The muon $g - 2$ and degenerate supersymmetry

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Abstract. A degenerate supersymmetric particle spectrum can escape constraints from flavor physics and at the same time evade limits from the direct searches. If such a spectrum is light enough, it can also account for the observed value of the anomalous magnetic moment of the muon. Inspired by this, we consider a scenario where all the soft terms have approximately a common mass scale while allowing for small splittings. We study this scenario considering the constraints from Higgs mass, various $B$ meson decays and the dark matter relic density. We find that, with superpartners ~ 800 – 1000 GeV, it is still possible to escape the present limits from the first run of LHC and flavor physics and can account for muon $g - 2$ within $2\sigma$.

1 Introduction

Low energy Supersymmetry (SUSY) has been considered as one of the most compelling solutions of the problem of naturalness of the electroweak scale. It provides dynamical origin of electroweak symmetry breaking and stabilizes the Higgs mass parameter against the large quantum corrections. In its simplest realization, this feature strongly prefers the existence of SUSY breaking scale below TeV. While the Large Hadron Collider (LHC) has conclusively established an existence of the Standard Model (SM) like Higgs boson with mass 126 GeV [1, 2], it hasn’t indicated any statistically significant signal of new physics yet.

The mass spectrum of sparticles is not clearly known. Because of the presence of several ideas of SUSY breaking and its mediation to the SM superpartners, no clear prediction for sparticle masses can be made. The SUSY mass spectrum is mostly discussed in the context of rather simplified models such as minimal supergravity or the constrained minimal supersymmetric standard model (CMSSM) or some extended versions of it. These models typically generate a widely spread SUSY mass spectrum that leads to experimentally detectable large visible or missing energies. Both the ATLAS and CMS detectors at the LHC have put significant constraints on these kind of spectrum [3, 4]. The gluino and squarks as heavy as 1.8 TeV have been ruled out by inclusive searches from the data collected with integrated luminosity 20.3/fb with 8 TeV center of mass energy by ATLAS [3]. Further in generic models, squarks up to 850 GeV and gluino up to 1.3 TeV have been ruled out if the lightest supersymmetric particle (LSP) is assumed to be massless [5]. There exist relatively weaker constraints on sleptons and electroweak gauginos due to their small production cross-sections at the LHC. In conclusion, the low energy SUSY with wide mass spectrum is seemingly pushed to a corner by the LHC and, according to some authors, losing its appeal as a solution to the hierarchy problem: see however Ref. [6–8], and references therein.

A very interesting possibility to evade the strong LHC bounds on low energy SUSY is to consider all the particles nearly degenerate in masses. The degeneracy among the sparticles and conservation of R-parity makes them hidden from the experimental searches because of small missing and/or visible energy release in sparticle decays. If the sparticle spectrum is compressed, in particular if the mass difference between the colored superpartners such as the squarks and gluino and LSP is very small, there will be less energetic jets/leptons/missing transverse energy leading to low detection efficiency. Such a weak signal would be hidden below the SM backgrounds. Indeed, there exist relatively weak constraints on these kind of spectrum from LHC run-I. For example, the lower limits on stop reduces to 300 GeV if the mass difference between stop and LSP is lower than the top quark mass [3]. A similar reduction on gluino mass constraint holds in case of compressed spectrum.

The phenomenological implications of degenerate SUSY spectra has been discussed in details in [9, 10]. It has been mostly studied from the context of direct searches and collider physics experiments. In [11], we study various indirect constraints and implications of (approximately) degenerate SUSY spectrum. Specifically, we
consider the bounds from B meson decays, Higgs mass and muon anomalous magnetic moment, i.e. \((g - 2)\), and their implications on compressed SUSY spectrum. Here, we mostly focus on the SUSY resolution of the \((g - 2)\) anomaly. At present, the discrepancy between the SM prediction \(a_\mu\) and experimental measurements \(a_\mu\) of muon anomalous magnetic moment is given by [12]

\[
\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (2.87 \pm 0.8) \times 10^{-9},
\]

where \(a_\mu \equiv \frac{1}{2}(g - 2)_\mu\). Clearly, there is more than 3\(\sigma\) deviation between the SM prediction and experimental observation in the value of \(a_\mu\). In this talk, we assume SUSY as the solution of this discrepancy and examine phenomenologically viable SUSY spectrum which can account for the muon magnetic moment anomaly.

2 Degenerate Supersymmetry

Before we discuss the various constraints on the degenerate SUSY, we briefly discuss the possible model origin of SUSY breaking for compressed SUSY spectra. Note that one requires degeneracy between only colored sparticles and the LSP in order to evade the most stringent LHC constraints. However models leading to such degeneracy frequently imply degeneracy among the complete spectrum including sleptons and electroweak gauginos. A well motivated class of models leading to such compressed spectrum are those where the SUSY breaking by extra spatial dimension based on the Scherk-Schwarz mechanism [13]. An example of such model is described in [14]. Another example is compactified on an orbifold \(S^1/Z_2\). The 5D N=1 (or equivalently, N=2 in 4D) SUSY is completely broken by \(Z_2\) parity and a non-trivial twist, parametrized by twist parameter \(\alpha\). One can also use twisting to break \(SU(2)_R\) global symmetry of two Higgs doublets. Denoting this twisting by parameter \(\gamma\), one obtains complete MSSM spectrum in terms of only three parameters: \(\alpha, \gamma\) and the compactification scale \(1/R\). At the leading order, the soft SUSY breaking terms are given as:

\[
M_1 = M_2 = M_3 = \frac{\alpha}{R},
\]

\[
m_{H_u}^2 = m_{H_d}^2 = m_Q^2 = m_U^2 = m_D^2 = m_{\tilde{L}}^2 = m_{\tilde{E}}^2 = \left(\frac{\alpha}{R}\right)^2,\]

\[
A = -2\frac{\alpha}{R} \mu = \frac{\gamma}{R} \mu B = -\frac{2\alpha \gamma}{R^2}.
\]

Here \(M_{1,2,3}\) represents the gaugino masses and \(m_i\) are the soft masses of various scalars in the MSSM. Note that the twisting which breaks SUSY is purely a geometrical effect and does not distinguish among flavours, colours and even between the gauginos and scalars. Further, \(\alpha\) and \(\gamma\) are real parameters and do not introduce SUSY CP and flavour effects. The renormalization group effects between the mediation scale \(1/R\) and SUSY breaking scale \(\alpha/R\) remains small if they are not very far from each other or \(\alpha/R \approx 1/R \approx \text{TeV}\). There exists also other variants of the above model with additional freedom among the Higgs sector parameters where the conditions in Eqs. (2,3,4) get modified depending on the location of the matter, gauge and Higgs fields in the extra dimension.

We now review the implications of degenerate soft masses on the physical mass spectrum of Degenerate Minimal Supersymmetric Standard Model (DMSSM). Adopting a phenomenological approach, we set a common mass scale \(M_D\) for the soft masses for gauginos, squarks and sleptons.

\[
M_1 \approx M_2 \approx M_3 \equiv M_D,
\]

\[
m_Q^2 \approx m_U^2 \approx m_D^2 \approx m_{\tilde{L}}^2 \approx m_{\tilde{E}}^2 \equiv M_D^2.
\]

We also parametrize

\[
|m_t| = k_{\mu} M_D^2, \quad \text{and} \quad m_{\tilde{X}}^2 = k_{\Lambda} M_D^2,
\]

where \(k_{\mu,\Lambda}\) are real and positive and are \(O(1)\) parameter while \(M_D\) is the mass of pseudoscalar Higgs. Note that the above relations can arise from the conditions in Eqs. (2,3,4) using specific choices of \(\alpha, \gamma\) and taking the radiative corrections into account.

The physical masses of first and second generations of sfermions turn out to be degenerate with \(M_D\) while the third generations of squarks and sleptons receive large corrections form large trilinear terms. The chargino and neutralino mass spectrum also remains degenerate with \(M_D\) if \(k_{\mu} = O(1)\). The Higgs spectrum implies one light CP even neutral Higgs which should be identified with the observed signal of Higgs boson while the other CP even state, a CP odd and the charged Higgs masses turn out to be \(\sqrt{2}k_{\mu} M_D\). For more details of physical mass spectrum of DMSSM, we refer reader to [11].

3 Muon \((g - 2)\) in DMSSM

The SUSY contributes to \((g - 2)_\mu\) at one loop through dominant neutralino-charge slepton and chargino-sneutrino loops. It is estimated as [15]:

\[
\Delta a_\mu = \frac{a m_\mu^2}{4\pi \cos^2 \theta_W (m_R^2 - m_\tilde{\tau}_1^2)} \frac{f_2(M_2^2/m_{\tilde{\tau}_1}^2) - f_2(M_1^2/m_{\tilde{\tau}_1}^2)}{m_{\tilde{\tau}_1}^2} + \frac{\alpha m_\mu^2}{4\pi \sin^2 \theta_W} \frac{f_2(M_2^2/m_{\tilde{\tau}_1}^2) - f_2(m_{\tilde{\tau}_1}^2/m_{\tilde{\tau}_1}^2)}{M_2^2 - \mu^2}.
\]

(8)

The loop integration function are given by

\[
f_2[x] = \frac{x^2 - 4x + 3 + 2\ln(x)}{(1 - x)^3} \xrightarrow{x \to 0} \lim_{x \to 0} f_2[x] = -\frac{2}{3},
\]

\[
f_3[x] = \frac{x^2 - 2x - 3 + \ln(x)}{(1 - x)^3} \xrightarrow{x \to 0} \lim_{x \to 0} f_3[x] = -\frac{1}{3}.
\]

(9)

Assuming the degeneracy \(M_1 \approx M_2 \approx m_{\tilde{\ell}} \approx m_{\tau} \approx M_D\), Eq. (8) gets simplified to

\[
\Delta a_\mu = \frac{a m_\mu^2}{4\pi \sin^2 \theta_W} \frac{\tan \beta}{M_D^2} \left(\frac{1}{(1 - x_r)^2} - \frac{2}{3} f_2(x_r^2) + \frac{1}{3\cot^2 \theta_W} \right)
\]

(10)

with \(x_r = \mu/M_D\). We choose the sign of \(\mu\) to be positive to account for the correct \(\Delta a_\mu\). Note that \(\mu = M_D\) is required by the complete degeneracy of neutralino and
in the presence of nonzero \( \epsilon \), as required by Eq. (1) at different levels of statistical significance. Our purpose here is only to delineate the ranges of parameters preferred by the magnetic moment constraint. As expected, low values of \( m_D \) (and hence, of sparticle masses) 300-500 GeV (400-1000 GeV) for low (high) values of \( \tan \beta \) are preferred at the 2\( \sigma \) level. Note also that the constraint weakens considerably if we allow for deviations up to 3\( \sigma \). We emphasize that although the direct constraints on squark and gluino masses are satisfied, much of the favoured parameter ranges in Fig. 1 are not viable because they do not yield the correct value of Higgs mass or violate other low energy constraints, as discussed in the next section.

4 Other indirect constraints on DMSSM

Next, we consider the constraints arising from the Higgs mass and most relevant \( B \) meson decays, such as \( B \to X_s \gamma \) and \( B \to \mu^+ \mu^- \) on the DMSSM. As it is well known, the tree level value of Higgs mass in MSSM is restricted to be \( \lesssim M_Z \) and large radiative corrections are needed to account for the observed Higgs mass. The dominant stop, sbottom and stau 1-loop correction to Higgs mass in the large \( \tan \beta \), limit is given by [16]

\[
\begin{align*}
    m_h^2 &= m_Z^2 \cos^2 2\beta + \delta m_h^2, \\
    \delta m_h^2 &= \frac{3}{4\pi^2} \frac{\tan \beta}{\epsilon_b} \frac{\mu^4}{\delta m_b^2} \left( 1 + \epsilon_b \tan \beta \right) m_b^2 \\
    &- \frac{1}{4\pi^2} \frac{\tan \beta}{\epsilon_b} \frac{\mu^4}{\delta m_b^2} \left( 1 + \epsilon_b \tan \beta \right) m_b^2 \\
    &\approx \frac{3}{4\pi^2} \frac{\tan \beta}{\epsilon_b} \frac{\mu^4}{\delta m_b^2} \left( 1 + \epsilon_b \tan \beta \right) m_b^2. \tag{11}
\end{align*}
\]

where \( X_t = A_t - \mu \cot \beta \). The various \( \epsilon \) parameters and \( M_{\text{SUSY}} \) are given in [11] in the limit of degenerate SUSY mass spectrum.

The window 124.4-125.8 GeV in Higgs mass is allowed by experimental measurements at 3\( \sigma \) [17]. In addition to this, we allow \( \pm 2 \text{ GeV} \) as a theoretical uncertainty in the estimation of the Higgs mass due to the simplified 1-loop expression we use. Therefore the conservative Higgs mass range we consider is 122.4 - 127.8 GeV.

We evaluate 1-loop contribution to \( B \to X_s \gamma \) in the DMSSM. It is parametrized as:

\[
R_{bsy} = \frac{\text{BR}(B \to X_s \gamma)}{\text{BR}(B \to X_s \gamma)_{\text{SM}}} = 1 - 2.45 C_{\gamma}^{NP} - 0.59 C_{s}^{NP}. \tag{12}
\]

Simplified expressions of the Wilson coefficients \( C_{7,8}^{NP} \) in the limit of degenerate SUSY spectrum are also given in [11]. In the presence of nonzero \( A_t \) (as it is required by Higgs mass) the largest contribution arises from the Higgsino-stop loop and it leads to large flavour violation.
The allowed room for new physics in this channel is [16]

\[ R_{\text{Bay}} = 1.02 \pm 0.10 . \]  \hspace{1cm} (13)

We also impose the \( B_s \to \mu^+ \mu^- \) constraints whose recent measurement at LHCb is in good agreement with the SM predictions and, therefore, leaves very little room for new physics. The DMSSM contribution to this channel is very sensitive to pseudoscalar Higgs mass and it is largest when \( k_A \leq 1 \), i.e. when the pseudoscalar Higgs is degenerate with SUSY particles.

After imposing all these constraints together with \((g-2)_\mu\), the results are shown in Fig. 2. For this analysis we consider \( k_\mu \approx k_A \approx 1 \) and various constraints are displayed in \( A_t - M_D \) plane for two values of \( \tan \beta \). As can be seen, large values of \( A_t \) is essentially required for correct Higgs mass. On the contrary, such large \( A_t \) introduces large flavour violations particularly in \( B \to X_s \gamma \) channel through stop-Higgsino loop. There exist a tiny room for \( M_D \approx 800 - 1000 \) GeV in case of large \( \tan \beta \) and \( A_t/M_D \approx 1.5 - 1.8 \) which can solve the current discrepancy in \((g-2)_\mu\) at 2\( \sigma \) being consistent with other constraints considered in this paper. Large regions are possible if we relax the \((g-2)_\mu\) requirement to be satisfied at 3\( \sigma \). Note though, that the size of the grey regions excluded by flavour constraints will be sensitive to our choice of \( k_A \) and \( k_\mu \).

5 Summary

The LHC constraints on SUSY can be evaded if the particle spectrum is approximately degenerate in masses. We show that degenerate sparticles as light as 800 - 1000 GeV can remain hidden from collider searches and can provide a viable solution to the discrepancy between the SM prediction and experimentally observed value of muon anomalous magnetic moment, also respecting various indirect constraints such as the Higgs mass and most relevant \( B \) meson decays. As we have shown using simplified analytical estimation, large \( A_t \) would be needed to reproduce the correct Higgs mass and large \( \tan \beta \) is favoured for large enough contribution to \((g-2)_\mu\). The \((g-2)_\mu\) at 2\( \sigma \) sets an upper limit on degenerate scale, \( M_D \leq 1 \) TeV and requires larger values of \( \tan \beta \).

We would like to emphasize that the results presented in this talk are based on simplified analytical formulas and assuming complete degeneracy in the soft masses. There can be various effects which can introduce small departure from the exact degeneracy in soft masses at weak scale. We therefore also perform a full numerical analysis introducing small deviation from the degeneracy and estimate various observables numerically using full 1-loop calculations. Also, we allow \( m_A \) to be a free parameter. Our preliminary results of such analysis are in qualitative agreement with the simplified analysis presented in this talk. However, there are significant quantitative changes mainly because of small deviations introduced from the exact degeneracy limit. We also discuss in detail the sparticle spectrum and dark matter constraints. The complete results of numerical analysis will be reported in [11].

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References