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Resummed Results for Hadron Collider Observables

Heather McAslan[1,a]

[1] *University of Sussex, Brighton, UK*

Abstract. Event shapes are invaluable QCD tools for theoretical calculations and experimental measurements. We revise the definition of these observables in e^+e^- annihilation and in hadron collisions, and give a review of the state-of-the-art results for their resummation. Then we detail how recent work on the resummation of event shapes in electron-positron annihilation can provide us with the tools to extend resummation of generic hadronic event shapes to NNLL accuracy. We match our findings to fixed-order results at NNLO accuracy, showing the sizeable effects of resummation in the relevant regions of phase space.

1 Event Shapes

Event shapes are measures of the hadronic energy flow in a system of particles. These observables are particularly useful for tuning Monte Carlo generators and extraction of the strong coupling α_s.

1.1 Event Shapes' Definitions in e^+e^-

Event shapes were originally defined in e^+e^- annihilation events. The canonical event shape, and a useful observable for encoding the physical geometries of different event types, is the thrust.

$$\tau \equiv 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \ , \tag{1}$$

where the vector $\vec{n}$ maximised by the sum defines the thrust axis, $\vec{n}_T$ and Q is simply the total centre of mass energy. The thrust axis divides the system into two hemispheres, $\mathcal{H}(1)$ and $\mathcal{H}(2)$, as shown in Fig. 1. The value of τ is equal to zero for events with two back-to-back jets, and equal to one for a spherically spread-out event, as seen in Figs. 1 and 2.
Using these hemispheres we can define jet broadening

$$B_T \equiv B_L + B_R \,, \tag{2}$$

where

$$B_L \equiv \sum_{i\in\mathcal{H}^{(1)}} \frac{|\vec{p}_i \times \vec{n}_T|}{2Q}\,, \quad B_R \equiv \sum_{i\in\mathcal{H}^{(2)}} \frac{|\vec{p}_i \times \vec{n}_T|}{2Q}\,. \tag{3}$$

[a] e-mail: h.mcaslan@sussex.ac.uk

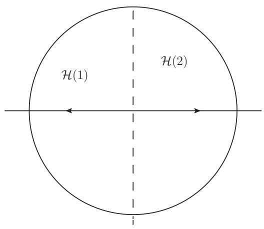

Figure 1. Pencil-like geometry of an event: $\tau = 0$

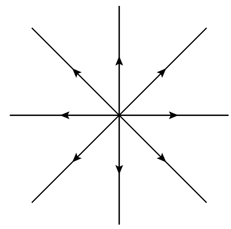

Figure 2. Spherical geometry of event: $\tau = 1$

1.2 Event Shapes' Definitions in Hadronic Collisions

Hadronic event shapes are defined analogously to those in e^+e^-. [1] However, they measure in the transverse plane, for boost invariance, and require a different choice of normalisation. Hadronic event shapes can be conveniently normalised by the sum of the transverse momentum of the final state particles. Transverse thrust is defined in analogy to thrust

$$\tau_\perp \equiv \max_{\vec{n_T}} \left(\frac{\sum_i |\vec{p}_{\perp i} \cdot \vec{n_T}|}{\sum_i |\vec{p}_{\perp i}|} \right), \tag{4}$$

where the maximum $\vec{n_T}$ defines the transverse thrust axis.
As before, we can define a jet broadening for hadronic collisions:

$$B_T \equiv \frac{1}{2P_\perp^2} \left(\sum_{i \in C^{(1)}} p_{\perp i} \sqrt{(\Delta\eta_{1,i})^2 + (\Delta\phi_{1,i})^2} + \sum_{i \in C^{(2)}} p_{\perp i} \sqrt{(\Delta\eta_{2,i})^2 + (\Delta\phi_{2,i})^2} \right), \tag{5}$$

where the normalisation $P_\perp^2 = \sum_i |\vec{p}_{\perp i}|$. Here $C^{(1)}$ and $C^{(2)}$ are the hemispheres defined by the transverse thrust axis, but excluding any particle i that is not within the measured 'central region'. The issue of constraining emissions in a central region of phase space (i.e. with small η) is discussed in the next section.

2 Globalness Issues and Solutions

Experimentally, measurements of QCD observables are made in the central region of the detector geometry where detector information is most extensive. This is implemented in event analysis by placing a cut on events outwith a central rapidity, $|\eta| < \eta_C$. When carrying out theoretical predictions, however, we want to avoid cutting on regions of phase space. Restricting the phase space of emissions causes large non-global logarithms to arise, which pose difficulties to resummation. The crucial interplay between theory and experiment calls for some compromise in the calculation/measurement of observables outside of the central region.

Techniques exist to suppress the effects of non-global effects. [2] One can define a new global observable which is split into a central piece and a piece sensitive to radiation away from the central region. For example, global thrust is defined

$$\tau_{\perp,\varepsilon} \equiv 1 - T_{\perp,\varepsilon} \equiv \tau_\perp + \varepsilon_{\bar{C}}, \tag{6}$$

where

$$\varepsilon_{\bar{C}} = \frac{1}{Q_{\perp,C}} \sum_{i \notin C} p_{\perp i} e^{-|\eta_i - \eta_C|}, \tag{7}$$

and

$$\eta_C = \frac{1}{Q_{\perp,C}} \sum_{i \in C} p_{\perp i} \eta_i \,, \tag{8}$$

and $\tau_\perp$ is the hadronic transverse thrust as given in Eq. (4). The normalisation $Q_{\perp,C}$ is the sum of transverse momenta of particles within the central region. The sum in Eqs (7) and (8) spans whichever objects we work with, be that individual particles or anti-k_t jets, for example. This parameterisation allows for a hadronic global thrust definition, sensitive to radiation in all regions of phase space. Global broadening, and other global event shapes, can be constructed in the same way.

There has been recent progress in techniques for tackling non-global resummation head-on. See for example [3] [4].)

3 Resummation

The infra-red and collinear safety of event shapes' definitions ensures that their cross-sections are finite in every order of perturbation theory. However, when the value of the event shape is small ($v \approx 0$ where e.g. $v \equiv \tau$, for thrust) the implicit constraint on QCD radiation being soft and collinear leads to large logarithms appearing at every order in α_s. These large logarithms are remnants of the cancellation of real and virtual singularities. The effective coupling becomes $\alpha_s \ln(1/v) \approx 1$ and of course the perturbative expansion is no longer reliable. The series is rearranged in terms of the dominant logarithms and the cumulative distribution is resummed via the following expression [5]

$$\Sigma(v) = \frac{1}{\sigma_0} \int_0^v dv' \frac{d\sigma(v')}{dv'} = e^{Lg_1(\lambda)+g_2(\lambda)} \mathcal{F}_{NLL}(\lambda), \qquad \lambda = \alpha_s(Q)\beta_0 \ln(1/v) \,, \tag{9}$$

where the exponential function is a Sudakov form factor, containing contributions arising from virtual corrections and unresolved real emissions. The $\mathcal{F}(\lambda)$ contains contributions from real resolved emissions. The $\{g_i(\lambda)\}$ are ordered in decreasing logarithmic-dominance, with $g_1(\lambda)$ resumming the leading logarithms - or terms of form $\alpha_s^n \ln^{n+1}(1/v)$ - $g_2(\lambda)$ resumming the next-to-leading logarithms - terms of form $\alpha_s^n \ln^n(1/v)$ - and so on. Eq. (9) is accurate to next-to-leading logarithmic order (NLL). Physically, truncating at NLL order corresponds to requiring that all radiation be soft and collinear, and widely separated in rapidity. To achieve NNLL accuracy one would also need to include $(\alpha_s/\pi)g_3(\lambda)$ in the exponent, and any NNLL contributions to the $\mathcal{F}$-function.

4 Resummation Tools

4.1 CAESAR

Resummation of event shapes (as well as other observables) to NLL accuracy has been carried out in the semi-numerical method of CAESAR, via the automated resummation of a generic observable. [5] CAESAR's generic observable is defined in the Born system, with an extra soft and collinear gluon (Fig. (3))

$$V(\{\tilde{p}\}, k) = d_\ell \left(\frac{k_t^{(\ell)}}{Q} \right)^{a_\ell} e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)}) \tag{10}$$

(Thrust can be recovered by setting $a_\ell = b_\ell = 1$.)
$\{\tilde{p}\}$ are the hard partons (or 'legs' ℓ) involved in the event, recoiled against the additional emission. Referring to fig. (3) they are the quark-antiquark dipole, $\{\tilde{p}\} = \{\tilde{p_1}, \tilde{p_2}\}$. k_t is the transverse momentum of the soft and collinear emission k, with respect to the dipole, with some normalisation Q. η is its

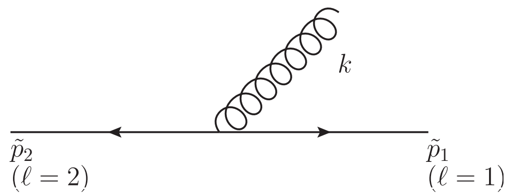

Figure 3. A system of a back-to-back quark-antiquark pair, and an additional gluon, k.

rapidity and ϕ its azimuth, also with respect to the dipole. The (a, b, d, g) parameterise each different observable.

CAESAR requires that the observable being resummed is continuously global, thereby prohibiting non-global logarithms. This implies that the scaling of the observable is the same everywhere, i.e. that $a = a_1 = a_2$. The observables must also satisfy recursive infrared and collinear (rIRC) safety. rIRC safety stipulates that the observable's scaling properties should be the same in the presence of any number of extra soft and/or collinear emissions, i.e. in the presence of a series of soft and collinear emisisons $(k_1, k_2, k_3, \ldots)$.

4.2 Resummation of Event Shapes to NNLL

CAESAR's NLL results are the current state-of-the-art for many event shapes. Recently there have been results appearing at NNLL using Soft Collinear Effective Theory, for example the transverse thrust [6], the beam thrust [7] and N-jettiness [8].

4.3 ARES (Automated Resummer of Event Shapes)

Using the formalism of CAESAR we have recently developed a resummation method (and accompanying code, soon to be made public) for event shapes in e^+e^- annihilation at NNLL order. [9] To achieve NNLL accuracy, we relax the kinematic assumptions made at NLL one by one: allowing one emission to roam in phase space, i.e. for a single hard-collinear or soft-wide angle emission to occur; allowing two emissions to be close in rapidity. Every other emission must obey the stronger NLL constraints. This reults in the calculation remaining purely NNLL accurate - we do no want to introduce subleading contributions that are not fully under our control. Each of these scenarios produces a correction to the real-emission function $\mathcal{F}$ in Eq. (9), giving $\mathcal{F}_{NNLL} = \mathcal{F}_{NLL} + \delta\mathcal{F}_i$. As mentioned in Sec. 3, we also include the function $g_3(\lambda)$ in the Sudakov factor's exponent. Thus the cumulative resummed distribution of an event shape v at NNLL is

$$\Sigma(v) = \frac{1}{\sigma_0}\int_0^v dv' \frac{d\sigma(v')}{dv'} = e^{Lg_1(\lambda)+g_2(\lambda)+\frac{\alpha_s(Q)}{\pi}g_3(\lambda)}\left[\mathcal{F}_{\rm NLL}(\lambda) + \frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\rm NNLL}(\lambda)\right] \tag{11}$$

Figures (4) and (5) show the effect of adding the ARES resummation to an NNLO fixed order Monte Carlo cross-section for thrust and broadening in e^+e^- (we use the generator `EERAD3` [10]). In the low τ and B_T regions the resummed results are dominant, as expected. It is clear that the fixed order result on its own does not provide a good prediction of the event shape cross-section in this region, and resummation is necessary. For larger values of the observable our resummation tends to the fixed order result, which is adequate in this region. There is full agreement to $O(\alpha_s^3)$ between the results from ARES and observables for which an analytic resummation was already available (thrust

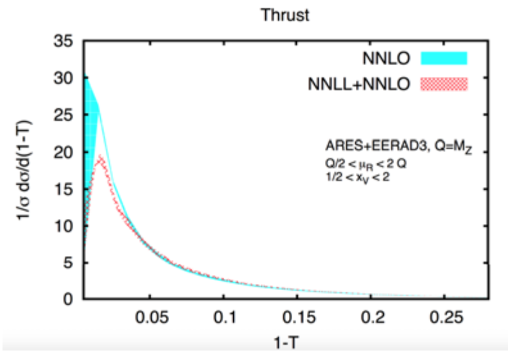

Figure 4. Effect of the resummation on the thrust distribution.

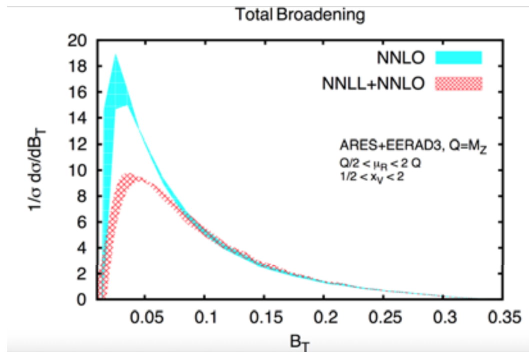

Figure 5. Effect of the resummation on the broadening distribution.

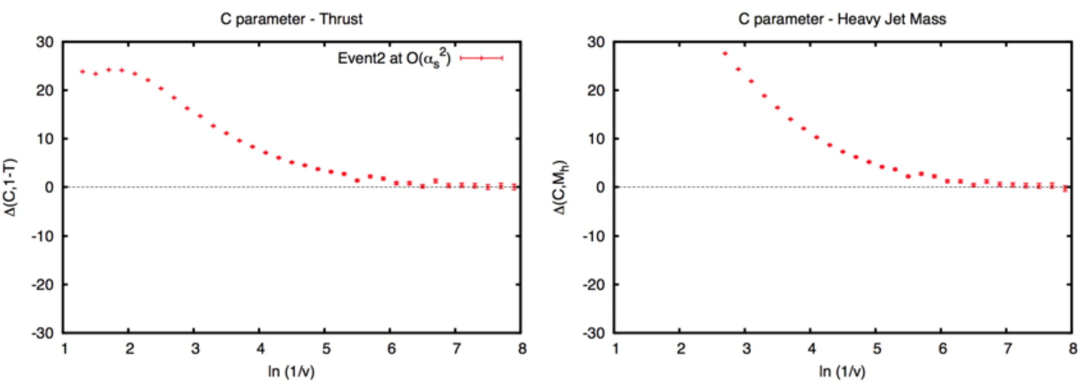

Figure 6. The difference between the expansion of our results and the full result must go to zero as $\ln(1/v) \to$ large positive values.

[13], total and wide broadenings[16], heavy jet mass[17]). To check our new results we compare our resummation to fixed order by expanding it in powers of the strong coupling. Figure (6) shows the difference between the fixed-order generator `Event2` [18] and the expansion of our resummation to order in α_s^2. To obtain stable distributions the difference of observables with the same k_t-scaling are plotted,

$$\Delta(v_1, v_2) = \left(\frac{1}{\sigma_0} \frac{d\sigma^{\mathrm{NLO}}}{d\ln\frac{1}{v_1}} - \frac{1}{\sigma_0} \frac{d\sigma^{\mathrm{NNLL}}|_{\mathrm{expanded}}}{d\ln\frac{1}{v_1}} \right) - \{v_1 \to v_2\}\,. \tag{12}$$

As expected, this difference tends to zero for low-v where the resummation is necessary to correctly capture the behaviour of the cross-section. This same trend can be seen for all of the observables we have resummed [9].

4.4 Hadronic Event Shapes in ARES

Having built the tools to carry out the resummation of a generic event shape in e^+e^- annihilation, we turn our attention to the resummation of hadronic event shapes at NNLL accuracy. A few adjustments are required in dealing with the extra beam information, however the resummation can be viewed as a combination of the same observable in e^+e^- annihilation and a beam part. For example, the resummation of the transverse thrust will be equivalent to the thrust in e^+e^- plus the beam thrust. Similarly, the hadronic jet broadening is equivalent to the jet broadening in e^+e^- plus the beam thrust.

We have developed a novel and general framework to calculate the resummed cross-section of observables to NNLL order, utilising the code to resum event-shape variables in $e^+e^- \to$ 2jets. We are currently working to extend this framework to allow the resummation of hadronic event shapes to NNLL accuracy.

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