Spectra for the $A = 6$ reactions calculated from a three-body resonance model

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Abstract. We develop a resonance model of the transition matrix for three-body breakup reactions of the $A = 6$ system and present calculations for the nucleon observed spectra, which are important for inertial confinement fusion and Big Bang nucleosynthesis (BBN). The model is motivated by the Faddeev approach where the form of the T matrix is written as a sum of the distinct Jacobi coordinate systems corresponding to particle configurations $(\alpha, n\text{-}n)$ and $(n, n\text{-}\alpha)$ to describe the final state. The structure in the spectra comes from the resonances of the two-body subsystems of the three-body final state, namely the singlet ($T = 1$) nucleon-nucleon (NN) anti-bound resonance, and the $N\alpha$ resonances designated the ground state ($J^\pi = \frac{3}{2}^-$) and first excited state ($J^\pi = \frac{1}{2}^-$) of the $A = 5$ systems ^{5}He and ^{5}Li. These resonances are described in terms of single-level, single-channel R-matrix parameters that are taken from analyses of NN and $N\alpha$ scattering data. While the resonance parameters are approximately charge symmetric, external charge-dependent effects are included in the penetrabilities, shifts, and hard-sphere phases, and in the level energies to account for internal Coulomb differences. The shapes of the resonance contributions to the spectrum are fixed by other, two-body data and the only adjustable parameters in the model are the combinatorial amplitudes for the compound system. These are adjusted to reproduce the observed nucleon spectra from measurements at the Omega and NIF facilities. We perform a simultaneous, least-squares fit of the tt neutron spectra and the ^{3}He^{3}He proton spectra. Using these amplitudes we make a prediction of the α spectra for both reactions at low energies. Significant differences in the tt and ^{3}He^{3}He spectra are due to Coulomb effects.

1 Introduction

Significant effort over many decades has been invested in measuring and analyzing the reactions of light nuclei ($A \leq 20$). These reactions are important for an array of applications in astrophysical (big bang nucleosynthesis and stellar evolution, for example) and terrestrial (inertial confinement fusion, criticality safety, and measurements standards) settings. Many systems however, particularly those involving unstable projectiles and targets remain unmeasured. There has been much interest and experimental activity lately at ICF facilities, such as OMEGA at the U. of Rochester, and the NIF at Livermore, to measure the particle spectra from three-body breakup reactions initiated by the laser implosion of capsules containing tritium (t =^{3}H), ^{3}He, or a mixture of the two. Here we describe

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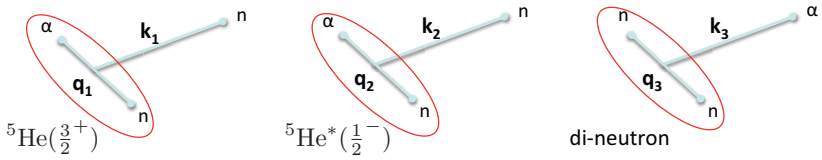

Figure 1: Configurations included in the three-body final state wave function showing the momentum Jacobi coordinates, $\mathbf{q}_i, \mathbf{k}_i$.

an R-matrix-based model for least-squares fitting these spectra that maintains some aspects of the Faddeev method for three-particle scattering. We discuss the simultaneous fit of the $n(p)$-spectra for the reaction $t(t,n)n\alpha$ (^{3}He(^{3}He, p)$p\alpha$) and give predictions for the α spectra.

The next section gives a brief introduction to the concepts of R-matrix theory and summarizes the resonance model, in which the T matrix is expressed as the sum of three Faddeev-like components. When specialized to the case of single-level terms, these expressions form the basis of our present model, which is similar to that of Brune *et al.* [1], but differs from it in detail. The application of these formulas to calculate the spectra of nucleons from the 3-body breakup of the reactions induced by $t+t$ and ^{3}He+^{3}He is described in Section 3. These calculations use charge-symmetric (CS) parameters for the quantities dominated by nuclear forces, but take into account the Coulomb differences in the resonant sub-systems. The conclusions from this study thus far and the outlook for future work are given in Section 4.

2 R-matrix based resonance model

R-matrix theory asserts that, due to the short range of nuclear forces, there exists a finite region of coordinate space beyond which the scattering wave function for an A-nucleon system assumes its known asymptotic form in terms of T-matrix amplitudes. Through the device of the Bloch operator, $\mathscr{L}_B$, acting at the surface enclosing that region, a complete set of eigenfunctions of the operator $H+\mathscr{L}_B$ can be used to expand the scattering wave function inside it as

$$\phi^+_{\mathbf{q}}(\mathbf{x}) = \sum_{\lambda} c_\lambda(\mathbf{q})\phi_\lambda(\mathbf{x}), \tag{1}$$

$$c_\lambda(\mathbf{q}) = \frac{1}{\sqrt{\pi}} \sum_{\lambda'} A_{\lambda\lambda'}\gamma_{\lambda'} O^{-1}(q)\mathscr{Y}^{j\pi}_{sl}(\hat{\mathbf{q}}). \tag{2}$$

In Eq. (2), $A_{\lambda\lambda'}$ is the level matrix, $\gamma_{\lambda'}$ is the reduced-width amplitude, O is the outgoing-wave solution, and $\mathscr{Y}^{j\pi}_{sl}$ is the spin-angle eigenfunction of total angular momentum and parity.

In the $A = 6$ reactions we consider here, a two-body initial state reacts to give a three-body final state. The assumed configurations and their three-momentum Jacobi coordinates in the final state are shown in Fig.1 for the case of the ^{6}He compound system relevant for the $T(t,n)n\alpha$ reaction. Analogous configurations obtain for the ^{6}Be compound system relevant for the ^{3}He(^{3}He, p)$p\alpha$ reaction. These three configurations take into account the the ^{5}He $J^\pi = \frac{3}{2}^+$ ground state, the ^{5}He* $\frac{1}{2}^-$ excited state and the di-neutron (nn) correlation. The pair-wise interactions in the final state are treated as in the Faddeev approach to three-particle scattering, but in these applications to the $A = 6$ systems, the initial state cannot be formed from a bound state of any pair of particles in the final state, as is required by the Faddeev theory. Hence, we call it “Faddeev-like."

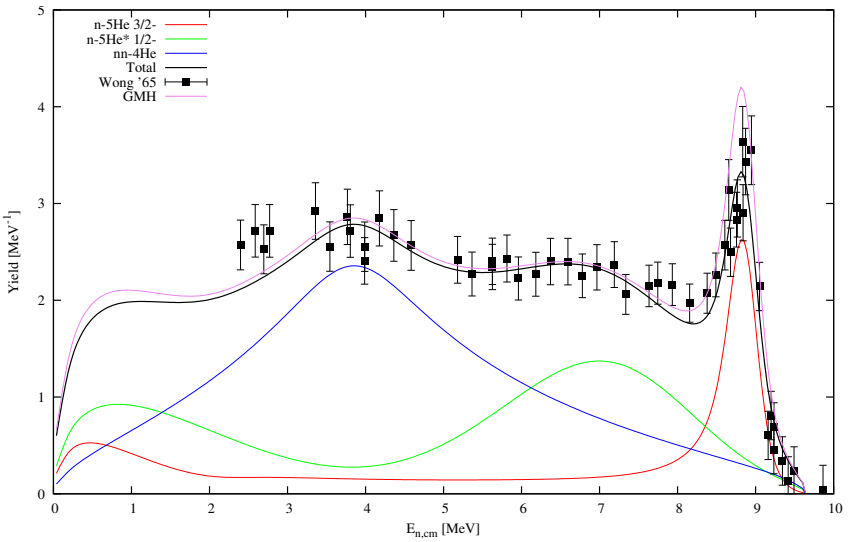

Figure 2: Neutron spectrum from the *tt* reaction at $E_t = 0.5$ MeV. The data are from Ref. [2], and the colored lines give the 3 Jacobi components of the spectrum.

The three-body breakup transition amplitude can be written as a sum of Faddeev-like components

$$T^{(3)} = \sum_{i=1}^{3} T^{(3)}_{\mathbf{q_i k_i k_0}}, \tag{3}$$

$$T^{(3)}_{\mathbf{q_i k_i k_0}} = -\pi \langle \chi_{\mathbf{q}_i} \chi_{\mathbf{k}_i} | V_i | \psi^+_{\mathbf{k}_0} \rangle, \tag{4}$$

where $\psi^+_{\mathbf{k}_0}$ is the scattering solution in the initial two-body state, and $\chi_{\mathbf{q}_i}$ and $\chi_{\mathbf{k}_i}$ are the plane-wave solutions for the momenta $\mathbf{q}_i$ and $\mathbf{k}_i$ conjugate to $\mathbf{x}_i$ and $\mathbf{r}_i$, respectively. The transition amplitude is related to the differential cross section as

$$\frac{d^3\sigma}{d\mathbf{k}'_1} = \frac{(2\pi)^4}{v_{\rm rel}} \int d^3k'_2 d^3k'_3 \left| T^{(3)} \right|^2 \delta(\epsilon_f - (\epsilon_i + Q)) \delta^{(3)}(\mathbf{k}'_1 + \mathbf{k}'_2 + \mathbf{k}'_3), \tag{5}$$

where $\mathbf{k}'_i$, $i = 1, 2, 3$ are the particle three-momenta in the final state and $v_{\rm rel}$ is the relative velocity of the inital state particles. Relating the plane-wave solution $\chi_{\mathbf{q}_i}$ on the left side of the matrix element [Eq.(4)] to the two-body scattering solution $\phi^-_{\mathbf{q}_i}$ through the Lippmann-Schwinger equation gives Eq. (4) as the distorted-wave T-matrix element

$$T^{(3)}_{\mathbf{q_i k_i k_0}} = -\pi \langle \phi^-_{\mathbf{q}_i} \chi_{\mathbf{k}_i} | V_i + V_i G^+_{0i} V_i | \psi^+_{\mathbf{k}_0} \rangle. \tag{6}$$

The R-matrix phenomenology is introduced by making in each of the Jacobi arrangements the expansion of Eqs. (1) and (2) for $\phi^-_{\mathbf{q}_i}$, giving for the 3-body breakup T matrix,

$$T^{(3)} = \sum_{i=1}^{3} \sum_{\lambda_i} c^{(i)}_{\lambda_i}(\mathbf{q}_i) \tilde{T}^{(2)}_{\lambda_i \mathbf{k}_i \mathbf{k}_0}, \tag{7}$$

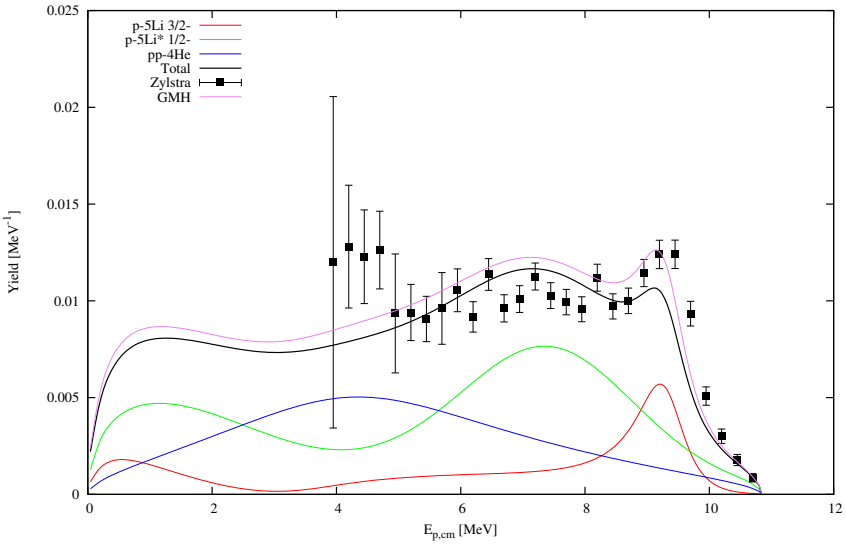

Figure 3: Proton spectrum from the ^{3}He^{3}He reaction at $E_{^3\mathrm{He}} = 0.33$ MeV. The data are from Ref.[3], and the colored lines give the 3 Jacobi components of the spectrum.

with

$$\tilde{T}^{(2)}_{\lambda_i \mathbf{k}_i \mathbf{k}_0} = -\pi \langle \phi^{(i)}_{\lambda_i} \chi_{\mathbf{k}_i} | V_i + V_i G^+_{0i} V_i | \psi^+_{\mathbf{k}_0} \rangle \tag{8}$$

$$= \sum_{Js'l'sl} \mathscr{Y}_{\lambda J s' l'}(\hat{\mathbf{k}}) O^{-1}_{l'}(k) R^{\bar{L}}_{\lambda s' l' s l}(\varepsilon) O^{-1}_{l}(k_0) \mathscr{Y}^*_{Jsl}(\hat{\mathbf{k}}_0), \tag{9}$$

a pseudo two-body off-energy-shell T-matrix element expressed in terms of the outgoing-wave R-matrix amplitudes $R^{\bar{L}}_{\lambda s' l' s l}(\varepsilon)$. In the applications described below, all the expansions are single-level ($\lambda = \lambda' = 1$), the resonance parameters in the c_λ are determined by outside data, and only the magnitudes of the R-matrix amplitudes $R^{\bar{L}}_{\lambda s' l' s l}(\varepsilon)$ are adjusted to fit the experimental spectra. The form of c_λ in the single-level approximation is

$$c_\lambda(\mathbf{q}) \sim \frac{1}{E_R - \frac{i}{2}\Gamma - \epsilon_q} e^{-i\phi_\ell} Y_\ell(\hat{\mathbf{q}}). \tag{10}$$

Here, (E_R, Γ) are the resonance parameters derived from the outside data and ϕ_ℓ is the hard-sphere phase shift.

3 Applications to A = 6 spectra

The single-level resonance parameters for the ground and first-excited states of ^{5}He and ^{5}Li were taken from analyses of $n\alpha$ and $p\alpha$ scattering; these are shown in the first three columns of Tables 4 and 5. Similarly, those for the singlet nn and pp resonances were taken from a charge-independent analysis of NN scattering.

The data fitting has been simplified by making several assumptions for this exploratory calculation. We neglect configuration mixing by considering just the magnitude-squared of each configuration

Channel (resonance)	J^{π}	E_R (MeV)	Γ(MeV)	$\lvert R^L \rvert$
^{5}He+n	$3/2^-$	0.99	0.65	0.157
^{5}He*+n	$1/2^-$	6.66	20.6	0.547
^{4}He+(nn)	0^+	-0.07	0.	0.338

Figure 4: Parameter values for $T(t,n)n\alpha$.

Channel (resonance)	J^{π}	E_R (MeV)	Γ(MeV)	$\lvert R^L \rvert$
^{5}Li+p	$3/2^-$	2.08	2.11	0.157
^{5}Li*+p	$1/2^-$	8.26	19.8	0.547
^{4}He+(pp)	0^+	1.56	0.	0.234

Figure 5: Parameter values for ^{3}He(^{3}He, p)$p\alpha$.

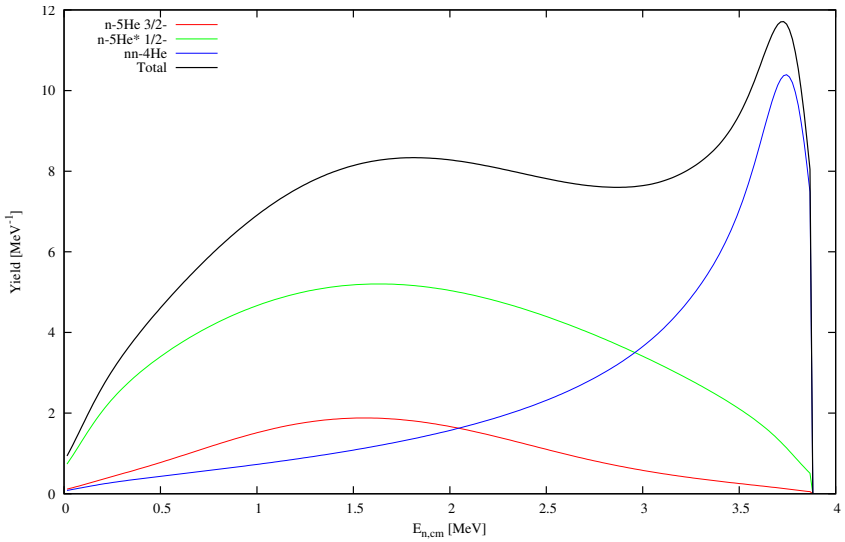

Figure 6: The α-spectrum for $T(t,n)n\alpha$.

amplitude in Eq.(7) and discarding interference terms. This permits the minimization by a least-squares approach. We have also neglected angular dependence of the di-neutron (nn) component of the wave function. The data have been fitted by taking into account the resolution broadening due to the instrument response functions. For the tt n-spectrum from Ref.[2] we use 0.250 MeV width in a Gaussian distribution; the p spectra was Gaussian broadened by 0.230 MeV.

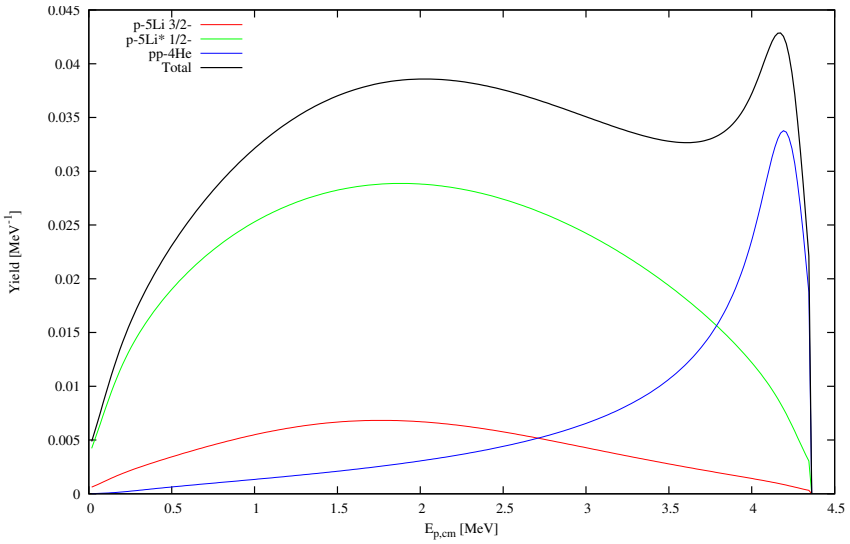

Figure 7: The α-spectrum for ^{3}He(^{3}He, p)$p\alpha$.

The R-matrix amplitudes R^L, in the right-most columns of the tables, were adjusted to simultaneously fit the tt n-spectra of Wong *et al.* [2] and the p-spectra of Zysltra[3], as shown in Figs.2 and 3. Charge symmetry has been used to constrain the R^L parameters in the first two rows of the tables to be the same values. The di-neutron and di-proton configuration parameters, in the third rows of the tables, are varied independently. The amplitudes determined in the simultaneous fit to the n and p spectra were used to predict the α-spectra for the tt and ^{3}He^{3}He reactions, as shown in Fig.6 and Fig.7.

4 Conclusions and outlook

The agreement with experimental $A = 6$ spectra indicates that the main features of the reactions are being captured within the present resonance model. Coulomb differences in the resonances are important, and lead to different branching ratios for the components of the spectra even with CS amplitudes. However, the peaks at the ends of the spectra coming from the ground states of ^{5}He and ^{5}Li are under-predicted, and the details of the interference terms could be improved.

Planned improvements include incorporation of data from ^{3}He t reactions from OMEGA[3] and the recent tt data determined at the National Accelerator Facility[5]. The fitting will be generalized from least-squares to allow the incorporation of configuration mixing. Improvements to the resonance model will be pursued to take into account angular dependence in the nn component of the wave function and dependence on the incident energy.

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