and $d_{e}$, while giving predictions within experimental reach [17].

![Figure 1. Predictions of the disoriented A-term scenario [17]. Left: $\mu \to e\gamma$ vs. $\Delta a_{\mu}$. Right: $d_{e}$ vs. $\Delta a_{\mu}$.](image-url)
3.2 Composite Higgs Models

Besides low-energy supersymmetry, a class of attractive SM extensions addressing the gauge hierarchy problem is provided by composite Higgs models [18, 19], where fermion masses and mixing angles are described by partial compositeness [20]. Light fermions obtain hierarchical masses from the mixing between an elementary sector and a composite one.

As a toy model, let us consider, for each SM fermion, a pair of heavy fermions allowing a Dirac mass term of the order of the compositeness scale and a mixing term with the SM fields [21]

$$ L_Y = -\sum_{i,j=1}^{3} \left( \bar{L}_i \Delta_{ij} L_R j - \bar{E}_R j \tilde{\Delta}_{ij} \tilde{E}_L i \right) + h.c. $$

$$ -\sum_{i,j=1}^{3} \left( \bar{L}_i m_i L_R j + \bar{E}_R j \tilde{m}_i \tilde{E}_L i \right) $$

$$ -\sum_{i,j=1}^{3} \left( \bar{R}_i \varphi Y^R_{i,j} \tilde{E}_L j + \bar{L}_i \varphi \tilde{Y^R_{i,j}} \tilde{E}_R j \right) + h.c. $$

(12)

The first line represents the mixing between the elementary composite sectors, the second line contains Dirac mass terms for the fermions of the composite sector and the third line shows the Yukawa interactions which are restricted to the composite sector only with $1 \leq |Y^R_{ih1}| |Y^L_{i1}| \leq 4\pi$. By integrating out the composite sector under the assumption $m_i = m_i$, $\tilde{m}_i = \tilde{m}$ and $m, \tilde{m} \gg v$, we get the SM-like Yukawa interaction

$$ L_{Y}^{eff} = -\tilde{\ell}_{L} \varphi y_{L}^{SM} \tilde{e}_{R} + ... \quad y_{L}^{SM} = X Y^R \tilde{X}^\dagger, $$

(13)

where $X \equiv \Delta m^{-1}$, $\tilde{X} \equiv \tilde{\Delta} \tilde{m}^{-1}$ and dots stand for contributions of higher order in $v/m$. The remarkable feature of this pattern is that hierarchical fermion masses and mixing angles can be explained by the mixing matrices $X$ and $\tilde{X}$, even in the presence of anarchical matrices $Y^R_{i,j}$ and $\tilde{Y}^R_{i,j}$. At the one-loop level, summing over the $h, Z$ and $W$ amplitudes, we get the main contribution to the electromagnetic dipole operator:

$$ A_{\mu e} \sim \frac{1}{16\pi^2} \frac{1}{m \tilde{m}} \left( X Y^R_{1,1} Y^* L \tilde{X}^\dagger \right)_\mu e. $$

(14)

If $Y^R_{1,1}$ and $Y^L_{1,1}$ are anarchical matrices we see that, in general, $y_{L}^{SM}$ and $A_{\mu e}$ are not diagonal in the same basis. From the present bounds on $BR(\mu \to e\gamma)$ and $d_e$ we find that, with reasonable assumptions on the mixing matrices $X$ and $\tilde{X}$, we need $m(Y)$ and $\tilde{m}(Y)$ well above 10 TeV, $Y$ denoting an average absolute value of the Yukawa matrices. If we postulate that $Y^R_{1,1} = 0$, charged lepton masses do not vanish while the leading order contribution (14) does and we find [22]

$$ BR(\mu \to e\gamma) \approx 3 \times 10^{-13} \left( \frac{1.5\text{ TeV}}{m} \right)^8 |Y^R_{1,1}|^8, $$

(15)

where $Y^R_{1,1}$ now stands for an average element of the anarchic matrix $Y^R_{1,1}$, implying that $\mu \to e\gamma$ saturates its current experimental bound for $m \approx 1.5$ TeV and $Y^R_{1,1} \approx 1$. A quite similar behavior is expected for the electron EDM. Indeed, setting $Y^R_{1,1} = 0$ and assuming $O(1)$ CP-violating phases, it turns out that [22]

$$ \frac{|d_e|}{e} \approx 10^{-27} \text{ cm} \left( \frac{3\text{ TeV}}{m} \right)^4 |Y^R_{1,1}|^4. $$

(16)

Moreover, it turns $\mu \to 3e$ and $BR(\mu^- Au \to e^- Au)$ are dominated by non-dipole operators and the correlation of eq. (4) is significantly violated as it is explicitly shown in fig. 2. This is a relevant result, as within composite Higgs models with $Y^R_{1,1} \neq 0$, as well as in supersymmetric scenarios, eq. (4) holds to an excellent approximation. In particular, we find [22]

$$ BR(\mu \to 3e) \approx 5 \times 10^{-13} \left( \frac{1\text{ TeV}}{m} \right)^4 |Y^R_{1,1}|^2, $$

(17)

$$ BR(\mu^- Au \to e^- Au) \approx 4 \times 10^{-13} \left( \frac{3\text{ TeV}}{m} \right)^4 |Y^R_{1,1}|^2. $$

(18)

![Figure 2. Branching ratio of $\mu \to 3e$ (left) and $\mu^- Au \to e^- Au$ (right) versus the branching ratio of $\mu \to e\gamma$ for $Y^R_{1,1} = 0$. The case of dominance of the dipole operator is shown in yellow.](image)
and therefore $\mu^- Au \rightarrow e^- Au$ is a better probe than $\mu \rightarrow 3e$ of the scenario in question.

4 Conclusions

Despite of the fact that the origin of flavor remains a major open problem, significant progress has been achieved in the phenomenological investigation of the sources of flavour symmetry breaking which are accessible at low energies, ruling out models with significant misalignments from the SM Yukawa couplings at the TeV scale.

The search for LFV in charged leptons is probably the most interesting goal of flavour physics in the next years (see table 1). The observation of neutrino oscillations has clearly demonstrated that lepton flavour is not conserved. The question is whether LFV effects can be visible also in other sectors of the theory. The most promising LFV low-energy channels are probably $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$, $\mu \rightarrow e$ conversion in Nuclei as well as $\tau$ LFV processes. The current experimental resolutions on cLFV processes are already testing territories of new physics (NP) models well beyond the LHC reach. On the other hand, with the expected sensitivities of next-generation experiments, cLFV will become the most powerful probe of NP signals at our disposal and the interrelationship among leptonic $g-2$, EDMs and cLFV will be of outmost importance to disentangle among different NP scenarios.

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References