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Pion Production from Proton Synchrotron Radiation under Strong Magnetic Field in a Relativistic Quantum Approach

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Abstract. We study pion production from proton synchrotron radiation in the presence of strong magnetic fields by using the exact proton propagator in a strong magnetic field and explicitly including the anomalous magnetic moment. Results in this exact quantum approach do not agree with those obtained in the semi-classical approach. Then, we find that the anomalous magnetic moment of the proton greatly enhances the production rate by about two orders magnitude, and that the decay width satisfies a robust scaling law.

1 Introduction

It is widely accepted that soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) correspond to magnetars [1], and that the associated strong magnetic fields may have a significant role in the production of high energy photons.

The magnetars have also been proposed [2, 3] as an acceleration site for ultra high-energy (UHE) cosmic rays (UHECRs), and a possible association between magnetar flares [4] and UHECRs has also been observed. Synchrotron radiation can be produced by high-energy protons accelerated in an environment containing a strong magnetic field. This process has been proposed as a source for high-energy photons in the GeV − TeV range [5–10]. The meson-nucleon couplings are about 100 times larger than the photon-nucleon coupling, and the meson production process is expected to exceed photon synchrotron emission in the high energy regime.

However, theoretical calculations were performed approximately in a semi-classical way [11–15] and each model gave a different result. In addition, these model could not give a momentum-distribution of a final pion.

In the semi-classical approach for the production of synchrotron radiation, the magnetic field strength is characterized by the curvature parameter, $\chi = E_i^3/(M^3 R_c)$ given in terms of the incident particle mass M, its energy E_i, and the curvature radius R_c. For protons propagating in a strong magnetic field, the value of χ can be written as

$$\chi = \frac{E_i^2}{M_p^3 R_c} = \frac{eBE_i}{M_p^3}, \tag{1}$$

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where M_p is the proton mass. Pion production is the dominant process compared to direct photon emission when $\chi \sim 0.01 - 1$ [15].

There are various semi-classical calculations [11–15] which give different results, but all of those models suggest that the decay width depends only on the parameter χ, not on the initial energy E_i and the strength of the magnetic field B individually, though this has not been confirmed.

In this work, we examine the scaling relation in the quantum calculation. Based upon this we can realistically estimate for the first time the results for a much larger Landau numbers from the results obtained from a smaller number of Landau levels.

Furthermore, we compare the results with those in the semi-classical approaches [14, 15]. In addition, we show the energy and angular distributions of emitted pions.

2 Formalism

Here, we briefly explain our formalism.

We assume a uniform magnetic field along the z-direction, $\boldsymbol{B} = (0, 0, B)$, and take the electromagnetic vector potential A^μ to be $A = (0, 0, xB, 0)$ at the position $\boldsymbol{r} \equiv (x, y, z)$. The relativistic proton wave function ψ is obtained from the following Dirac equation:

$$\left[\alpha_z p_z - i\alpha_x \partial_x + \alpha_y (p_y - eB_x) + M_p \beta - \frac{\kappa_p e}{M_p} B \Sigma_z\right] \psi(x, p_z, s) = E\psi(x, p_z, s), \tag{2}$$

where M_p is the proton mass, κ_p is the proton anomalous magnetic moment (AMM), e is the elementary charge, and E is the single particle energy written as with

$$E(n, p_z, s) = \sqrt{p_z^2 + (\sqrt{2eBn + M_p^2} - s\kappa_p eB/M_p)^2}. \tag{3}$$

To calculate the pionic decay width, furthermore, we use the pseudo-vector coupling interaction Lagrangian density as

$$\mathcal{L} = \frac{if_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \tau_a \psi \partial^\mu \phi_a, \tag{4}$$

where f_π is the pseudo-vector pion-nucleon coupling constant, m_π is the pion mass, and ϕ is the pion field. Then, we obtain the differential decay width of the proton as

$$\frac{d^3\Gamma_{p\pi}}{dq^3} = \frac{1}{8\pi^2 E_\pi} \left(\frac{f_\pi}{m_\pi}\right)^2 \sum_{n_f, s_f} \delta(E_f + E_\pi - E_i) W_{if}, \tag{5}$$

with

$$\begin{aligned} W_{if} &= \mathrm{Tr}\left\{\rho_M(n_i, s_i, q_z) O_\pi \rho_M(n_f, s_f, p_z - q_z) O_\pi^\dagger\right\}, \\ \rho_M &= \frac{\left[E\gamma_0 + \sqrt{2eBn}\gamma^2 - p_z\gamma^3 + M_p + (\kappa_p eB/M_p)\Sigma_z\right]}{4E} \left[1 + \frac{s\left(\kappa_p eB/M_p + p_z \gamma_5 \gamma^0 - E\gamma_5 \gamma^3\right)}{\sqrt{(2eBn + M_p^2)}}\right], \end{aligned}$$

where $q \equiv (q_0; \boldsymbol{q})$ is a momentum of a emitted pion. In the above equation, f_n is the harmonic oscillator wave function with the principle quantum number n, and

$$\begin{aligned} O_\pi &= \gamma_5 \left\{\left[\mathcal{M}\left(n_i, n_f\right) \frac{1 + \Sigma_z}{2} + \mathcal{M}\left(n_i - 1, n_f - 1\right) \frac{1 - \Sigma_z}{2}\right] \left[\gamma_0 q_0 - \gamma^3 q_z\right] \right. \\ &\quad \left. - \left[\mathcal{M}\left(n_i, n_f - 1\right) \frac{1 + \Sigma_z}{2} + \mathcal{M}\left(n_i - 1, n_f\right) \frac{1 - \Sigma_z}{2}\right] \gamma^2 q_T \right\}, \end{aligned} \tag{6}$$

where $q_T \equiv \sqrt{q_x^2 + q_y^2}$, and $\mathcal{M}(n_1, n_2)$ is the harmonic oscillator (HO) overlap integral defined as

$$\mathcal{M}(n_1, n_2) = \sqrt{eB} \int dx f_{n_1}\left(\sqrt{eB}x + \frac{q_T}{2\sqrt{eB}}\right) f_{n_2}\left(\sqrt{eB}x - \frac{q_T}{2\sqrt{eB}}\right). \quad (7)$$

3 Results

Now, we show numerical results for pion emission.

In Fig.1, we show the total pionic decay widths of the proton with $p_{iz} = 0$ when $B = 10^{18}$G (a) and $B = 5 \times 10^{17}$G (b) as functions of the Landau level n_i. These decay widths are averaged over the proton spin, $\Gamma = [\Gamma(n_i, +1) + \Gamma(n_i, -1)]/2$. The solid and dot-dashed lines represent the decay widths of the proton with and without AMM, respectively. We see that the decay width with AMM is about 50 times larger than that without AMM.

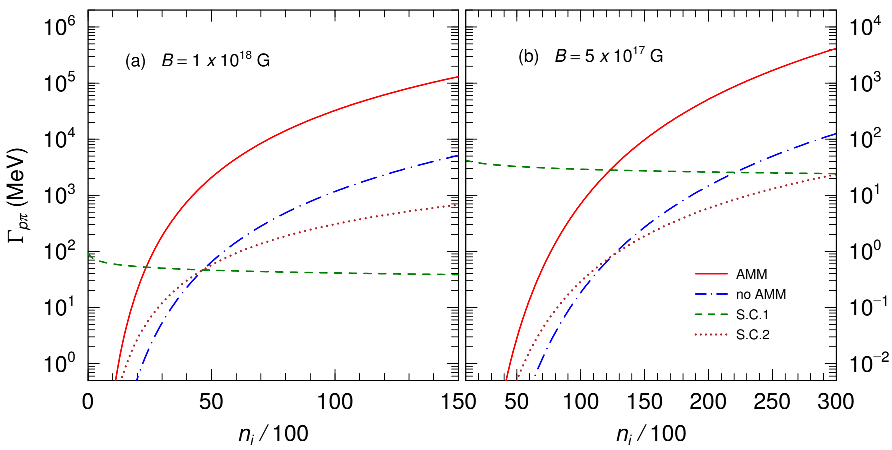

Figure 1. Pionic decay width of protons versus the initial Landau level at $p_{iz} = 0$ for $B = 5 \times 10^{17}$G. The decay width is averaged over the proton spin. The solid and dot-dashed lines represent the decay widths of the proton with and without the AMM, respectively. The dashed and dotted lines indicate the results in the semi-classical approaches of Refs. [14] and in [15], respectively.

For comparison, furthermore, we also plot results in the semi-classical approaches of Ref. [14] (dashed line) and Ref. [15] (dotted line). The results in the semi-classical calculations do not agree with those in the quantum calculation, particularly when the AMM is included.

In order to study the AMM effect more clearly, we next examine the HO overlap integral $M(n_i, n_f)$ defined in Eq. (7). In Fig. 2 we show the n_f-dependence of $M(n_i, n_f)$ when $n_i = 46$. The solid circle and triangle in the inset represent the peak positions in the cases with $s_i = -s_f = -1$ and $s_i = -s_f = 1$ with the AMM included. The open diamond indicates the case of a spin-flip $s_i = -s_f$ without including AMM.

$M(n_i, n_f)$ shows an oscillating behavior, but only the strength after the last peak contributes to the results. In this region the strength rapidly decreases with increases of $n_i - n_f$. A pion cannot be produced in the free kinematics, $B = 0$, because of energy momentum conservation. Under the influence of a magnetic field, momentum conservation is not satisfied, so that a pion can be produced in the kinematical condition far from the free kinematics, where $M(n_1, n_2)$ rapidly decreases.

The AMM gives a repulsive potential for $s_i = -1$ and attractive for $s_i = 1$. The transition with $s_i = s_f = -1$ introduces an additional energy to be consumed in the pion production. Thus, the difference between the initial and final Landau-levels is shifted toward a smaller number. In addition, the transition with $s_i = s_f = 1$ reduces the production energy. In this kinematical region a small difference between the initial and final Landau-levels significantly changes the transition strength. Therefore, the AMM plays the important roles of increasing greatly the pionic decay width when $s_i = s_f = -1$ and to decrease it $s_i = s_f = 1$.

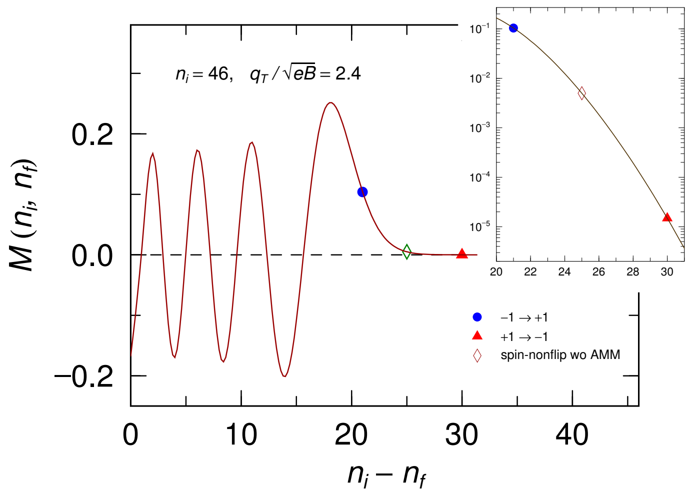

Figure 2. The transition strength function $M(n_i, n_f)$, Eq. (7), when $n_i = 46$. The solid circle and triangle show the peak positions in the cases $s_i = -s_f = -1$ and $s_i = -s_f = 1$, and the opened diamond indicates the case of the spin-flip $s_i = -s_f$ when $\kappa_p = 0$.

In Fig. 3 we present the decay widths, $\Gamma_{p\pi}(n_i, n_f)$ as functions of $(n_i - n_f)/n_i$ when the parameter χ is fixed. The upper panels show the results with the AMM included for $\chi = 0.02$ (a1) and $\chi = 0.07$ (a2), and the lower panels show the results without the AMM for $\chi = 0.02$ (b1) and $\chi = 0.07$ (b2). The dashed, dot-dashed, solid and dotted lines show the results with initial Landau numbers, $n_i = 5 \times 10^3$, 2×10^4, and 6×10^4, respectively. In all results the initial and final proton spins are set to be $s_i = -s_f = -1$ because this is the dominant contribution.

First we note that the dependence of $(n_i - n_f)/n_i$ for the different cases are almost completely in agreement when χ is fixed. There are however various semi-classical calculations [11–15] which give different results, but all of the models indicate that the decay width depends only on the parameter χ, not on the initial energy E_i and the magnetic field B. Thus, we confirm that this scaling relation is satisfied in the quantum calculations, independently of the AMM.

In the all results the peak positions are $(n_i - n_f)/n_i \approx 0.3 - 0.4$. As χ increases, the peak position is shifted to a larger value, and the peak width becomes broader.

When comparing the results with the AMM included (a1, a2) and those without the AMM (b1, b2) we can see that an AMM slightly shifts the peak position to smaller values of $(n_i - n_f)/n_i$, and that the peak width is slightly broader than in the case without the AMM.

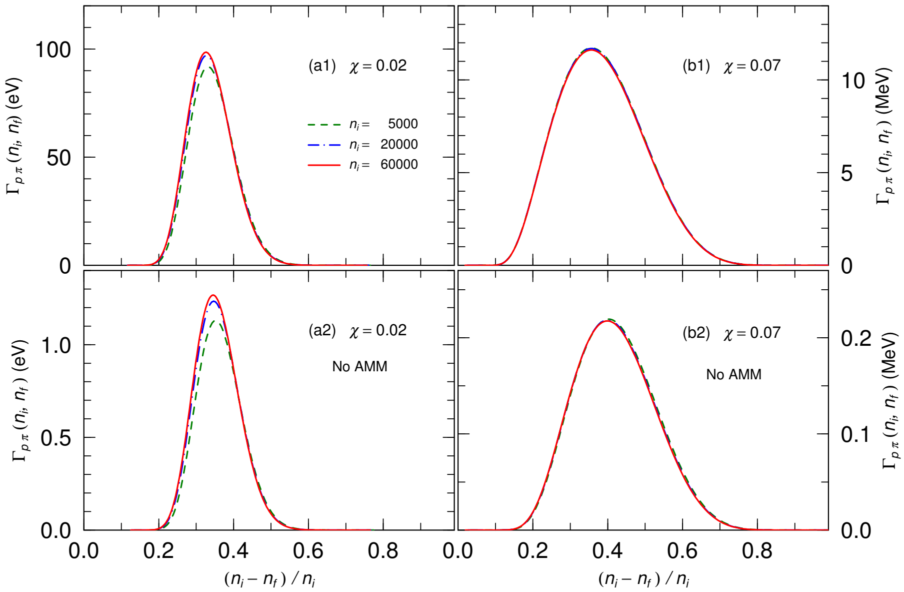

Figure 3. (Color online) Pionic decay widths of protons when $s_i = -s_f = -1$ as a function of $(n_i - n_f)/n_i$ for $\chi = 0.02$ (a1) and $\chi = 0.07$ (b1). The bottom panels (a2) and (b2) show the results without the AMM for $\chi = 0.02$ and $\chi = 0.07$, respectively. The dashed, dotted-dashed and solid lines represent the results with initial Landau number, $n_i = 5 \times 10^3$, 2×10^4 and 6×10^4, respectively.

Moreover, we see that the peak position is shifted by including the AMM with the scaling relation being satisfied. This result indicates that the AMM remains important for any magnetic field strength and proton energy.

As written in Ref. [16], the very large effect of the AMM comes from the shift of this peak position, i.e. where the HO overlap integral $\mathcal{M}(n_i, n_f)$ changes rapidly. For $n_i \approx 48$ and the shift of $n_i - n_f$ to 2, the absolute value of $\mathcal{M}$ increases by about a factor of 100. When the magnetic field is weaker or the initial energy becomes larger, n_i increases, and the AMM effect is expected to be smaller. When χ is fixed, however, the AMM effect remains and plays an important role in any regime.

Furthermore, we can see that $(n_i - n_f)/n_i \approx 0.3 - 0.4$, namely the Landau level difference in the pion emission between the initial and final states is of the same order of the initial and final Landau levels, $(n_i - n_f) \sim n_i \sim n_f$.

In the adiabatic limit it can be assumed that the relative momentum between the final proton and the pion is zero, and that the two particles have the same velocity. In this limit the ratio between these two energies is the same as the mass ratio: $E_\pi/E_f \approx m_\pi/M_p$, and the final proton and pion energy become

$$E_f \approx \frac{M_p}{M_p + m_\pi} E_i, \quad E_\pi \approx \frac{m_\pi}{M_p + m_\pi} E_i. \tag{8}$$

The initial proton energy is very large $E_i \gg M_p$, $E_{i,f} \approx \sqrt{2n_{i,f}eB}$, so that the following relation is given in the adiabatic and high energy limit as $\sqrt{n_i} - \sqrt{n_f} \approx (m_\pi/M_p)\sqrt{n_i}$. This leads to

$$n_i - n_f \approx \frac{M_p^2 - (M_p - m_\pi)^2}{M_p^2} n_i \approx 0.28 n_i. \tag{9}$$

The actual πN-interaction is the p-wave, and the relative momentum is not zero even in the adiabatic limit, so that the actual value of $n_i - n_f$ is larger than the above value; this argument is consistent with the present results.

In the usual semi-classical approximations it is assumed that $n_i - n_v \ll n_i$. This assumption is equivalent to the adiabatic limit when a produced particle is massless such as a photon. Thus, the assumption, $n_i - n_v \ll n_i$, is not satisfied when the emitted particle is massive, so that its results do not agree with the results in the quantum calculations.

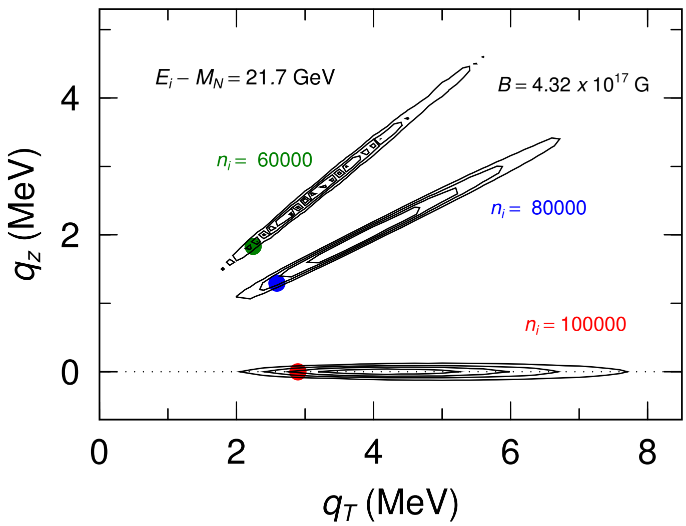

Figure 4. (Color online) The contour plot of the differential pionic luminosity at the initial spin state $s_i = -1$, the initial kinetic energy $E_i - M_p = 21.7$GeV and the magnetic field, $B = 4.32 \times 10^{17}$G ($\chi = 0.07$) with initial Landau number, $n_i = 100,000$ ($p_{iz} = 0$), 80,000 and 60,000. Each lines show the relative strength. The vertical and horizontal axises are the z-component and the transverse component of the emitted pion momentum.

In Fig. 4, we show luminosity distribution of the emitted pion, $d^3I/dq_\pi^3 = E_\pi d^3\Gamma/dq_\pi^3$, with the contour plots, when the initial proton has the initial spin $s_i = -1$ and the initial energy $E_i - M_p = 21.7$GeV and $n_i = 100,000$, 80,000, 60,000 when $B = 4.32 \times 10^{17}$G; the state with $n_i = 100,000$ corresponds to that with $p_{iz} = 0$ and $\chi = 0.07$. The pion momentum is distributed narrowly for the polar angle, but is distributed widely in the absolute value.

The solid circles indicate the pion momentum in the adiabatic limit, $q_z = r_\pi p_{iz}$ and $q_z = r_\pi\sqrt{2n_i eB}$ with $r_\pi = m_\pi/(M_p + m_\pi)$. We see that the absolute value of the emitted pion momentum is almost larger than that in the adiabatic limit, and that the direction of the emitted pion momentum is almost same as that of the initial nucleon momentum.

As the magnetic field decreases, and the initial Landau number increases, the width of the angular distribution becomes narrower, and this width for the polar angle must become negligible in the zero limit of the magnetic field.

4 Summary

We have calculated the pion synchrotron radiation from high energy protons propagating in strong magnetic fields in a microscopic quantum field theoretical framework. We solved the Dirac equation in a strong magnetic field and obtained the proton propagator from its solution. Then, we derived the pionic decay width of the propagating proton in a fully relativistic and quantum mechanical way.

We find out that the anomalous magnetic moment has a very large effect which enlarges the emission rate by about 50 times, and that the polar angles of the emitted pion and the final pion are almost the same as that of the initial proton, when the proton energy is 10 GeV and the magnetic field is 5×10^{17}G. One can not know these effects before performing the quantum calculations.

Furthermore, we have shown that the results in our quantum approach do not agree with those in the usual semi-classical approximations. In the usual semi-classical approximation it is assumed that $n_i - n_\nu \ll n_i$, where the HO overlap function $\mathcal{M}$ can be approximately written with the Airy function. This assumption is not satisfied when a produced particle is massive,

In actual magnetars the surface magnetic field is known to be of order $B \sim 10^{15}$G . In the present method we did not perform a calculation for such a magnetic field strength because of the large number of Landau levels involved: $n_i \sim 10^{12} - 10^{13}$.

The present work suggests a better way to treat pion production. The result of $\Gamma(n_i, n_f)$ depends only on χ and $\Delta n_{if}/n_i$ demonstrates that one can calculate $\Gamma(n_i, n_f)$ for values of $n_i \sim 10^{4-5}$ and scale those results to more realistic conditions. Also, when $p_{iz} \neq 0$, the decay width can be obtained from a Lorenz transformation along z-direction.

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