Nuclear Poincaré cycle synchronizes with the incident de Broglie wave to predict regularity in neutron resonance energies

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Abstract. In observed neutron resonances, long believed to be a form of quantum chaos, regular family structures are found in the s-wave resonances of many even-even nuclei in the tens keV to MeV region [M.Ohkubo, Phys. Rev. C **87**, 014608(2013)]. Resonance reactions take place when the incident de Broglie wave synchronizes with the Poincaré cycle of the compound nucleus, which is composed of several normal modes with periods that are time quantized by inverse Fermi energy. Based on the breathing model of the compound nucleus, neutron resonance energies in family structures are written by simple arithmetic expressions using S_n and small integers. Family structures in observed resonances of ^{40}Ca+n and ^{37}Cl+n are described as simple cases. A model for time quantization is discussed.

1 Introduction

In neutron-nucleus interactions, a compound nucleus (CN) is formed with excitation energy $E_x = S_n + E_n$, where S_n is neutron separation energy $\sim$ 8 MeV, and E_n is the neutron kinetic energy in the center of mass system (CMS). In the compound nucleus, many degrees of freedom will be excited and mixed to form very complicated states or so-called chaos. Statistical distributions of many observed neutron resonance data are in good agreement with predictions of the Random Matrix Theory (RMT), therefore, the neutron resonance region is long believed to be a form of quantum chaos where no regularity in level spacings/energies is expected.

However, as is seen in every field of science, different methods of analysis extract different features of the system under scrutiny. Many facts contradicting RMT have been reported in observed resonance energies/spacings over the past five decades that relate the frequent appearance of special spacings by D_{ij} (spacings between two arbitrary levels) analysis or by Fourier-like analysis [1].

In order to understand the mechanism of neutron resonance reactions including special spacings/energies, we developed the “Breathing Model" of the compound nucleus, analogous to classical resonance phenomena where nuclear Poincaré cycles synchronize with the incident de Broglie wave at resonances. This model predicts resonance energies by simple arithmetic expressions with good accuracy. In section 2 of this article, the concept of the Breathing Model is described. In section 3, theoretical insight into time behaviors of normal modes uses time quantization by inverse Fermi energy. In section 4, simple cases of two normal modes found in observed resonance data are shown. In section 5, regularity in resonance energies are described. In section 6, a preliminary time quantization

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mechanism of a ‘hot chain’ is described. In section 7, transition from simple CN to complex CN are described. Discussions and conclusion are found in section 8.

2 Breathing model of the compound nucleus in resonance reactions

In the neutron-nucleus reaction, the wave packet of an incident neutron is divided into two components: (1) pass-by component without interaction; and (2) penetration component that penetrates the CN, excites many normal modes, and appears at the CN surface after every recurrence time τ_{rec} or the “Poincaré period" τ_{PP}. Interference takes place between the two components, and the observed cross-sections are the result of the interference between the two wave packet components. We define the initial “coalescent phase" as the wave crest of the pass-by component, the gather, and the flare up of penetration components on the CN surface (doorway) during a short time $\sim \tau_0$ (described below). This coalescent phase appears repeatedly with a time period τ_{rec} which depends on the ensemble of normal modes excited at resonance. This behaves as a scattering center for the pass-by component during the resonance lifetime $\sim 2\pi\hbar/\Gamma$, where Γ is the resonance width. At resonances, the incident de Broglie frequency synchronizes with the nuclear Poincaré cycle as illustrated in Fig. 1. If the mechanism above does not exist, an energy independent potential scattering cross-section (~ 10 *rmb* $= 10 \times 10^{-28}\text{m}^2$) will be observed.

3 Theoretical insight

The penetration component excites the CN to energy $S_n(\sim 8$ MeV). As the lifetime of the CN formed by neutron resonance is relatively long, many excited degrees of freedom can be approximated by an oscillator ensemble rewritten as an ensemble of normal modes. Several combinations of normal modes are possible. The energy and time period of i-th mode are E_i and $\tau_i = 2\pi\hbar/E_i$, respectively. For M normal modes,the recurrence periods τ_{rec} are integer multiples of τ_i.

$$\tau_{rec} = n_i\tau_i, \qquad n_i\text{:integer}(i = 1, 2, ..., M), \tag{1}$$

where the influence of angular uncertainty ~ 1 rad is negligibly small for the lifetime of the resonance. From Eq. (1), τ_i/τ_j $(i, j = 1, 2, ..., M)$ are integer ratios. Therefore, τ_i $(i = 1, 2, ..., M)$ are restricted to be integer multiples of unit time τ_0, or time quantization by τ_0. For unit time τ_0, it is plausible to use the inverse Fermi energy,

$$\tau_0 = 2\pi\hbar/G = 1.2 \times 10^{-22}\text{s}, \tag{2}$$

where $G \sim 34.5$ MeV is almost the Fermi energy(the maximum energy of the nucleon in nuclear potential). Hence, the time period of the normal mode is $n_i\tau_0$, and its energy is G/n_i where n_i is an integer.

Although not all details are known, the Hamiltonian and wave functions of the CN described by M normal modes are formally written as

$$H = H_1 + ... + H_M, \tag{3}$$

$$\Psi(x, t) = \psi_1(x, t) \otimes ... \otimes \psi_M(x, t). \tag{4}$$

The time periodic recurrence of $\psi_i(x, t)$ and $\Psi(x, t)$ are

$$\psi_i(x_i, t) = \psi_i(x_i, t + n_i\tau_0), \tag{5}$$

$$\Psi(x, t) = \Psi(x, t + \tau_{rec}). \tag{6}$$

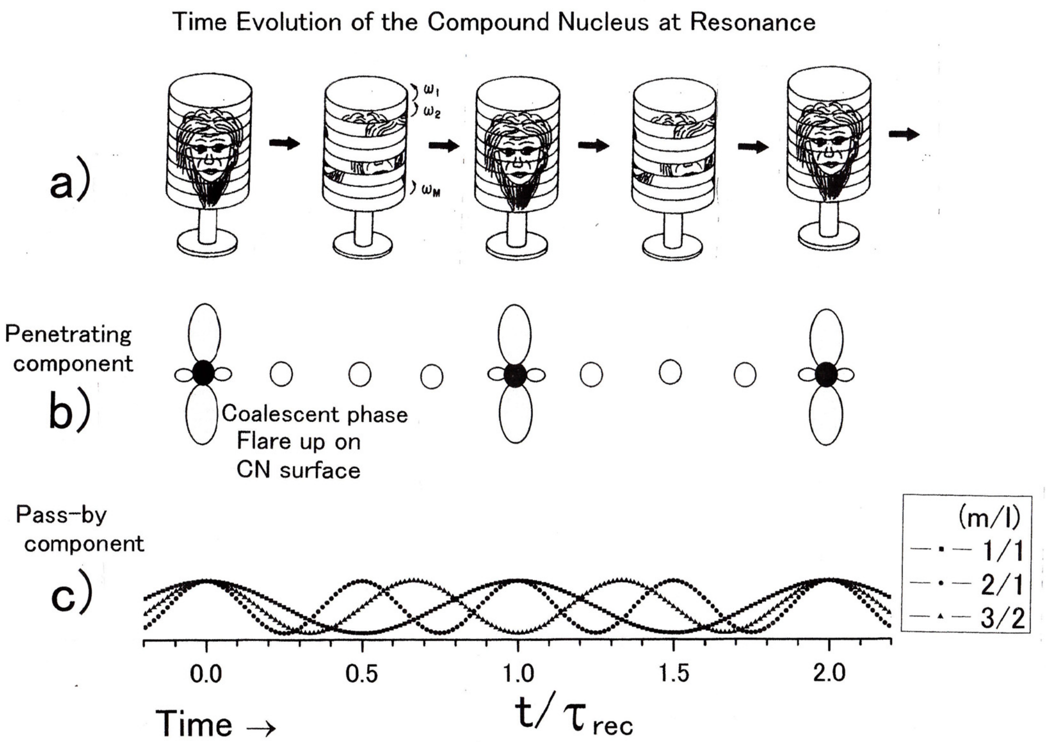

Figure 1. Time evolution of the compound nucleus at resonance. (a) A toy model is used to show recurrence of many oscillators rotating with arbitrary frequencies. (b) Penetration component, which excites CN and recurs (flare up on CN surface) at coalescent phases with the Poincaré periodicity, synchronizing with the pass-by component (c) at resonance.

The nuclear Poincaré period τ_{pp} or recurrence time τ_{rec} is defined as

$$\tau_{pp} = \tau_{rec} = LCM(n_1, ..., n_M)\tau_0, \tag{7}$$

where LCM is the least common multiple for a set of integers $(n_1, ..., n_M)$.

The recurrence energy E_{rec} is defined as

$$E_{rec} = \frac{G}{LCM(n_1, ..., n_M)}. \tag{8}$$

The excitation energy of the CN caused by the penetration component S_n is written as

$$S_n = E_1 + ... + E_M = G\left(\frac{1}{n_1} + ... + \frac{1}{n_M}\right) = G\sum_{i=1}^{M}\frac{1}{n_i}. \tag{9}$$

Neutron resonances take place when the period of the incident wave (pass-by component) $\tau_w(= 2\pi\hbar/E_n)$ are in simple integer ratios (k/m) of the nuclear Poincaré period τ_{rec}.

$$\tau_w = (k/m)LCM(n_1, ..., n_M)\tau_0. \tag{10}$$

That is, the possible resonance energies E_n in CMS are

$$E_n = (m/k)E_{rec}, \tag{11}$$

composing a family structure, where k, and m are small integers.

4 Simple cases

Despite expectations of complex features of CN, many simple cases are found in observed resonance data where only two normal modes are excited. The ratio S_n/E_n are in a simple form

$$\frac{S_n}{E_n} = (n_1 + n_2)\left(\frac{k}{m}\right). \tag{12}$$

and S_n is

$$S_n = \left(\frac{1}{n_1} + \frac{1}{n_2}\right)G. \tag{13}$$

In this case, many observed resonances dispose in vicinity of (IVO)

$$E_n = \left(\frac{m}{k}\right)E_{rec} = \left(\frac{m}{k}\right)\frac{G}{n_1 n_2} = \left(\frac{m}{k}\right)\frac{S_n}{n_1 + n_2}, \tag{14}$$

composing a family structure.

4.1 ^{40}Ca+n

In s-wave resonances in ^{40}Ca+n [2], we found that $S_n/E_n = 17(k/m)$ for many resonances (^{40}Ca (I)) which provided the clue to this problem [3]. Also, S_n is written as $S_n = 8362$ keV $= (17/70)G = (1/7 + 1/10)G$ where $G = 34434$ keV. Among 40 s-wave resonances observed in $0 \leqslant E_n^{obs} \leqslant 2500$ keV region, 15 resonances are IVO $E_n^C = (S_n/17)(m/k) = 491.9(m/k)$ keV with $(k, m \leqslant 10)$, where (m/k) are 1/3 , 3/7, 1/2, 7/10, 8/9, 1/1, 10/9, 7/6, 5/4, 4/3, 7/4, 2/1, 3/1, 4/1 and 9/2. For 10 resonances, the deviations $\delta = E_n^{obs} - E_n^c$ are within a region of 5 keV width.

The statistical probability of the appearance of the pseudofamily are calculated assuming a random distribution of $n_l = 40$ levels in the energy region $R = 2330$ keV [1]. Candidate cases (m/k) $(k, m \leqslant 10)$ are counted on a (k, m) plane to be $B = 53$. For a level placed at random in the region, the probability of being on a candidate region is $p = (\epsilon B)/R = 0.114$. For 40 levels placed at random, the expected number of integer ratios is $n_l p = 4.6$. The probability for 10 levels out of 40 being integer ratios is calculated by the binomial distribution: $P_r(10, 40, p) = {}_{40}C_{10}p^{10}(1-p)^{30} \approx 0.0083 = 0.83\%$, and the sum $P_s = \sum_{j=10}^{40} P_r(j, 40, p) \leqslant 1.2\%$.

In ^{40}Ca+n in the same energy region, another family coexists showing $S_n/E_n = 39(k/m)$. Many resonances dispose IVO $E_n^c = (S_n/39)(m/k) = 214(m/k)$ keV, called ^{40}Ca (II).

Similar regular family structures are found in s-wave resonances in target nuclei of ^{54}Cr,(I)(II), ^{64}Ni, ^{90}Zr, and ^{208}Pb.

For resonances including mixed J^π, similar family structures are found in target nuclei of ^{26}Mg, ^{37}Cl, ^{48}Ca, ^{50}Ti, and ^{58}Fe.

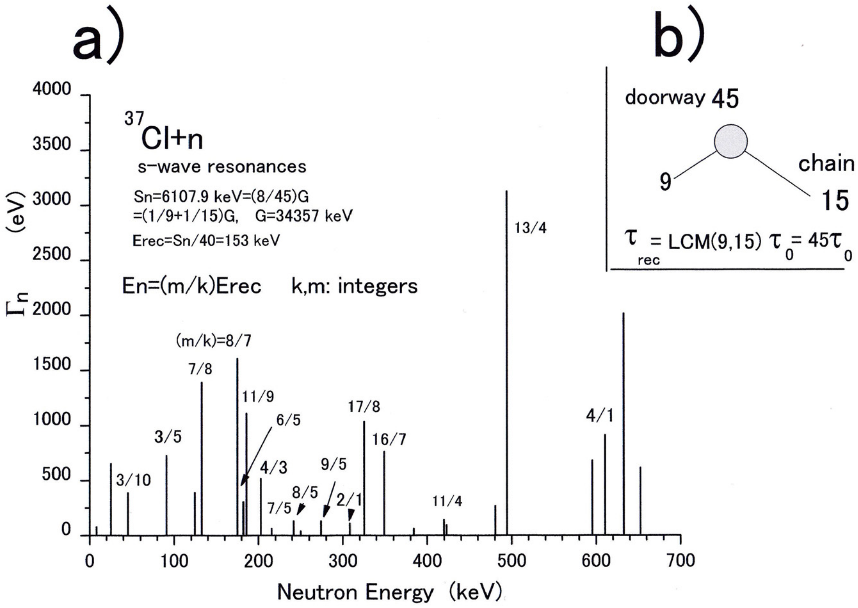

Figure 2. a) ^{37}Cl+n resonances. Neutron width Γ_n vs. incident neutron energy up to 700 keV [4]. S_n = 6107.9 keV can be written as $(8/45)G = (1/9 + 1/15)G$, where G = 34357 keV. To meet energy region requirements, a recurrence energy $E_{rec} = (1/225)G = S_n/40$ = 152.7 keV is used. b) Two normal modes (hot chains) of periods $9\tau_0$ and $15\tau_0$ couple to the doorway (circle) where coalescent phases appear with a time period of $45\tau_0$ repeatedly.

4.2 ^{37}Cl+n

As an example, the regular family structures of s-wave resonances in ^{37}Cl+n are described below where 1^+ and 2^+ resonances coexist. Original data from Sayer et al. [4] observed up to 1 MeV. The separation energy S_n is rewritten as S_n = 6107.9 keV = $(8/45)G = (1/9 + 1/15)G$ where G = 34357 keV. The recurrence energy is $E_{rec} = G/45 = S_n/8$ = 763.5 keV. In observed values S_n/E_n, a factor 40 appears for many resonances. Then $S_n = (40/225)G$ and one fifth harmonics of $E_{rec} = S_n/40$ = 152.7 keV are used. Many observed resonance energies E_n^{obs} dispose IVO E_n^c = $152.7(m/k)$ keV. Below $E_n \leqslant 500$ keV, 23 resonances are observed.

For 10 resonances, m/k are; 3/10, 3/5, 7/8, 8/7, 6/5, 4/3, 7/5, 8/5, 9/5, and 2/1 as illustrated in Fig.2a. Among these resonances, deviations $\delta = E_n^{obs} - E_n^c$ are within a width of 0.8 keV for 6 resonances. The probability of the appearance of these dispositions is calculated similar to the ^{40}Ca (I) described above. The parameters are: $n_l = 23$ in the region R = 500 keV, ϵ = 0.8 keV, $B = 55$, and $p = \epsilon B/R = 0.088$. $P_r = {}_{23}C_6 p^6(1-p)^{17} = 0.0097 = 0.97$ % and P_s are calculated to be 1.3 %.

5 Regularity in resonance energies

The probability of the appearance of the regular family structures P_s are quite small for several nuclei. In Table II of Ohkubo[1], P_s are listed for seven cases of four target nuclei, ^{40}Ca,(I)(II), ^{54}Cr,(I)(II), ^{64}Ni, and ^{90}Zr, where the average P_s is 1.3 %. In addition, P_s is 1.3 % for ^{37}Cl. We have investigated neutron resonances of about 40 light and magic nuclides. If we assume random dispositions of levels for all 40 nuclides, the expectation of it is less than one nuclide. However, it appears in 5 nuclides. The probability U of appearing in 5 nuclides among 40 nuclides with $P_s = 1.3\%$ is calculated by the binomial distribution as

$$U = {}_{40}C_5(0.013)^5(0.987)^{35} \approx 1.5 \times 10^{-4}. \tag{15}$$

The occurrence of a phenomenon with very small probability means a failure of the random hypothesis, and the existence of regular family structures are verified with the statistical significance level of $\sim 10^{-4}$. Though the random level ensemble substitutes for the RMT ensemble, this conclusion will not be changed for RMT ensemble.

6 Time quantization of normal modes

Time quantization of normal modes is inevitable for resonance reactions because of the finite time restriction of τ_{rec} as described in Eqs. (1)and (2). For the mechanism of time quantization of normal modes, we consider “hot chains" excited in CN. For example in ^{40}Ca(I), two normal modes of $7\tau_0$ and $10\tau_0$ are excited. These normal modes work as a time-delay-system for the penetration components. For the time-delay-system of $7\tau_0$, we consider a hot chain composed of seven elements. An element of the chain couples to the doorway, where the penetration component, called fire, enters and exits, and interference takes place with the pass-by component. The fire transfers to the neighbouring element in unit time τ_0, and the fire circulates in the chain in $7\tau_0$, repeatedly. Similar mechanisms work for the $10\tau_0$ chain. Fires in two chains gather and flare up at the doorway to form a scattering center for the pass-by component with a period $LCM(7, 10) = 70\tau_0$ repeatedly during the lifetime of the resonance.

7 From simple to complex

The simple cases above are for light or magic nuclei where neutron resonance level densities are not very high. Few normal modes couple to the doorway as illustrated in Fig. 2(b) and Fig. 3(a). For medium and heavy nuclei where level densities are high, many (M) normal modes couple to the doorway as illustrated in Fig. 3(b). M are estimated to be $M \leqslant 10$ for observed resonances [5]. Several different resonance families coexist in the same energy region and display complex features. For the resonances in the eV region, $LCM(n_1, ..., n_M)$ will be increased to $10^4 \sim 10^7$, and family structure analysis will be an interesting and difficult problem.

8 Discussion and Conclusion

The classical analogy of resonance phenomena is valid for neutron resonance reactions with some modification of physical concepts, i.e., the nuclear Poincaré cycle synchronizes with the incident de Broglie wave. Resonance families derived by arithmetic expressions in Eq. (11), agree well with the observed resonance energies. Time quantization of normal modes by τ_0 is essential to this work and simplifies the excitation modes in the energy region $\sim S_n$. Since τ_0 deviates about ±1% from family to family depending on the internuclear interactions, the value $G = 2\pi\hbar/\tau_0 \sim 34.5$ MeV deviates consequently.

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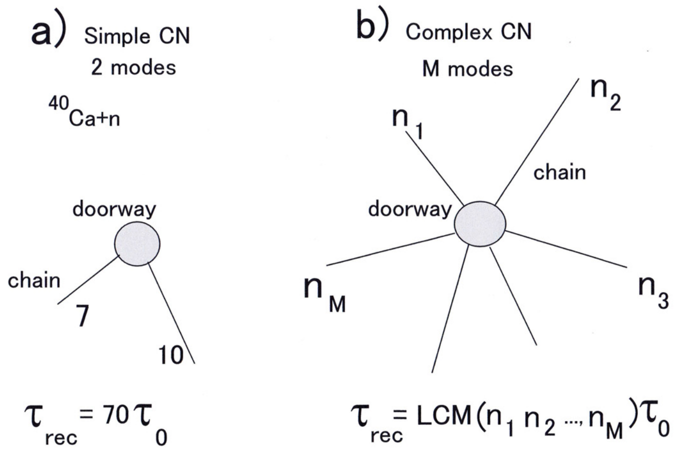

Figure 3. (a) Simple case of ^{40}Ca+n compound nucleus. Two normal modes (hot chains) of periods $7\tau_0$ and $10\tau_0$ couple to the doorway (center circle) where coalescent phases appear with time period $70\tau_0$ repeatedly. (b) Complex compound nucleus. Many ($M \leqslant 10$) normal modes couple to the doorway where coalescent phases appear with time period $\tau_{rec} = LCM(n_1, ..., n_M)\tau_0$, repeatedly.

Time behaviors of the compound nucleus are extracted by analysing energy structures of neutron resonance levels. This is mathematically analogous to determining crystal structures from neutron diffraction patterns using a neutron plane wave $\exp(i(kx - \omega t))$, where k is wave vector, x is a space coordinate, ω is frequency, and t is time. The crystal diffractions are in the (k, x) domain, scattering from atoms at space-periodic lattices, whereas the neutron resonances are in the (ω, t) domain, scattering from time-periodic coalescent phases on the CN surface.

The traditional idea of quantum chaos in the neutron resonance region is not correct. For a long time, it was surmised that regularities in resonance positions/spacings did not exist because of the success of the Random Matrix Theory, supported by many observed neutron resonance data analysed by traditional statistical methods. These methods, however, could not detect strong correlations of family structures as described in this article. To this point, it must be stressed that traditional statistical analyses have no ability to distinguish between chaos and regularity. Our results offer a new explanation for a regular system for this region.

On the other hand, rules on strengths and J^π of each resonance, which will depend on spatial components of wave functions, are not known at present. However, this approach seems to be promising to investigate nuclear dynamic structures in discrete energy regions.

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