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Nuclear lifetime measurements from data with independently varying observation times

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Abstract. A method is presented for obtaining ground-state and isomeric lifetimes from storage-ring data. The method published by Schmidt *et al.* is extended to the problems and solutions associated with applying their method to storage-ring data. These developed procedures are applied to real experimental data for ^{193}Re, and a mean-lifetime value of 37^{+9}_{-7} s is obtained. This is consistent with a previous measurement.

1 Introduction

As experiments start probing nuclei further away from the line of stability there is an inevitable drop in production cross-sections [1]. Therefore, obtaining estimates of the mean lifetimes of nuclei or nuclear states where poor or few measurements are available is becoming an important problem to address. In this paper, methods of lifetime measurement are investigated for use with storage-ring data. Different scenarios are explored through Monte-Carlo methods and tested on real data; the case of ^{193}Re is discussed.

This paper will focus on a particular application to storage-ring data, although in principle it can be applied to any experiment where the time of a single event can be measured. The data for ^{193}Re was produced using in-flight separated projectile fragmentation of a ^{197}Au beam on a ^{9}Be target. For details of the experiment, see Ref. [2]. The reaction products were injected into the Experimental Storage Ring (ESR) [3] and measured over many revolutions; the techniques used for these measurements are not discussed here but can be found in Refs. [4, 5].

A method for measuring lifetimes that has been used successfully in the past and been applied to storage-ring data [2, 6] is that presented by Schmidt *et al.* [7]. In Ref. [7], the mean lifetime is derived through what we have termed 'the unrestricted mean-lifetime method'. Here, each nucleus or nuclear state is measured from some start time to the point at which it is seen to decay, either by direct measurement, i.e. observation of a γ-ray, β or α particle, or inferred, as in the case of the storage-ring where a change in ion-orbital frequency implies a decay has occurred. In the case of storage-ring data, multiple ion events can be used, as long as each individual ion can be observed until it decays, and the individual times of survival measured [8]. A number (n_d) of these decays are measured and the observed 'survival' times summed together to give a total observation time (t_d); thus, simply dividing the two numbers gives the mean lifetime. The error on such a measurement is explained in Ref. [7] and it is implicit that this method can only be used when the observation time of each event is unrestricted, i.e. the each ion is observed until it decays.

However, in many situations with storage-ring data, an unrestricted observation time cannot be obtained, therefore an alternative method needs to be used to evaluate the lifetime. One such method using decay curves has been presented in Akber *et al.* [9]. However, this has limited applicability when only a few ions are observed as the decay curve is not well defined. Another method for extracting information from these data is suggested below.

2 Restricted Mean-Lifetime Method

In some experiments a fixed time for the observation will be used, for example, a 60-s measurement time. The unrestricted mean-lifetime method can be adjusted such that some information can be gained from the lifetime. As with the unrestricted version, the estimate of τ is obtained by dividing the total observation time by the number of decays seen. A Monte Carlo simulation was used to investigate the effectiveness of this method. Multiple simulations have been performed, but for the purposes of this paper τ is fixed at 60.4 s, and measurements are taken every 3.5 s; these numbers are chosen to approximate the conditions found in the data for ^{193}Re. A measurement corresponds to checking if the ion is still observed in the relevant time bin

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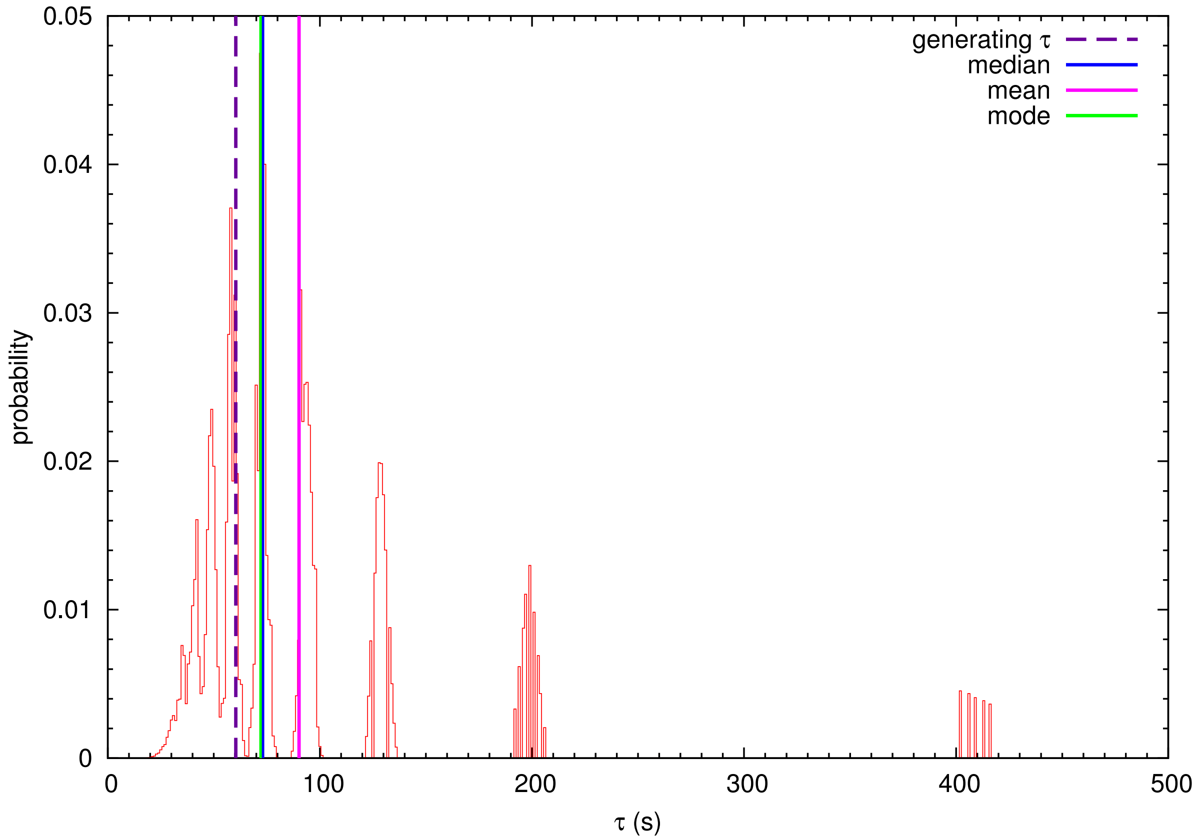

Figure 1. The probability density function obtained from the Monte Carlo simulation using τ = 60.4 s, n = 20, and b = 6 (for a total observation time of 6 × 3.5 s = 21.0 s). The number of counts reflects the likelihood of obtaining the given τ from the average time method. The purple/dashed line shows the generating τ used in the simulation. The green line shows the maximum (mode) of the distribution, whilst the blue line indicates the median. The mean is marked by the magenta line.

– i.e. has not decayed. A number of different parameters can be varied, such as the initial population of ions n, and the number of possible time bins b (the largest possible observation time for one ion is thus $t_b \times b$ s, where t_b is the size of the time bin). The probability of decay in one time bin is $1-\exp(-t_b/\tau)$, and a random number is generated to decide whether the ion has decayed in that time bin. This is repeated b times, and a time t is incremented by 3.5 s at each iteration where no decay is observed. If a decay is observed, the current t is added to a cumulative total t_{obs}, and the number of observed decays n_d is increased by 1. If no decay is observed, then $t_b \times b$ is added to the total t_{obs}. This process is repeated n times, and the final lifetime estimate is taken as

$$\overline{\tau} = \frac{t_{obs}}{n_d}.$$

By repeating this entire calculation numerous times, a probability density function can be found by histogramming the obtained mean-lifetimes ($\overline{\tau}$), as shown in Fig. 1. This represents the relative likelihood of obtaining an estimate of τ (along the x-axis) from the previously outlined method. Note the oscillatory behaviour in Fig. 1; each individual peak corresponds to a particular number of decays (since many ions survive for the full observation time). If n_d is low, the values of τ will be high, but as n_d becomes higher, the peaks overlap and form a smooth curve. The magnitude of each individual peak corresponds to the likelihood of that particular n_d occurring.

Once a histogram has been generated, an important question is which value best describes the outcomes of sampling from the distribution. For example, in the case of a gaussian curve, the mean is chosen as both the expected value, most likely outcome, and median. In less-ideal cases the decision may be unclear. The mean may be the most obvious choice as the expected value, however in some cases the mean is in fact impossible to obtain as an outcome. The mode, being the most likely outcome is also a possible choice, however cases where a maximum in the broad function corresponds to a minimum in the oscillations can result in a mode that either does not exist or is far away from the next most likely peak. For the purposes of this paper the median is chosen, as 50% of estimates will be above the median, and 50% below. See Fig. 1 for a comparison of the different averages.

As b and n increase, the probability density tends to a continuous distribution, and the difference between the actual τ and the median decreases, leading to estimates that are in general closer to the true τ. To account for the difference between the measured τ and the median of the dis-

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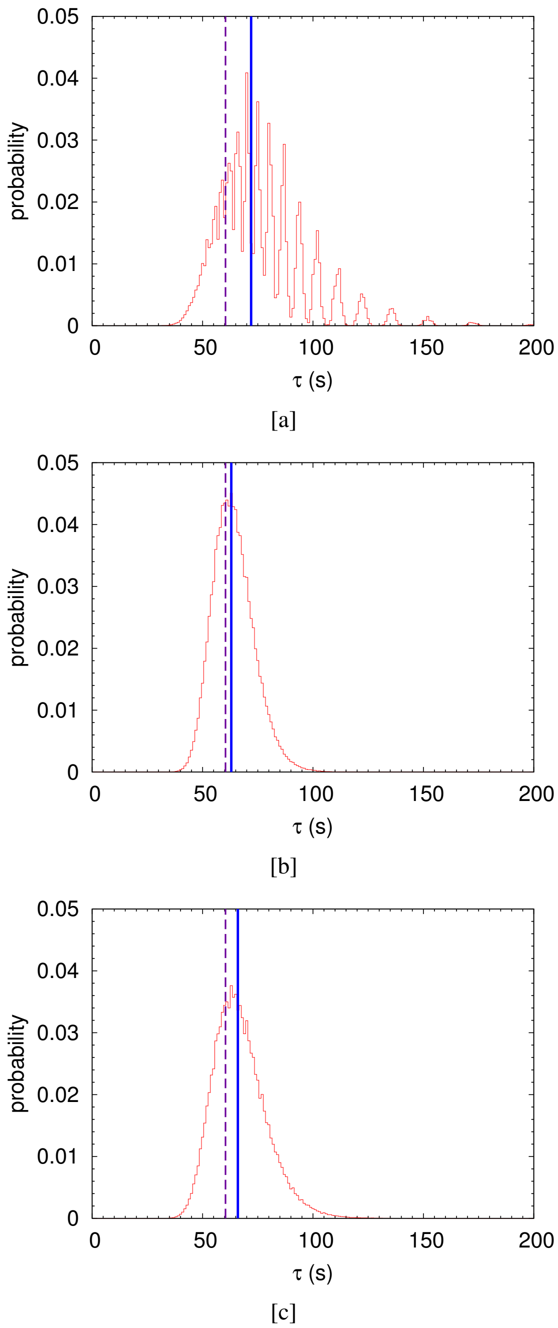

Figure 2. Panel (a) shows the histogram for the conditions $b = 6$, $n = 70$. The oscillations and skew can be clearly seen, with a significant difference between the median (blue) and generating τ (purple/dashed). Panel (b) shows the probability distribution produced by using $b = 21$, $n = 70$. In this case the method is clearly a much better estimate, with a smooth histogram and a median close to the generating τ. Panel (c) is produced using $b = 6$ for 35 ions, and $b = 21$ for the remaining 35. Qualitatively, this is a combination of (*a*) and (*b*), the distribution is much smoother than (*a*), but with a higher variance and skew than (*b*).

tribution a method has been developed for correcting the initial average time estimate to recover a best fit lifetime (Section 4).

There is uncertainty in the initial estimate obtained from experimental observations. The lifetime of each ion is not measured precisely due to the bin size. It is unknown where in the relevant bin the ion decays, and so each time measurement has an error of 3.5 s (the ^{193}Re case is discussed here). Thus the (experimental) error of $\overline{\tau}$ is given by

$$\Delta\overline{\tau} = \frac{\sqrt{3.5^2 + 3.5^2 + ... + 3.5^2}}{n_d} = \frac{3.5\sqrt{n}}{n_d}.$$

It is important to distinguish this experimental error from the statistical error inherent in the method itself. The first is a function of experimental limitations (namely precision of measurement), whilst the second is a result of the probabilistic nature of decay. This statistical error will be examined in Section 4.

3 Independently Varying Observation Time

A more common and widely applicable scenario is that the observation time is not fixed, but has a varying time window. The reason for this varying time window is the nature of the experiment itself. Generally, when an experiment is run, a region of interest can be monitored online, and then measurement times changed such that certain ions of interest can be observed for long time periods. This has a knock-on effect that the regions that are not monitored will also have the new measurement time. It is then a possibility that ions not in the observation region will be trapped for this extended period and more data taken. It is clear that these data can be reduced to the previous (fixed) case by cutting each observation off to the shortest observation time. However, this removes information that could otherwise have been used; what is proposed here is to use this additional information to obtain a more precise result.

Similar to before, a Monte Carlo simulation can be run, this time varying the observation times in a pre-defined configuration. For example, for a total population of 20 ions, the simulation can run the first 10 with $b = 5$, the next 5 with $b = 9$, 3 with $b = 15$, and the final 2 with $b = 30$. This has an effect of smoothing the probability density function such that it starts to tend to a continuous distribution. Figure 2c shows an example of this. Whilst this situation is not ideal, it is preferred to the fixed observation window. Although having a larger number of observation bins, larger observation window or more ions makes the most marked improvement, it is clear that the addition of a varying window length substantially cleans the probability distribution. However, in all cases a correction will need to be performed to obtain a correct estimate of the lifetime. This is discussed in Section 4.

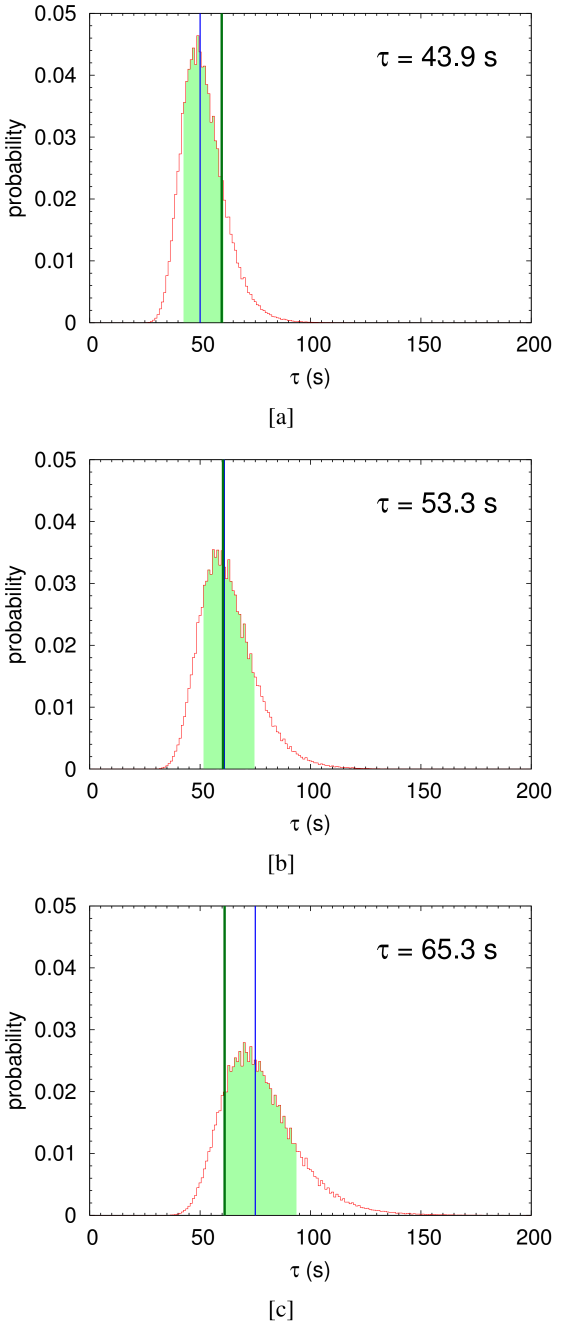

Figure 3. Probability distributions used to evaluate the uncertainties on the corrected value. The green line shows the initial uncorrected lifetime (and the blue line shows the median of the distribution, the shaded area is 68% of the distribution (1σ either side of the median). The number shown in each panel is the generating τ used to produce the histogram. Panel (a) shows the lower uncertainty estimate of the corrected τ. The lower uncertainty estimate is $\tau = 43.9$ s, i.e. the value used to generate the distribution. Panel (b) shows the estimate for the corrected τ. Panel (c) shows the upper uncertainty estimate of the corrected τ. All distributions are generated with the observation time configuration of the ^{193}Re data.

4 Correcting the Initial Mean-Lifetime Estimate

A methodology has been produced to correct for the inherent inaccuracy of the scenarios discussed above. The real experimental case of ^{193}Re has been used to as an example of how this can be applied to gain a genuine result. In the case of ^{193}Re, 74 single ion events were measured, with observation times ranging between 21.0 s and 234.5 s [10]. To obtain a real value for τ (the mean-lifetime) an adjustment to the initial mean-lifetime estimate must be made. To achieve this, a recursive Monte Carlo simulation is run. The initial estimate of $\tau = \tau_0 = \frac{t_{obs}}{n_d}$ is used to generate a histogram similar to Fig. 2 (c). The median of the distribution (m_0) is then compared with τ_0, and an offset $a_1 = m_0 - \tau_0$ evaluated. The Monte Carlo simulation is then run again with the new lifetime estimate of $\tau = \tau_1 = \tau_0 - a_1$. The recursive steps are summarised below:

$$a_{i+1} = m_i - \tau_0,$$

$$\tau_{i+1} = \tau_i - a_i,$$

with each median calculated from the histogram generated by setting $\tau = \tau_i$. The observation times used for each simulation are the same as those in the experimental data. After a number of steps, the result is a τ which produces a histogram with a median at τ_0, the initial estimate (Fig. 3b). This τ is the corrected value for the mean-lifetime of the ion.

To find error estimates for the corrected τ, the highest and lowest τ values for which it is reasonable to obtain an initial mean-lifetime estimate of τ_0 are found. For each histogram generated, define $l_i < m_i$ and $u_i > m_i$ such that 34% of the distribution area (1σ) is between l_i and m_i, and 34% (1σ) between m_i and u_i. Note that for a particular probability distribution, if $l_i \leqslant \tau_0 \leqslant u_i$ then it is reasonable to obtain an initial mean-lifetime estimate of τ_0. Conversely, if $l_i > \tau_0$ or $u_i < \tau_0$, it is unlikely ($> 1\sigma$) that τ_0 is obtained. Thus the highest reasonable τ is that which generates a distribution such that $l_i = \tau_0$. Similarly, the lowest reasonable τ is that which gives $u_i = \tau_0$.

If there is experimental error in τ_0, the high and low bounds of τ_0 are τ_{0h} and τ_{0l}. Then the highest reasonable (corrected) τ gives a distribution with $l_i = \tau_{0h}$, and the lowest reasonable τ gives $u_i = \tau_{0l}$. Figures 3(a) and (c) show this.

To find the lifetimes that generate the appropriate distributions, two more recursive processes are run, one converging to $\tau_0 = l_i$, and one to $\tau_0 = u_i$. This is achieved by having $a_{i+1} = l_i - \tau_0$, or $a_{i+1} = u_i - \tau_0$.

This process has been used on the 74 single ion observations of ^{193}Re. The initial τ_0 estimate obtained (before correction) is 60.4 ± 0.7 s. After applying the correction developed in section 4, a laboratory frame lifetime of $\tau = 53^{+12}_{-10}$ s was deduced. This lifetime needs to be further corrected to account for relativistic effects corresponding to the ion velocity of $\frac{v}{c} = 0.714$. The final value obtained is 37^{+9}_{-7} s; this can be compared to the only other result we know of, an unpublished value in Ref. [11] of 51^{+8}_{-8} s. Both values agree within uncertainties.

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5 Discussion

This method has clear advantages over others which may be used for similar data sets. In comparison to curve fitting procedures as seen in [9], the corrected mean-lifetime method allows for all measured data to be used, without requiring all ions to be observed for the same period of time. This is particularly advantageous when only small numbers of decays are observed. It is also more widely applicable than methods which require an unrestricted lifetime. However, the limitations of the method are clear considering Figs. 1 and 2 (a). The discrete nature of the probability distributions for low n and b mean that any measurement obtained is inherently unreliable. Whilst a rough estimate of τ may be obtained, the applicability of the recursive correction outlined in Section 4 is less clear when the histogram is not smooth. In these cases it is not clear that the median is the correct average to use, however, the mode is equally problematic if a minimum in the fine oscillations coincides with the maximum of the broad function. The smoothing effects are difficult to quantify due to the large number of parameters, in particular, the wide variety of observation time configurations. However, on a case-by-case basis, the applicability of this method can be evaluated by modelling the observation time configuration observed in the experiment. An analytical solution to the problem is a subject for further study and beyond the scope of this paper.

6 Conclusion

The unrestricted mean-lifetime method has been adjusted to both restricted observation times, and independently varying observation times. The method allows all measured data to be used, as opposed to other methods which require a fixed observation time. A recursive algorithm corrects for the bias in the methods, and can be applied on an individual basis to specific experimental observation time configurations. The results obtained from this method for the ^{193}Re data are consistent with those obtained previously.

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