

AIRCRAFT WAKE VORTEX PARAMETRIZATION BASED ON 1.5- μm COHERENT DOPPLER LIDAR DATA

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ABSTRACT

A strategy of measurement by a 1.5- μm pulsed coherent Doppler lidar “Stream Line” has been developed, and a method for estimation of aircraft wake vortices from the lidar data has been proposed. The principal possibility of obtaining the information about the vortex situation over an airport airfield with the Stream Line lidar has been demonstrated.

1. INTRODUCTION

An aircraft flight is always accompanied by the formation of a pair of vortices. An aircraft wake can extend to many kilometers and constitute a danger for other aircrafts. A pulsed coherent Doppler lidar (PCDL) is now the best tool for the experimental study of wake vortices. The 2- μm PCDL is used most widely for measurement of wake vortex parameters (vortex axis coordinates and circulation) [1]. These parameters were estimated from lidar data by different methods, which are described in [2-5].

The necessary condition for lidar measurements of aircraft vortex parameters is the rather high spatiotemporal resolution of raw data and the high signal-to-noise ratio (SNR), which can be provided by the 2- μm PCDL in the atmospheric boundary layer. In contrast to the 2- μm PCDL [6], the Stream Line 1.5- μm PCDL developed by HALO Photonics [7] has the 20 times lower probing pulse energy [8]. Therefore, for the Stream Line lidar, the SNR defined as the ratio of the mean power of lidar return signal to the mean power of noise in the frequency band of 50 MHz at the same concentration of aerosol particles is at least several tens times lower than SNR of the 2- μm PCDL.

In this paper, we study the possibility of obtaining the information about an aircraft wake from data measured by the Stream Line lidar. A measurement strategy is developed, and a method for estimation of aircraft vortex axis coordinates and circulation is proposed. The results of test

measurements by the Stream Line lidar at the air field of the Tomsk Airport are presented.

2. METHODOLOGY

To obtain the information about aircraft wake vortices from data of PCDL, the scanning of the probing beam in the vertical plane perpendicularly to the aircraft flight line is used during the measurements. Figure 1 shows the measurement geometry for the case of up-down beam scanning, that is, the elevation angle φ variable in time.

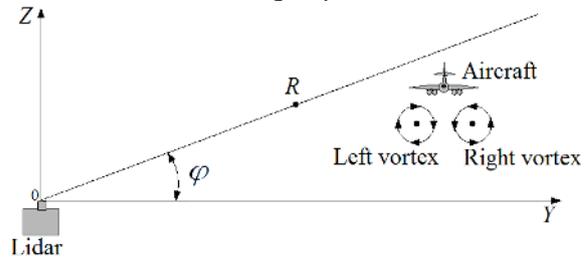


Fig. 1. Geometry of measurement by PCDL scanning in the vertical plane.

The raw data measured by the Stream Line lidar are an array of estimates of correlation functions of the normalized complex lidar signal $\hat{C}(lT_s, R_k, \varphi_m; n')$, where $l = 0, 1, \dots, 6$, $T_s = B^{-1}$ is the sampling interval, $B = 50$ MHz is the frequency bandwidth, $R_k = R_0 + k\Delta R$ is the distance from the lidar, $k = 0, 1, \dots, K - 1$, $\Delta R = cT_s / 2 = 3$ m, c is the speed of light, $\varphi_m = m\Delta\varphi$ is the elevation angle, $m = 0, 1, \dots, M - 1$, $\Delta\varphi = \omega_s \Delta T$ is the scanning angle resolution for the rate ω_s , $\Delta T = N_a / f_p$ is the time of measurement, N_a is the number of laser pulses used for accumulation [9], $f_p = 15$ kHz is the pulse repetition frequency, and $n' = 1, 2, 3, \dots$ is the scan number. If the lidar signal is normalized to the square root of the mean noise power, then the estimate of the signal-to-noise ratio can be determined as $\hat{\text{SNR}}(R_k, \varphi_m; n') = \hat{C}(0, R_k, \varphi_m; n') - 1$.

Using Eq. (5) from [9], we can obtain the array of estimates of Doppler spectrum $\hat{S}_D(V_l, R_k, \varphi_m; n')$

from $\hat{C}(IT_s, R_k, \varphi_m; n')$, where $V_{l'} = (l' - L'/2)\Delta V$, $l' = 0, 1, \dots, L' - 1$, L' is the number of spectral channels, which can be taken arbitrarily large by adding $L' - 7$ complex zeros to the array $\hat{C}(IT_s, R_k, \varphi_m; n')$, $\delta V = (\lambda/2)B/L'$, and $\lambda = 1.5 \mu\text{m}$ is the wavelength. Since SNR has very low values in the case of the Stream Line lidar, the estimates of spectral moments [2] and velocity envelopes [3] cannot be obtained from $\hat{S}_D(V_{l'}, R_k, \varphi_m; n)$ with the acceptable accuracy. Therefore, to extract the information about aircraft wake vortices from the data measured by this lidar, we obtained estimates of radial velocity $\hat{V}_r(R_k, \varphi_m; n')$ at the point of maximum of the Doppler spectrum, that is, $\max\{\hat{S}_D(V_{l'}, R_k, \varphi_m; n')\} = \hat{S}_D(\hat{V}_r, R_k, \varphi_m; n')$.

The focusing of the probing beam to some distance F allows the SNR in the vicinity of the focus to be increased significantly. Our numerous experiments for the case of the Stream Line lidar have shown that at $F = 300 \text{ m}$ the $\text{SNR}(R_k)$ is equal, on average, to 6 dB at the point of maximum $R_k = F$, to 14 dB at $R_k = 100 \text{ m}$, and to -16 dB at $R_k = 600 \text{ m}$. If in this case $N_a = 1500$ pulses are used for the accumulation of data, then for distances $100 \text{ m} \leq R_k \leq 600 \text{ m}$ the probability P_b of the bad estimate of the radial velocity does not exceed 10^{-4} and the bias of $\hat{V}_r(R_k, \varphi_m; n')$ is very close to zero.

Our studies have allowed us to determine the parameters optimal for measurements of vortices generated by a landing aircraft at a height lower 50 m by the Stream Line lidar: the distance between the lidar and the runway of about 300 m, $F = 300 \text{ m}$, $N_a = 1500$, $\omega_s = 2^0/\text{s}$, and $M = 76$. Taking into account that $f_p = 15 \text{ kHz}$, the measurement time is $\Delta T = 0.1 \text{ s}$, the scanning angle resolution is $\Delta\varphi = 0.2^0$, the maximum elevation angle is $\varphi_{M-1} = (M-1)\Delta\varphi = 15^0$, and the scan duration is $T_{\text{scan}} = M\Delta T = 7.6 \text{ s}$.

Let the array of lidar estimates of the radial velocity $\hat{V}_r(R_k, \varphi_m; n')$ contain the information about aircraft wake vortices starting from the scan number $n' = n_0 + 1$ and up to $n' = n_0 + N$. To avoid

the influence of the background wind, we obtain the data array

$$\hat{V}_r^{\Delta\varphi}(R_k, \varphi_m; n) = \hat{V}_r(R_k, \varphi_m; n_0 + n) - \hat{V}_r(R_k, \varphi_m; n_0), \quad (1)$$

where $n = n' - n_0 = 1, 2, \dots, N$. Figure 2(a) exemplifies the distribution $\hat{V}_r^{\Delta\varphi}(R_k, \varphi_m; 2)$ obtained from the data measured by the Stream Line lidar at the air field of the Tomsk Airport, as a B737 aircraft crossed the scanning plane at a height of 37 m. We can see that at distances from 250 to 330 m from the lidar, the pair of wake vortices affects the radial velocity.

To estimate coordinates of axes of the right and left wake vortices from the data analogous to those shown in Fig. 2(a), we introduce the functions $E(R_k; n)$, $\varphi_{\max}(R_k; n)$, and $\varphi_{\min}(R_k; n)$ determined by the following algorithm:

$$\max_{\varphi} \{\hat{V}_r^{\Delta\varphi}(R_k, \varphi_m; n)\} = \hat{V}_r^{\Delta\varphi}(R_k, \varphi_{\max}; n), \quad (2)$$

$$\min_{\varphi} \{\hat{V}_r^{\Delta\varphi}(R_k, \varphi_m; n)\} = \hat{V}_r^{\Delta\varphi}(R_k, \varphi_{\min}; n), \quad (3)$$

$$E(R_k; n) = [\hat{V}_r^{\Delta\varphi}(R_k, \varphi_{\max}; n)]^2 + [\hat{V}_r^{\Delta\varphi}(R_k, \varphi_{\min}; n)]^2, \quad (4)$$

where the maxima and minima fall within the angle range φ_m for every distance R_k .

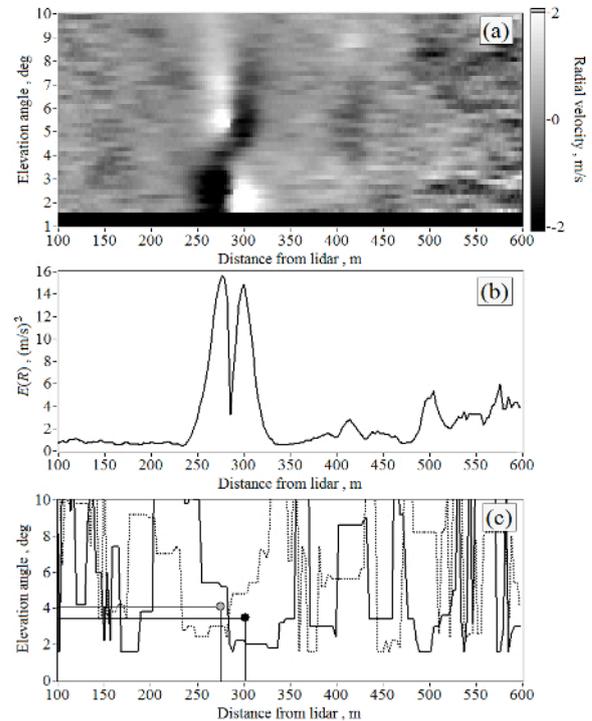


Fig.2. Distribution of the difference of estimates of radial velocities $\hat{V}_r^{\Delta\varphi}(R_k, \varphi_m; 2)$ (a) and the corresponding functions (b): $E(R_k; 2)$, (c): $\varphi_{\max}(R_k; 2)$ (solid curve) and $\varphi_{\min}(R_k; 2)$ (dashed curve).

Figures 2(b) and 2(c) show $E(R_k;2)$, $\varphi_{\max}(R_k;2)$, and $\varphi_{\min}(R_k;2)$ obtained from the data depicted in Fig. 2(a). We can see that $E(R_k;2)$ has two pronounced peaks. The results of our numerical investigation for the Stream Line lidar (when the longitudinal size of the sensing volume is around 30 m) indicate that the positions of the far and near peaks are almost identical with the distances between the lidar and the axes of the right $R_{C1}(n)$ and left $R_{C2}(n)$ aircraft vortices, while $[\varphi_{\max}(R_{C1};n) + \varphi_{\min}(R_{C1};n)]/2$ and $[\varphi_{\max}(R_{C2};n) + \varphi_{\min}(R_{C2};n)]/2$ are very close to the angular coordinates of the axes of the right $\varphi_{C1}(n)$ and the left $\varphi_{C2}(n)$ vortices, respectively.

The algorithm for estimation of coordinates of the axis of the i -th aircraft vortex ($i=1$ for the right vortex and $i=2$ for the left one) can be the following. For the n -th scan, we first find the position of absolute maximum of the function $E(R_k;n)$ in the entire range of distances $R_k \in [R_0, (K-1)\Delta R]$, that is, we use the procedure

$$\max_R \{E(R_k;n)\} = E(R_{\max};n). \quad (5)$$

Two vortices generated by an aircraft rotate in the opposite directions (see Fig.1). Consequently, if $\varphi_{\max}(R_{\max};n) > \varphi_{\min}(R_{\max};n)$, then $R_{\max} = R_{C2}$. Other-wise, $R_{\max} = R_{C1}$.

Assume that the condition $\varphi_{\max}(R_{\max};n) > \varphi_{\min}(R_{\max};n)$ is true. Then the coordinates of the axis of the left vortex $\{R_{C2}(n), \varphi_{C2}(n)\}$ in the polar coordinate system are determined as

$$\begin{aligned} \{R_{C2}(n) = R_{\max}, \varphi_{C2}(n) = \\ = [\varphi_{\max}(R_{\max};n) + \varphi_{\min}(R_{\max};n)]/2\}. \end{aligned} \quad (6)$$

The position of the second peak R'_{\max} , by analogy with Eq. (5), should be sought for in the range of distances $R_k \in [R_{\max} + b_0/2, (K-1)\Delta R]$, where $b_0 = (\pi/4)B_A$ is the theoretical initial distance between the vortex axes, and B_A is the aircraft wingspan [10], and then

$$\begin{aligned} \{R_{C1}(n) = R'_{\max}, \varphi_{C1}(n) = \\ = [\varphi_{\max}(R'_{\max};n) + \varphi_{\min}(R'_{\max};n)]/2\}. \end{aligned} \quad (7)$$

If $\varphi_{\max}(R_{\max};n) < \varphi_{\min}(R_{\max};n)$ and $R_{\max} = R_{C1}(n)$, then $R'_{\max} = R_{C2}(n)$ is sought for in the range $R_k \in [R_0, R_{\max} - b_0/2]$. In this case, $\varphi_{C1}(n) = [\varphi_{\max}(R_{\max};n) + \varphi_{\min}(R_{\max};n)]/2$

and $\varphi_{C2}(n) = [\varphi_{\max}(R'_{\max};n) + \varphi_{\min}(R'_{\max};n) + \varphi_{\min}(R_{\max};n)]/2$. In Figure 2(c), the black and grey circles show the coordinates of the axes of, respectively, the right and left aircraft vortices as obtained from the data of Figure 2(a).

The elevation angle $\varphi_m^{(n)}$ is a function of time t measured from the instant, when the aircraft crosses the scanning plane $\varphi_m^{(n)} \equiv \varphi(t)$. From the equality $\varphi(t_n^{(i)}) = \varphi_{Ci}(n)$, we can determine the time, when the sensing beam crosses the axis of the i -th aircraft vortex for the n -th scan. The value of $t_n^{(i)}$ is the age of the i -th vortex. Using the obtained arrays of $R_{Ci}(n)$ and $\varphi_{Ci}(n)$, we can represent the coordinates of the vortex axes as functions of time in the Cartesian coordinate system

$$\begin{aligned} \mathbf{r}_{Ci}(t_n^{(i)}) = \{Z_{Ci}(t_n^{(i)}) = \\ = R_{Ci}(n) \sin[\varphi_{Ci}(n)], Y_{Ci}(t_n^{(i)}) = R_{Ci}(n) \cos[\varphi_{Ci}(n)]\}. \end{aligned} \quad (8)$$

The next step in processing of lidar data is obtaining estimates of the circulation of the right $\hat{\Gamma}_1(t_n^{(1)})$ and left $\hat{\Gamma}_2(t_n^{(2)})$ vortices. We propose to determine the vortex circulation through minimization of the functionals $\rho(\Gamma_1)$ and $\rho(\Gamma_2)$, that is, using the procedure

$$\min\{\rho(\Gamma_i)\} = \rho(\hat{\Gamma}_i), \quad (9)$$

where

$$\rho(\Gamma_i) = \sum_m [V_r^0(R_{Ci}, \varphi_m; n) - \bar{V}_r(R_{Ci}, \varphi_m | \Gamma_1, \Gamma_2)]^2, \quad (10)$$

$\bar{V}_r(R_{Ci}, \varphi_m | \Gamma_1, \Gamma_2)$ is the radial velocity calculated theoretically at arbitrary values of circulation of the right Γ_1 and left Γ_2 vortices. Since the estimates of the radial velocity are obtained from the position of maximum of the estimates of the Doppler spectrum, the analytical formula for the model function $\bar{V}_r(R_{Ci}, \varphi_m | \Gamma_1, \Gamma_2)$ cannot be derived. This problem can be solved in the following way.

Based on the theory developed in [4] and taking into account that the temporal profile of the probing pulse power is close to the Gaussian one, we have derived the equation for the correlation function of the complex lidar signal $C(IT_s, R_{Ci}, \varphi_m)$. Based on calculation of $C(IT_s, R_{Ci}, \varphi_m)$ at different values of Γ_1 and Γ_2 , we obtain the Doppler spectra $S_D(V_r, R_{Ci}, \varphi_m)$ and then, using the procedure $\max\{S_D(V_r, R_{Ci}, \varphi_m)\} = S_D(\bar{V}_r, R_{Ci}, \varphi_m)$, we find $\bar{V}_r(R_{Ci}, \varphi_m | \Gamma_1, \Gamma_2)$.

The estimates $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ are obtained by the iterative procedure.

3. RESULTS

To test the described method for estimation of the wake vortex parameters from data measured by the Stream Line lidar, we have conducted an experiment at the airfield of the Tomsk Airport on 9 July 2014. The lidar was installed 315 m far from the runway. For the time of continuous measurement by this lidar from 05:00 to 07:10 LT, four B737-800 aircraft crossed the plane of scanning by the probing beam at a height of about 37 m.

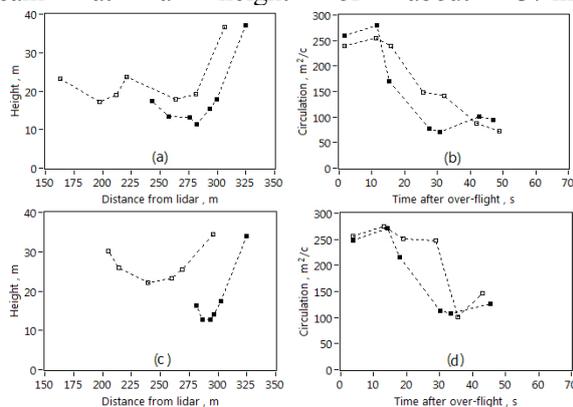


Fig.3. Trajectories of axes (a, c) and evolution of circulation (b, d) of the left (open squares) and right (closed squares) vortices generated by B737 aircraft over the airfield of the Tomsk Airport on 9 July 2014 at 05:09:21 (a, b) and 07:02:20 (c, d).

Figure 3 shows the final results of processing of the data measured by the Stream Line lidar. According to the data of Fig. 3, the estimated initial distance between the axes of the right and left vortices $\hat{b}_0 = |\mathbf{r}_{C1}(t_1^{(1)}) - \mathbf{r}_{C2}(t_1^{(2)})|$ is, on average, 25 m, which is close to the theoretical value $b_0 = 27$ m for this type of aircraft, while the estimate of the initial vortex circulation $\hat{\Gamma}(t_1^{(i)})$ is, on average, $250 \text{ m}^2/\text{s}$, which is close to the theoretical value of circulation obtained from calculation by the equation for the initial circulation $\Gamma_0 = M_A g / (\rho_a b_0 V_A)$ [10], where M_A is aircraft mass, and V_A is the true velocity of aircraft, g is gravitation acceleration, and ρ_a is air density.

The trajectories and circulations of vortices shown in Figure 3 are characteristic in the case of vortex generation at low altitudes [4]. The

direction and speed of vortex transport by the cross wind correspond to the weather conditions of the experiment. The lifetime of the wake generated by aircraft of this type is about 1 min.

4. CONCLUSIONS

Thus, in this work we have developed the strategy of measurement by the Stream Line lidar and proposed the method for processing of data measured by this lidar in order to estimate the wake vortex parameters. The method has been tested in numerical and field experiments. The experimental results depicted in Figure 3 indicate the principal possibility of obtaining the information about the presence of aircraft wake vortices in the vicinity of a runway and the vortex strength (circulation) from data measured by the Stream Line lidar.

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