

Multi-quasiparticle high- K isomeric states in deformed nuclei

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Abstract. In the past years, we have made many theoretical investigations on multi-quasiparticle high- K isomeric states. A deformation-pairing-configuration self-consistent calculation has been developed by calculating a configuration-constrained multi-quasiparticle potential energy surface (PES). The specific single-particle orbits that define the high- K configuration are identified and tracked (adiabatically blocked) by calculating the average Nilsson numbers. The deformed Woods-Saxon potential was taken to give single-particle orbits. The configuration-constrained PES takes into account the shape polarization effect. Such calculations give good results on excitation energies, deformations and other structure information about multi-quasiparticle high- K isomeric states. Many different mass regions have been investigated.

1 Introduction

Atomic nuclei can be excited by breaking paired nucleons [1–4]. If the unpaired nucleons couple to a high angular momentum (usually resulting in a large angular momentum projection onto the symmetry axis of the deformed nucleus, named K), the excited state can be an isomer due to the forbiddenness of electromagnetic decays from high- K to low- K states [1, 2]. For example, the $K^\pi = 8^-$ isomers with the configuration $\pi^2\{7/2^+[404] \otimes 9/2^- [514]\}$ systematically occur in ^{170–184}Hf, with half-lives that range from nanoseconds to hours. The well-known 31-yr $K^\pi = 16^-$ isomer at 2.4 MeV in ¹⁷⁸Hf is still attracting interest for possible applications. An abundance of high- K isomers have been observed across the entire chart of nuclides [1, 2] including superheavy (see Ref. [5] and references therein) and drip-line mass regions [6, 7].

Theoretically, previous calculations were usually performed assuming that isomers have the same deformation as the ground state of the nucleus (see, e.g., [8]). However, high- Ω orbits (Ω is defined as the nucleon angular momentum projection onto the symmetry axis of the deformed nucleus) usually have a strong deformation-driving force, that would make the isomer shape deviate from that of the ground state. Shape polarization is more obvious in soft nuclei [9–12]. Furthermore, the pairing is another crucial factor that drastically affects the excitation energy of multi-quasiparticle states. The reduction of pairing in broken-pair multi-quasiparticle states leads to different pairing energies from the ground state. Configuration-constrained PES calculation takes into account these as-

pects in a self-consistent manner with respect to deformation and pairing [3, 13, 14].

2 Model

We start from the non-axial deformed Woods-Saxon (WS) potential and use the universal potential parameters [15], with the monopole pairing strength G determined by the average gap method [16]. The quadrupole pairing can also be included [17] but its effect on the excitation energies of multi-quasiparticle states is small [18]. Due to nucleon pair(s) broken in multi-quasiparticle states, the pairing correlation is reduced remarkably in high- K states. To avoid the spurious phase transition encountered in the BCS approach, we approximate particle number projection by means of the Lipkin-Nogami (LN) pairing method. In the macroscopic-microscopic model, the total energy of a state contains a macroscopic part that is calculated by the liquid-drop model and a microscopic part that can be obtained by the Strutinsky shell correction. This leads to the energy of a configuration that is given by [3]

$$E_{LN} = \sum_{j=1}^S e_{k_j} + \sum_{k \neq k_j} 2V_k^2 e_k - \frac{\Delta^2}{G} - G \sum_{k \neq k_j} V_k^4 + G \frac{N-S}{2} - 4\lambda_2 \sum_{k \neq k_j} (U_k V_k)^2, \quad (1)$$

where e_k is the energy of the k -th single-particle orbit, and V_k^2 (or U_k^2) is the occupation (or unoccupation) probability of the orbit in the BCS wave function. G is the pairing strength, and Δ is the pairing gap. S is the seniority

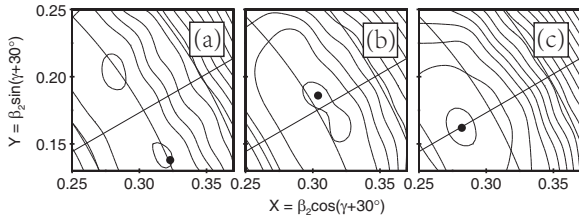


Figure 1. Calculated potential energy surfaces for the lowest-energy two-quasiparticle states in (a) ^{104}Zr ($K^\pi = 5^-, \nu_2^{5-} [532] \otimes \nu_2^{5+} [413]$), (b) ^{108}Zr ($K^\pi = 4^-, \nu_2^{7-} [523] \otimes \nu_2^{7+} [411]$), and (c) ^{110}Zr ($K^\pi = 6^-, \nu_2^{5+} [402] \otimes \nu_2^{7-} [523]$), as a function of quadrupole deformation parameters β_2 and γ , with each value minimized with respect to β_4 . The energy interval between contours is 200 keV.

of the given type of nucleon, i.e., the number of unpaired protons or neutrons (indicated by k_j), and N is the number of protons or neutrons. λ_2 is a Lagrange multiplier that brings the second-order (ΔN^2) correction to the fluctuation of the particle number in the BCS wave function. Adiabatic blockings are achieved by tracking the specific single-particle orbits by calculating their average Nilsson numbers that evolve smoothly with changing deformation [3].

The potential energy surface (PES) is calculated in a lattice of quadrupole deformations (β_2, γ) with a hexadecapole β_4 variation [3]. The β_4 deformation is determined by minimizing the energy at each (β_2, γ) point. The γ deformation plays an important role in the description of nuclear collective rotation [17, 19, 20] and high- K excitations [21–25]. The γ deformation results in K -mixing (K is a conserved number for axially symmetric shape [26–28]). In heavy mass regions, higher-order deformations, e.g., β_6 and β_8 , or reflection-asymmetric octupole deformation β_3 can appear [29–31]. Oblate isomers have also been predicted [32]. The pairing strength is readjusted by including mean-field and blocking effects [33]. The readjustment is crucial to reproduce experimental excitation energies.

3 Calculations

We have investigated multi-quasiparticle high- K states in different mass regions. Furthermore, PES calculations give excitation energies, deformations, g -factors, and deformation softness. The configurations can be analysed by comparing with experimental observations. Figure 1 shows configuration-constrained PES's for the lowest-energy two-quasiparticle high- K states in neutron-rich zirconium isotopes [34]. It can be seen that the calculated PES's show energy minima (giving the deformations of the states) and situations of deformation softness. The information is useful for analyses of the structure of a nucleus. Both triaxial deformation and softness can lead to K mixing and hence affect the decay properties of the multi-quasiparticle state.

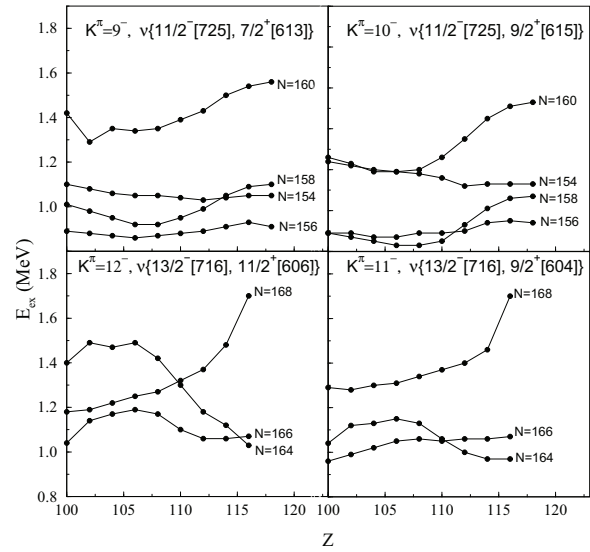


Figure 2. Calculated excitation energies for the predicted two-quasineutron high- K states of superheavy nuclei.

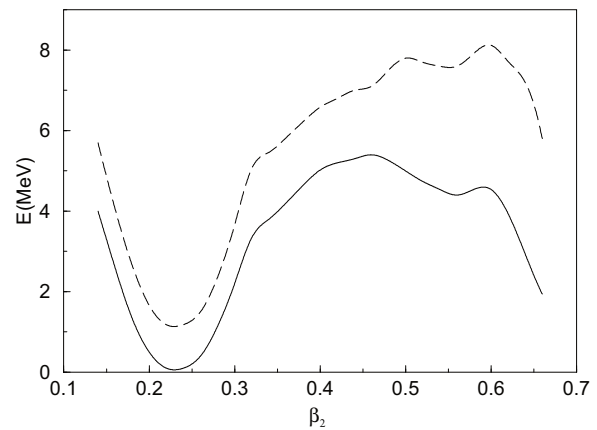


Figure 3. Calculated potential energies versus β_2 deformation in ^{256}Fm . The solid and dashed lines represent the ground state and the $\pi_2^{7+} [633] \otimes \nu_2^{7-} [514]$ isomer, respectively. At each β_2 point, the energy has been minimized with respect to the γ and β_4 deformations.

It is interesting that high- K isomers can have longer lifetimes than corresponding ground states [35]. This is due to the K selection rule in electromagnetic transitions. We have made a systematic investigation of possible high- K isomers in superheavy nuclei [5]. Figure 2 predicts possible two-quasineutron states in superheavy nuclei. The calculated excitation energies are low, around 1.0 – 1.5 MeV. The low-lying high- K states should be isomers that can be observed experimentally. Both calculations [5, 38] and experiments [35–37] show that high- K isomerism can enhance the stability of superheavy nuclei, that is important for the experimental study of the heaviest nuclei. Figure 3 displays the calculated energy curves as a function of deformation for the ground state and a

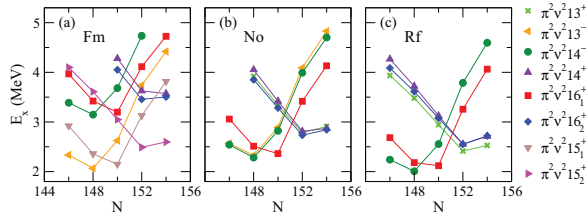


Figure 4. Predicted excitation energies for four-quasiparticle high-K states in Fm, No, and Rf nuclei. Configurations are $\pi^2\nu^2 13^+ \{\pi 1/2^- [521], \pi 9/2^+ [624], \nu 7/2^+ [613], \nu 9/2^- [734]\}$, $\pi^2\nu^2 13^- \{\pi 7/2^+ [633], \pi 7/2^- [514], \nu 5/2^+ [622], \nu 7/2^+ [624]\}$, $\pi^2\nu^2 14^+ \{\pi 7/2^- [514], \pi 9/2^+ [624], \nu 5/2^+ [622], \nu 7/2^+ [624]\}$, $\pi^2\nu^2 14^- \{\pi 7/2^- [514], \pi 9/2^+ [624], \nu 3/2^+ [622], \nu 9/2^- [734]\}$, $\pi^2\nu^2 16^+ \{\pi 7/2^- [514], \pi 9/2^+ [624], \nu 7/2^+ [624], \nu 9/2^- [734]\}$, $\pi^2\nu^2 16^- \{\pi 7/2^- [514], \pi 9/2^+ [624], \nu 7/2^+ [613], \nu 9/2^- [734]\}$, $\pi^2\nu^2 15^+ \{\pi 7/2^+ [633], \pi 7/2^- [514], \nu 7/2^+ [624], \nu 9/2^- [734]\}$, $\pi^2\nu^2 15^- \{\pi 7/2^+ [633], \pi 7/2^- [514], \nu 7/2^+ [613], \nu 9/2^- [734]\}$.

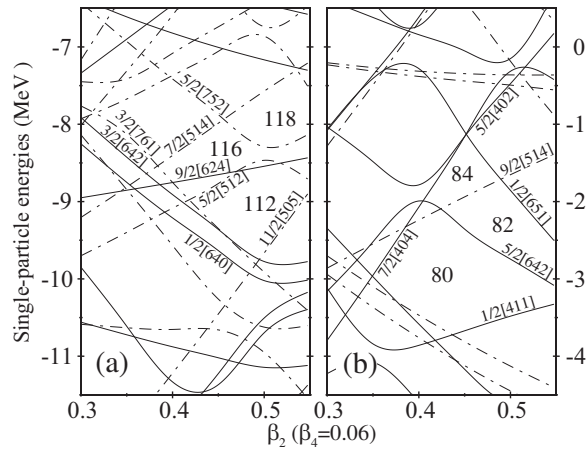


Figure 5. Neutron (a) and proton (b) single-particle levels plotted against β_2 deformation ($\beta_4 = 0.06$, $\gamma = 0^\circ$).

$K^\pi = 7^- \pi 7/2^+ [633] \otimes 7/2^- [514]$ isomer in ^{256}Fm . We see that the isomer has a higher and wider barrier than the ground state. It implies that the isomer should be more stable against fission which is the main decay mode of the heaviest nuclei. Figure 4 predicts some low-lying four-quasiparticle high-K states in Fm, No and Rf isotopes. Experiments have observed two-quasiparticle and four-quasiparticle isomers in ^{254}No [36]. Our configuration-constrained PES calculations give good descriptions of the experimental excitation energies and configuration assignments [5, 38]. It has also been found that high-spin isomerism can enhance the stability of drip-line nuclei [6], which may extend the borders of the nuclide chart.

High-K states at extremely elongated shapes present another interesting topic related to nuclear isomerism. A large axially-symmetric prolate deformation would provide a favoured condition to preserve the intrinsic angular momentum K , and hence enhance the stability of the metastable state. Experiments have observed many su-

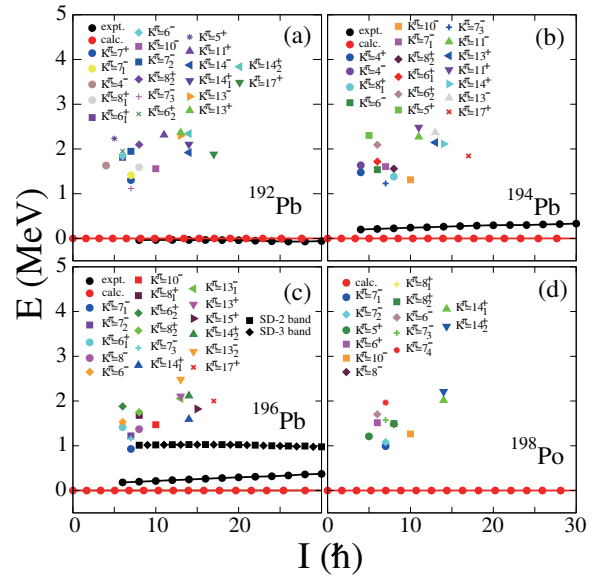


Figure 6. Excitation energies relative to the calculated yrast SD band. Experimental data are in black. A variety of multi-qp states are shown for ^{192}Pb [(a)], ^{194}Pb [(b)], ^{196}Pb [(c)], and ^{198}Po [(d)].

perdeformed (SD) rotational bands. The most well-known mass region for the SD rotation is around $A \sim 190$. It is an interesting open question whether multi-quasiparticle high-K states exist at superdeformation and can be both populated and detected in experiment. Figure 5 shows that there are many high- Ω single-particle orbits in the SD range of $\beta_2 \approx 0.4 - 0.5$. If SD high-K states are located at low energies (relative to SD rotational bands), they would be populated.

With the configuration-constrained PES method, we have performed systematic calculations to search for possible SD multi-quasiparticle high-K states in the mass $A \sim 190$ region [39]. Figure 6 gives predicted SD two-quasiparticle high-K states. Their excitation energies are about 1.0 – 2.0 MeV higher than the calculated yrast SD rotational band. Figure 7 shows a scheme for electromagnetic transitions from possible SD high-K bands, which may provide guidance for future experiments.

4 Summary

We have performed deformation-pairing self-consistent configuration-constrained potential-energy-surface calculations for multi-quasiparticle high-K states in different mass regions. The calculations show that high-K states exist systematically in the superheavy mass region. The calculated excitation energies of high-K states can compete with those of collective rotational states. Experimentally several high-K isomers have been identified in the superheavy region. The present calculations can well reproduce the experimental data. We have also predicted superdeformed high-K states in the $A \sim 190$ region where superdeformed rotational bands have been observed. The calculated excitation energies are about

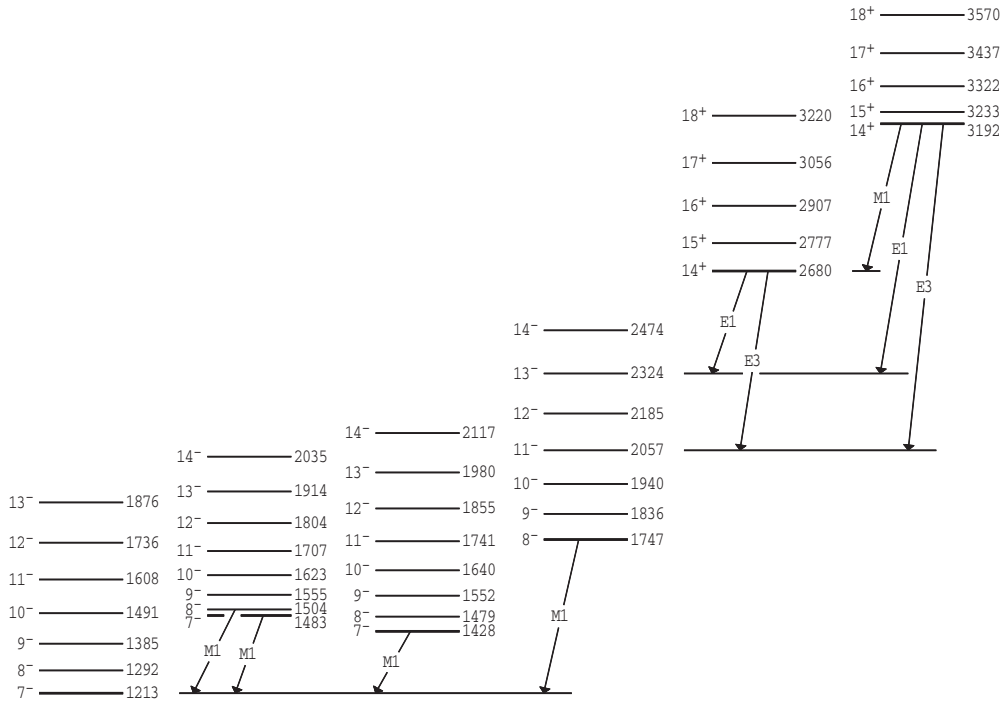


Figure 7. Notional decay scheme of calculated high- K SD bands in ^{196}Pb , showing the main decay paths expected between bands. Energies are in keV, relative to the SD minimum.

1.0 – 2.0 MeV above the observed yrast superdeformed rotational band. The predictions can provide useful guidance for experiments. The collective rotations of multi-quasiparticle high- K states have also been calculated by the configuration-constrained total Routhian surface method [40, 41].

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