NONUNIFORM ATMOSPHERIC SCATTER OF SHORT-PULSE OPTICAL RADIATION

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ABSTRACT

The results of calculation of the mean intensity of spatially partially coherent pulsed optical beam scattered by an atmospheric layer are presented. It is shown that in contrast to cw radiation, which scatters uniformly, the scatter of pulsed beam becomes nonuniform for shorter pulses. For the femtosecond pulse duration the backscattered radiation is localized near the strictly backward direction in the area with transverse dimensions comparable with the initial size of the sensing beam.

1. INTRODUCTION

The diffractive broadening of broadband pulsed optical beams as a function of the pulse duration has been studied in [1-5], and it has been shown that as the pulse duration decreases, the diffractive broadening of the beam becomes smaller than that of continuous wave (cw) beams. In the limit of zero pulse duration (δ - pulse), the beam diffractive broadening does not occur at all [3-5]. In lidar problems the atmospheric scatter of laser radiation is one of the key elements [6, 7]. In this paper, the scatter of short pulse optical beams by an atmospheric layer is analyzed.

2. METHODOLOGY

Let a laser beam of pulsed radiation with the initial complex amplitude distribution $U(0,\boldsymbol{\rho}',t)$ after propagation of the path L in the direction of the Z axis in the atmosphere is scattered by an atmospheric layer. The backscattered radiation is received by a photodetector in the plane $Z=0$.

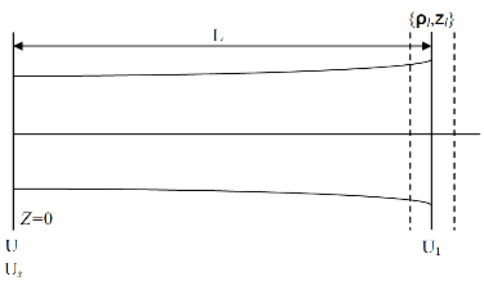

Fig. 1. Propagation and scattering geometry.

The complex amplitude of the field incident onto a scattering particle can be written in the form

$$U_1(z_l,\boldsymbol{\rho}_l,t)=\int d^2\boldsymbol{\rho}' U\left(0,\boldsymbol{\rho}',t-\frac{z_l}{c}\right)G(0,\boldsymbol{\rho}';z_l,\boldsymbol{\rho}_l), \quad (1)$$

where

$G(0,\boldsymbol{\rho}';z_l,\boldsymbol{\rho}_l)=\dfrac{ke^{ikz_l}}{2\pi i z_l}\exp\left\{i\dfrac{k}{2z_l}\left(\boldsymbol{\rho}_l-\boldsymbol{\rho}'\right)^2\right\}\exp(i\psi_1)$ is the Green's function of the parabolic equation for the complex amplitude of the field propagating in free space from point $(0,\boldsymbol{\rho}')$ to point $(z_l,\boldsymbol{\rho}_l)$, ψ_1 – is a random phase of the field due to turbulence, z_l is the longitudinal coordinate of the scattering particle, $\boldsymbol{\rho}_l=\{x,y\}$ is the radius vector determining its position in the transverse plane, $k=2\pi/\lambda=\omega/c$, λ is wavelength, ω is frequency, c is the speed of light, t is time, i is imaginary unit. The initial field of the sensing pulsed beam satisfies the condition

$$U(0,\boldsymbol{\rho}',t)=U_0(\boldsymbol{\rho}')A(t). \quad (2)$$

The complex amplitude of the field scattered by the particle, in the plane $Z=0$ can be written as

$$U_{sl}(0,\boldsymbol{\rho},t)=\alpha_l U_1\left(z_l,\boldsymbol{\rho}_l,t-\frac{2z_l}{c}\right)G\left(z_l,\boldsymbol{\rho}_l;0,\boldsymbol{\rho}\right)e^{i\psi_2}, \quad (3)$$

where $G\left(z_l,\boldsymbol{\rho}_l;0,\boldsymbol{\rho}\right)$ is the Green's function and ψ_2 is a turbulent phase for the backward propagation from the scattering particle with coordinates $(z_l,\boldsymbol{\rho}_l)$ to point $(0,\boldsymbol{\rho})$ in the receiving plane, α_l is the backscattering amplitude.

The total field in the plane $(0,\boldsymbol{\rho})$ can be found through summation of the right-hand side of Eq.(3) over all scattering particles N_s in the layer

$$U_s=\sum_{l=1}^{N_s}U_{sl}. \quad (4)$$

According to [3-5], the strength of the electric field $E_1(z_l,\boldsymbol{\rho}_l,t)=e^{i\omega_0 t}U_1(z_l,\boldsymbol{\rho}_l,t)$ incident on the particle can be written in the form

$$E_1(z_l,\boldsymbol{\rho}_l,t)=\int d\omega\, U(z_l,\boldsymbol{\rho}_l,\omega)\exp\left\{-i(\omega-\omega_0)t\right\}, \quad (5)$$

where

$$U(z_l,\boldsymbol{\rho}_l,\omega)=\int d\boldsymbol{\rho}'\, U(0,\boldsymbol{\rho}')G(0,\boldsymbol{\rho}';z_l,\boldsymbol{\rho}_l), \quad (6)$$

$$U(0,\boldsymbol{\rho}',\omega)=\frac{1}{2\pi}U_0(\boldsymbol{\rho}')\int dt\, e^{i\omega t}A(t)=P(\omega)U_0(\boldsymbol{\rho}'),\quad (7)$$

ω_0 is the carrier frequency.

$$A(t)=\int d\omega\, P(\omega)e^{-i\omega t}. \quad (8)$$

Upon the use of Eqs. (3) and (5)-(7) in Eq. (4), we obtain the following equation for the strength of the electric field scattered by an atmospheric layer $E_s(0,\boldsymbol{\rho},t)=e^{i\omega_0 t}U_s(0,\boldsymbol{\rho},t)$ in the plane $Z=0$:

$$E_s(0,\boldsymbol{\rho},t)=\sum_{l=1}^{N_s}\alpha_l\int d\omega\, P(\omega)\exp\{-i(\omega-\omega_0)t\}$$
$$\times\int d\boldsymbol{\rho}'U_0(\boldsymbol{\rho}')G(0,\boldsymbol{\rho}';z_l,\boldsymbol{\rho}_l)G(z_l,\boldsymbol{\rho}_l;0,\boldsymbol{\rho})e^{i\psi_1+i\psi_2}. \quad (9)$$

For a thin scattering layer (short pulse) $|z_l-L|<<L$, we can take $z_l\approx L$ in Eq. (3), keeping only the dependence on z_l in the fast oscillating factor e^{ikz_l} [8]. Then the Green's functions in Eq. (9) take the form

$$G(0,\boldsymbol{\rho};z_l,\boldsymbol{\rho}_l)=\frac{ke^{ikz_l}}{2\pi iL}\exp\left\{i\frac{k}{2L}(\boldsymbol{\rho}-\boldsymbol{\rho}_l)^2\right\}. \quad (10)$$

Equation (9) allows us to write the following equation for the function of mutual coherence of the electric field strength of the partially coherent pulsed radiation $\Gamma_2(0,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,t_1,t_2)=\langle E_s(0,\boldsymbol{\rho}_1,t_1)E_s^*(0,\boldsymbol{\rho}_2,t_2)\rangle$ scattered by an atmospheric layer:

$$\Gamma_2(0,\boldsymbol{\rho}_{1,2};t_{1,2})=\sum_{l=1}^{N_s}\sum_{l'=1}^{N_s}\alpha_l\alpha_{l'}\int d\omega_{1,2}\langle P(\omega_1)P^*(\omega_2)\rangle$$
$$\times\exp\{-i(\omega_1-\omega_0)t_1+i(\omega_2-\omega_0)t_2\} \quad (11)$$
$$\times\int d\boldsymbol{\rho}'_{1,2}\langle U_0(\boldsymbol{\rho}'_1)U_0^*(\boldsymbol{\rho}'_2)\rangle G(0,\boldsymbol{\rho}'_1;z_l,\boldsymbol{\rho}_l)G(z_l,\boldsymbol{\rho}_l;0,\boldsymbol{\rho}_1)$$
$$\times G^*(0,\boldsymbol{\rho}'_2;z_{l'},\boldsymbol{\rho}_{l'})G^*(z_{l'},\boldsymbol{\rho}_{l'};0,\boldsymbol{\rho}_2)\langle e^{i(\psi_1+\psi_2-\psi_3-\psi_4)}\rangle,$$

where according to Eq. (7)

$$\langle P(\omega_1)P^*(\omega_2)\rangle=\frac{1}{(2\pi)^2}\int dt_{1,2}\langle A(t_1)A^*(t_2)\rangle$$
$$\times\exp\{i[(\omega_1t_1)-(\omega_2t_2)]\}, \quad (12)$$

the angular brackets $\langle ...\rangle$ denote averaging over an ensemble of random realizations. At the equal coordinates $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\mathbf{R}$, Eq. (11) describes the intensity distribution of the scattered radiation in the source plane $Z=0$.

According to [9], for the arbitrary probability distribution of the initial position of scattering particles in space, the terms with $l\neq l'$ in Eq. (11) can be neglected because of the fast oscillating terms e^{ikz_1} in the Green's functions (10), if the scale of variation of the probability density function far exceeds the wavelength of the scattered field. The scales of turbulent inhomogeneities of the velocity field in the atmosphere far exceed the wavelength of optical waves, so we can use this approximation and restrict our consideration to only the terms with $l=l'$ in summation in Eq. (11). That is, the right-hand side of Eq. (11) $\sum_{l=1}^{N_s}\sum_{l'=1}^{N_s}\alpha_l\alpha_{l'}\Phi(z_l,z_{l'};\boldsymbol{\rho}_l,\boldsymbol{\rho}_{l'})$ takes the form $\sum_{l=1}^{N_s}|\alpha_l|^2\Phi(z_l,\boldsymbol{\rho}_l)$.

Then, for the averaging over microphysical factors, we can use the relation [10]

$$\left\langle\sum_{l=1}^{N_s}|\alpha_l|^2\Phi(z_l,\boldsymbol{\rho}_l)\right\rangle_a=$$
$$=\int_0^\infty dz_l\int d\boldsymbol{\rho}_l\rho_c(z_l)\sigma_\pi(z_l)\Phi(z_l,\boldsymbol{\rho}_l), \quad (13)$$

where $\rho_c(z_l)$ is the concentration of scattering particles, $\sigma_\pi(z_l)=\langle|\alpha_l|^2\rangle$ is the differential scattering cross section. Averaging over turbulent phase is carried out in accordance with known rules [6, 11]. But, as to impact of turbulence, we will neglect it. In what follows we will consider limiting case of "far zone", where the Fresnel number $\Omega_0=(k_0a^2)/L$ of transmitting aperture with radius a for the carrier frequency $k_0=\omega_0/c$ is much less than unity and turbulent broadening of sensing beam, at least, on inclinated and height paths is less than diffractive one [11]. As a result, Eq. (11) acquires the form

$$\Gamma_2(0,\boldsymbol{\rho}_{1,2};t_{1,2})=\int_0^\infty dz_l\rho_c(z_l)\sigma_\pi(z_l)\int d\omega_{1,2}\langle P(\omega_1)P^*(\omega_2)\rangle$$
$$\times\exp\{i(\omega_1-\omega_0)t_1+i(\omega_2-\omega_0)t_2\}\int d\boldsymbol{\rho}_l$$
$$\times\int d\boldsymbol{\rho}'_{1,2}\langle U_0(\boldsymbol{\rho}'_1,\omega_1)U_0^*(\boldsymbol{\rho}'_2,\omega_2)\rangle G(0,\boldsymbol{\rho}'_1;z_l,\boldsymbol{\rho}_l)$$
$$\times G(z_l,\boldsymbol{\rho}_l;0,\boldsymbol{\rho}_1)G^*(0,\boldsymbol{\rho}'_2;z_{l'},\boldsymbol{\rho}_{l'})G^*(z_{l'},\boldsymbol{\rho}_{l'};0,\boldsymbol{\rho}_2). \quad (14)$$

In Eq. (14), we can use the Gauss models for the functions of spatial and temporal correlation of the initial field [12, 13]

$$\langle A(t_1)A(t_2)\rangle=A_0^2\exp\left\{-\frac{(t_1+t_2)^2}{2T^2}-\frac{(t_1-t_2)^2}{T_0^2}\right\}, \quad (15)$$

$$\langle U_0(\boldsymbol{\rho}'_1)U_0^*(\boldsymbol{\rho}'_2)\rangle=U_0^2\exp\left\{-\frac{\boldsymbol{\rho}_1'^2+\boldsymbol{\rho}_2'^2}{2a^2}-\frac{(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)^2}{\rho_0^2}\right\}. \quad (16)$$

where $\frac{1}{T_0^2}=\frac{1}{4T^2}+\frac{1}{T_c^2}$, T is the pulse duration, T_c is the coherence time of the source field, a is radius, F is the wavefront curvature radius, and

ρ_0 is the radius of spatial coherence of the beam field in the initial plane.

Upon the integration over the thickness of the scattering layer [8], coordinates of the initial plane $\boldsymbol{\rho}_1'$ and $\boldsymbol{\rho}_2'$, and the plane of the scattering layer $\boldsymbol{\rho}_l$ in Eq.(14), we obtain for $\Gamma_2(\mathbf{R},\boldsymbol{\rho},t,\tau)$, where $\mathbf{R}=\dfrac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2}$, $\boldsymbol{\rho}=\boldsymbol{\rho}_1-\boldsymbol{\rho}_2$, $t=(t_1+t_2)/2$, $\tau=t_1-t_2$, the expression in a form of a 2-D integral.

This expression allows one to obtain a formula for the mean intensity of the pulsed radiation scattered by an atmospheric layer $I(\mathbf{R})$, if put $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2$. In particular, for sensing beam with $T_0=T$ in a far zone $\Omega_0<<1$ we obtain the following equation for the mean intensity

$$I(\mathbf{R})=I(R)=\Gamma_2\left(R,0,t=\frac{L}{c},0\right)=$$
$$C\,\omega_0 T\int d\Omega_{1,2}\,\frac{\Omega_1^2-\Omega_2^2}{Q_{\Omega F}}\times \tag{17}$$
$$\times\exp\left\{-(\omega_0 T)^2\left[\Omega_2^2+\frac{1}{4}(\Omega_1+1)^2\right]+\frac{R^2}{a_g^2}\left[i\frac{\Omega_2}{\Omega_0}-\frac{\Omega_2^2 a_\rho}{\Omega_0^2 Q_{\Omega F}}\right]\right\},$$

where $C=\dfrac{1}{2}\beta_\pi(a/L)^4U_0^2A_0^2$, $\beta_\pi=\rho_c\sigma_\pi$ denotes the backscattering coefficient, $Q_{\Omega F}=\Omega_1^2+\Omega_2^2a_\rho-i2\Omega_2a_\rho\Omega_0^{-1}$, $a_\rho=4(a/\rho_0)^2+1$, $a_g=L/(k_0a)$.

Equation (17) allows us to calculate the mean intensity of the field of scattered beam for any pulse duration. At $\omega_0T\to\infty$, it transforms into the equation corresponding to cw radiation $I=\beta_\pi\left(a^4/L^4\right)U_0A_0/2$, where there is no dependence on the vector **R**, that is, the intensity of the scattered cw radiation is distributed uniformly over the transverse plane. In contrast to the cw radiation, the integrand (17) at the finite values of the parameter ω_0T, which characterizes the pulse duration, keeps the dependence on the spatial coordinate **R**. That is, the uniform scattering of the cw radiation becomes nonuniform in the case of the pulsed sensing radiation.

3. RESULTS

Figure 2 shows the intensity distributions of the beam with $\Omega_0=0.1$ scattered by the atmospheric layer. The calculations were based on Eq.(17) without regard for the constant *C*. It follows from the figure that the diffuse uniform scattering of cw optical radiation ($\omega_0T=10^8$) becomes nonuniform as the pulse duration decreases.

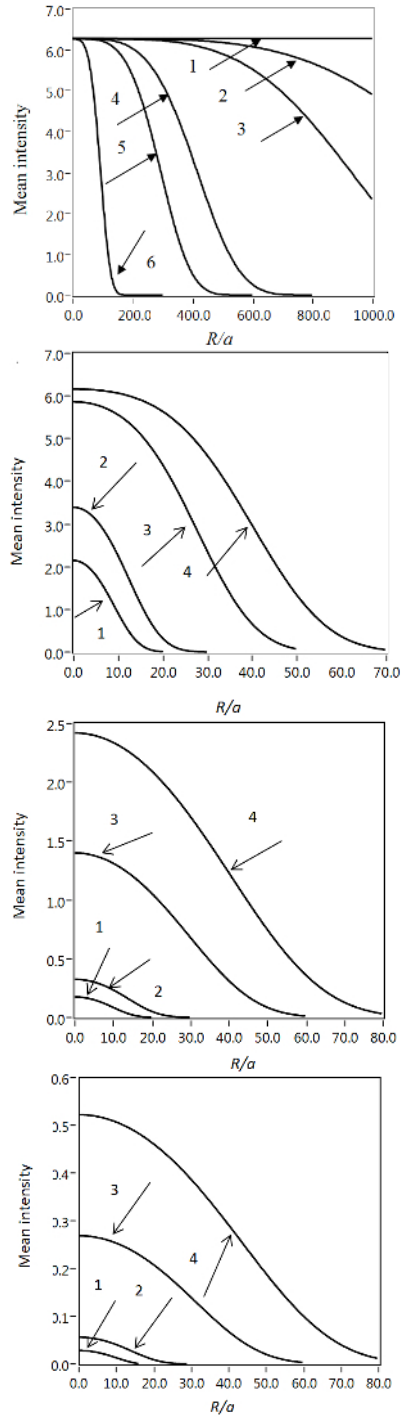

Fig. 2. Intensity distribution of the scattered collimated beam, Ω_0=0.1: a) $\omega_0T=10^8$ (1), 10^5 (2), $5\cdot10^4$ (3), 10^4 (4), $5\cdot10^3$ (5), 500(6); b), c), d) ω_0T=5 (1), 10 (2), 50 (3), 10^2 (4); $a/\rho_0=0$(a, b), 2(c), 5(d).

As the parameter $\omega_0 T$ decreases, the scattered radiation localizes in the smaller vicinity of the strictly backward direction (compare Figs.2a and 2b), and for femtosecond pulses $\omega_0 T = 5$ the scattered radiation is localized in the transverse plane near the axis of the sensing beam within a few its initial dimensions.

Deterioration of the initial spatial coherence of the pulsed sensing beam has no effect on the localization of the scattered radiation near the beam axis (Figs. 2c, 2d). In the full accordance with Eq. (19), as the radius of spatial coherence of the initial field ρ_0 decreases, only the absolute value of the mean intensity decreases, while the scale of its decrease in the transverse plane remains unchanged.

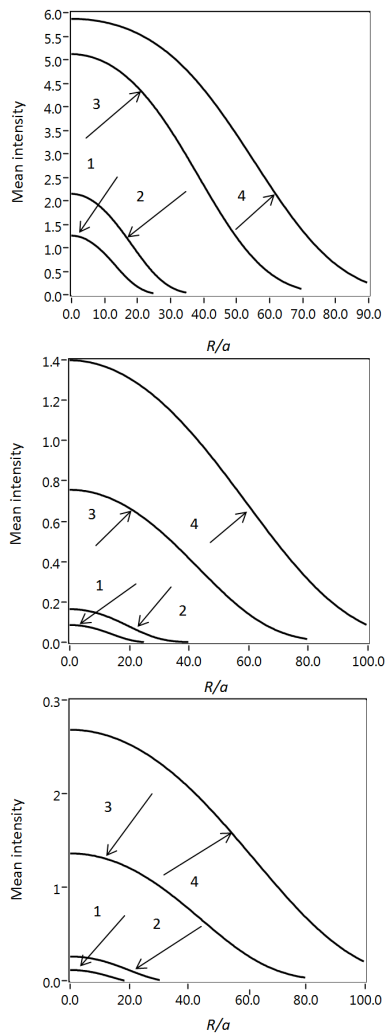

Fig. 3. Intensity distribution of the scattered collimated beam, $\Omega_0 = 0.05$: $\omega_0 T = 5$ (1), 10 (2), 50 (3), 10^2 (4); $a/\rho_0 = 0$(a), 2(b), 5(c).

The decrease of the Fresnel number of the transmitting aperture Ω_0 diminishes the effect of nonuniformity of scattering of the pulsed radiation (Fig.3).

It can be seen from Fig. 3 and Fig. 2 that for $\Omega_0 = 0.05$ and $\omega_0 T = 5$ the scattered radiation is localized near the axis of the sensing beam in the area with transverse dimensions approximately two times greater than that for $\Omega_0 = 0.1$.

Deterioration of the initial spatial coherence of sensing beam does not lead to changes of transverse dimensions of this area (Figs. 3b and 3c). This leads to only the significant decrease in the intensity of the scattered radiation.

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REFERENCES

[1] I.V. Zaloznaya, A.V. Falits, Atmospheric and Oceanic Optics **22**, pp.590-594 (2009).
[2] L.O. Gerasimova, I.V. Zaloznaya, Atmospheric and Oceanic Optics **24**, pp.185-189 (2011) [in Russian]
[3] V.A. Banakh, Opt. Lett. **36**, pp.4554-4556 (2011).
[4] V.A. Banakh, L.O. Gerasimova, I.V. Zaloznaya, O.V. Tikhomirova, Atmospheric and Oceanic Optics **26**, pp.185-189 (2013).
[5] V.A. Banakh, L.O. Gerasimova, Atmospheric and Oceanic Optics **26**, pp.5-10 (2013) [in Russian]
[6] V. A. Banakh, V.L. Mironov, "Lidar in a turbulent atmosphere", Artech House. Boston & London, 1987, 185 p.
[7] V. Banakh, I. Smalikho, "Coherent Doppler wind lidars in a turbulent atmosphere", Artech House. Boston-London. 2013. 250 p.
[8] V.A. Banakh, Atmospheric and Oceanic Optics **20**, pp.271-274 (2007).
[9] B. Crosignani, P. Di Porto, M. Bertolotti, "Statistical Properties of Scattered Light", Academic Press, New York-San Francisco-London, 1975, 206 p.
[10] A. Ishimaru, "Wave Propagation and Scattering in Random Media", Academic Press, 1978, 572 pp.
[11] V.E. Zuev, V.A. Banakh, V.V. Pokasov, "Optics of turbulent atmosphere", Leningrad, Gidrometeozdat, 1988, 270 p. [in Russian].
[12] I.P. Christov, Optica Acta **33**, pp.63-72 (1986).
[13] A.I. Kon, V.I. Tatarskii, Radiophysics and Quantum Electronics. **15**, pp.1187-1192 (1972).