The SM extensions with additional light scalar singlet, nonrenormalizable Yukawa interactions and \((g - 2)\)\(\mu\)

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Abstract. We consider the SM extension with additional light real singlet scalar, right-handed neutrino and nonrenormalizable Yukawa interaction for the first two generations. We show that the proposed model can explain the observed \((g - 2)\) muon anomaly. Phenomenological consequences as flavour violating decays \(\tau \rightarrow \mu\mu\mu, \mu\mu e, \mu e e\) are briefly discussed. We also propose the \(U_R(1)\) gauge generalization of the SM with complex scalar singlet and nonzero right-handed charges for the first two generations.

1 Introduction

The discovery of the neutrino oscillations [1, 2] means that at least two neutrino have nonzero masses. The minimal extension of the SM with nonzero neutrino masses is the \(\nu\)MSM [3, 4]. In this model one adds to the SM three additional massive Majorana(right-handed) fermions \(\nu_R,\ i = 1,2,3\). Due to seesaw mechanism [3, 5] after the spontaneous \(SU_L(2) \otimes U(1)\) electroweak symmetry breaking the neutrinos acquire masses \(m_{\nu_i} = \frac{m_{Di}^2}{M_{Ri}}\). Here \(m_{Di}\) are the Dirac neutrino masses and \(M_{Ri}\) are the masses of the \(\nu_R,\) neutrinos. The \(\nu\)MSM has a candidate - the lightest Majorana neutrino with a mass \(M_{\nu_R} \leq O(50)\) KeV - for dark matter. Besides, the model with light Majorana neutrino can solve the problem of the baryon asymmetry in our Universe [4].

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In this report which is based mainly on Refs.[6] we consider the extension of the $\nu$MSM with additional scalar field and nonrenormalizable Yukawa interaction for the first two generations. We show that the SM extension with additional light real singlet field can explain the $(g - 2)$ muon anomaly. Phenomenological consequences of the proposed model as flavour violating decays $\tau \rightarrow \mu\mu\mu, \mu e e$ are briefly discussed. We also propose the $U_R(1)$ gauge generalization of the SM with complex scalar singlet and nonzero righthanded charges for the first two generations.

2 The $\nu$MSM extension with additional real scalar isosinglet and nonrenormalizable Yukawa interaction

In this section we consider the extension of the $\nu$MSM with additional scalar field and nonrenormalizable Yukawa interaction for the first two generations [6]. The Lagrangian of the model has the form

$$L_{tot} = L_{SM} + L_{Qd\phi} + L_{Qu\phi} + L_{Le\phi} + L_\phi + L_{\nu R}. \tag{1}$$

Here

$$L_{Qd\phi} = -\frac{h_{Qd\phi,i,k}}{M} \bar{Q}_L i \partial_\mu \phi d_R k + H.c., \tag{2}$$

$$L_{Qu\phi} = -\frac{h_{Qu\phi,i,k}}{M} \bar{Q}_L i H \phi u_R k + H.c., \tag{3}$$

$$L_{Le\phi} = -\frac{h_{Le\phi,i,k}}{M} \bar{L}_L i H \phi e_R k + H.c., \tag{4}$$

$$L_\phi = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{M^2_\phi}{2} \phi^4, \tag{5}$$

$$L_{\nu R} = i \tilde{\nu}_{Rj} \partial_\mu \nu_{Rj} - \frac{M_{\nu R}}{2} \nu_{Rj} \nu_{Rj} + h_{Li j} \bar{L}_i H \nu_{Rj} + H.c.), \tag{6}$$

where $L_{SM}$ is the SM Lagrangian, $\nu_{Rj}$ - are the Majorana neutrinos and $L_1 = (\nu_{eL}, e_L), L_2 = (\nu_{\mu L}, \mu_L), L_3 = (\nu_{\tau L}, \tau_L), e_{R1} = e_R, e_{R2} = \mu_R, e_{R3} = \tau_R, Q_{L1} = (u_L, d_L), Q_{L2} = (c_L, s_L), Q_{L3} = (t_L, b_L), d_{R1} = d_R, d_{R2} = s_R, d_{R3} = b_R, u_{R1} = u_R, u_{R2} = c_R, u_{R3} = t_R, \bar{H} = (-(H^-)^*, (H^0)^*)^1. \quad$ The main peculiarity of the model (1-6) is the use of nonrenormalizable Yukawa interactions (2-4)\(^2\). Here we consider the particular case of the general model (1 - 6) with nonzero renormalizable Yukawa interaction only for

\(^1\)Here $H = (H^0, H^-)$ is the SM Higgs doublet and $M$ is some high energy scale .

\(^2\)The nonrenormalizable Yukawa interactions have been considered in Refs.[7].
the third fermion generation. We assume that the masses of the first two light generations arise due to nonrenormalizable interactions (2-4). We impose the discrete symmetry

$$\phi \to -\phi,$$

$$e_{Rk} \to -e_{Rk} \ (k = 1, 2),$$

$$d_{Rk} \to -d_{Rk} \ (k = 1, 2).$$

The discrete symmetry (7 - 9) restricts the form of nonrenormalizable interactions (2-4), namely

$$h_{Qdφ,i\bar{j}} = h_{Quφ,i\bar{j}} = 0, \quad (10)$$

$$h_{Leφ,i\bar{j}} = 0, \quad (11)$$

As a consequence of the (7-9) the renormalizable SM Yukawa interaction with the first two quark and lepton generations vanishes and the fermions of the first two generations acquire masses only due to nonrenormalizable interactions (2-4).

We can consider the nonrenormalizable interactions (2 - 4) as some effective interactions arising from renormalizable interactions. For instance, the interaction (2) can be realized in renormalizable extension of the SM with additional scalar field $φ$ and new massive quark $SU(2)_L$ singlet fields $D_R$, $D_L$ with a mass $M_D$ and $U(1)$ hypercharges $Y_{D_L} = Y_{D_R} = -\frac{1}{3}$. The interaction of new quark fields $D_R$, $D_L$ with ordinary quarks and the neutral scalar field $φ$ is

$$L_{qDφ} = -c_i\bar{Q}_L\bar{H}_D - k_j\bar{D}_Ld_{Rj}\phi + H.c.. \quad (12)$$

In the heavy $D$-quark mass limit $M_D \to \infty$ we obtain the effective interaction (2) with

$$\frac{h_{Qdφ,i\bar{j}}}{M} = \frac{c_i k_j}{M_D}. \quad (13)$$

Analogously we can consider nonrenormalizable interaction (4) as an effective interaction which arises in renormalizable extension of the SM with additional scalar field and new massive lepton $SU(2)_L$ singlet fields $E_R$, $E_L$ with $Y_{E_L} = Y_{E_R} = 1$ The interaction of new lepton fields $E_R$, $E_L$ with ordinary quarks and the neutral scalar field $φ$ is

$$L_{Eφ} = -d_i\bar{L}_L\bar{H}_E - f_j\bar{E}_Le_{Rj}\phi + H.c.. \quad (14)$$
In the heavy $E$-lepton mass limit $M_E \to \infty$ we obtain the effective interaction (4) with
\[
\frac{h_{Le,ij}}{M} = \frac{d_i f_j}{M_E} .
\] (15)

After the spontaneous $SU(2)_L \otimes U(1)$ electroweak symmetry breaking the Yukawa interaction of the scalar field with charged leptons takes the form
\[
L_{ll} = -\bar{h}_{Le,ik} \bar{e}_L \phi e_R + H.c.,
\] (16)

where
\[
\bar{h}_{Le,ik} = h_{Le,ik} \frac{<H>}{M},
\] (17)

$i = 1, 2, 3, k = 1, 2$ and $e_{L1} = e_L, e_{L2} = \mu_L, e_{L3} = \tau_L, <H> = 174\text{ GeV}$. Nonzero vacuum expectation value for the real field $\phi$ generates nonzero lepton masses for electrons and muons while the mass of $\tau$-lepton arises due to renormalizable Yukawa coupling. The lepton mass matrix and the Yukawa lepton $\phi^\prime = \phi - <\phi>$ interactions are
\[
L_{ll} = -\bar{h}_{Le,ik} <\phi> \bar{e}_L e_R - m_\tau \bar{e}_{L3} e_{R3} + H.c.,
\] (18)
\[
L_{ll\phi} = -\bar{h}_{Le,ik} \bar{e}_L \phi^\prime e_R + H.c.,
\] (19)

where $i = 1, 2, 3$ and $k = 1, 2$. The mass terms (18) and the interaction (19) have different flavour structure that leads to the tree level flavour changing transitions like $\tau \to \mu + \phi$, $\tau \to e + \phi$ and as a consequence to the flavour violating decays like
\[
\tau^- \to \mu^- + \phi^*, \mu^\prime \mu^-, \mu^\prime \mu^-,
\] (20)
\[
\tau^- \to \mu^- + \phi^* \to \mu^- e^+ e^-,
\] (21)
\[
\tau \to e^- + \phi^* \to e^- \mu^+ \mu^-,
\] (22)
\[
\tau \to e^- + \phi^* \to e^- e^+ e^-.
\] (23)

At present state of art we can‘t predict the value of flavour violating Yukawa couplings $\bar{h}_{Le31}, \bar{h}_{Le32}$.

The interaction (19) leads, in particular, to the additional one loop contribution to muon magnetic moment due to $\phi^\prime$ scalar exchange, namely [8]
\[
\Delta a_\mu = \frac{h_{Le,22}^2 m_\mu^2}{8\pi^2 M_\phi^2} \int_0^1 \frac{x^2 (2 - x)}{(1 - x)(1 - \lambda^2 x) + \lambda^2 x},
\] (24)
where $\lambda = \frac{m_\mu}{m_\phi}$. In the limit $M_\phi >> m_\mu$

$$\Delta a_\mu = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_\phi^2} \bar{h}_{Le,22}^2 \ln \left( \frac{M_\phi}{m_\mu} \right) - \frac{7}{12} .$$

(25)

The precise measurement of the anomalous magnetic moment of the positive muon from the Brookhaven AGS experiment [9] gives a result which is $3.6\sigma$ higher than the Standard Model (SM) prediction

$$a_{\mu}^{exp} - a_{\mu}^{SM} = (288 \pm 80) \cdot 10^{-11} ,$$

(26)

where $a_\mu \equiv \frac{g-2}{2}$. Using the formulae (25, 26) we find that for $m_\phi = (100, 10, 1, 0.5) \text{ GeV}$ the muon $g - 2$ anomaly can be explained for

$$\bar{h}_{Le,22}^2 = (1.6 \pm 0.5) \cdot 10^{-2} \text{ for } m_\phi = 100 \text{ GeV} ,$$

(27)

$$\bar{h}_{Le,22}^2 = (2.6 \pm 0.8) \cdot 10^{-4} \text{ for } m_\phi = 10 \text{ GeV} ,$$

(28)

$$\bar{h}_{Le,22}^2 = (6.2 \pm 1.9) \cdot 10^{-6} \text{ for } m_\phi = 1 \text{ GeV} .$$

(29)

$$\bar{h}_{Le,22}^2 = (2.6 \pm 0.8) \cdot 10^{-6} \text{ for } m_\phi = 0.5 \text{ GeV} .$$

(30)

For the opposite limit $m_\mu \gg m_\phi$

$$\Delta a_\mu = \frac{3h^2_{Le,22}}{16\pi^2} ,$$

(31)

and as a consequence of (26, 31) we find that

$$\bar{h}_{Le,22}^2 = (1.5 \pm 0.5) \cdot 10^{-7} .$$

(32)

As in the SM the Yukawa couplings $\bar{h}_{Le,ii}$ are proportional to the lepton masses. It means that the interaction of the $\phi$ scalar with electrons is weaker than the interaction of the $\phi$ scalar with muons by factor $m_\mu/m_e \approx 200$ and the contribution of the $\phi$ scalar to the electron magnetic moment is suppressed at least by factor $(m_e/m_\mu)^2$ in comparison with the muon magnetic moment even for superlight $m_\phi \ll m_e$ scalar. For instance, for $m_\phi = 1 \text{ GeV}$ the contribution of the $\phi$ scalar to the electron magnetic moment is

$$(\Delta a_e)_\phi = (0.16 \pm 0.05) \cdot 10^{-17}$$

(33)
that is much smaller the bound from $a_e$ [10]

$$
\Delta a_e = a_e^{Exp} - a_e^{SM} = (-1.06 \pm 0.82) \cdot 10^{-12}
$$

(34)

Due to the suppression factor $\frac{m_e}{m_\mu}$ for electon Yukawa coupling in comparison with muon Yukawa coupling the search for light $\phi$ scalar in electron fixed target experiments or $e^+e^-$ experiments is very problematic but not hopeless.\(^3\) The search for very light $\phi$ scalar in $\pi \rightarrow (\phi \rightarrow e^+e^-)\gamma$ decay is possible but again we have additional suppression factor $(\frac{m_e-m_\mu}{m_\mu})^2 \sim O(10^{-2})$. Light scalar particle $\phi$ with a mass $m_\phi \lesssim 1 \text{ GeV}$ decaying into muon pair can be searched for at CERN SPS secondary muon beam in full analogy with the search for new light vector boson $Z'$ [11].

### 3 The $U(1)$ gauge generalization of the model with real scalar field

Here we outline one of possible generalizations of the model (1-6). In the proposed generalization instead of real scalar $\phi$ we use complex scalar $\Phi$ and new abelian gauge group $U_R(1)^4$ with nonzero charges for right handed fermions of the first and second generations, namely

$$
Q_X(u_R) = Q_X(c_R) = Q_X(\nu_{eR}) = Q_X(\nu_{\mu R}) = -Q_X(d_R) = -Q_X(s_R) = -Q_X(e_R) = -Q_X(\mu_R).
$$

(35)

The nonrenormalizable Yukawa interactions of the first and second generation fermions in full analogy with the (2-4) interactions take the form

$$
L_{Q_d\Phi} = -\frac{h_{Q_d\Phi,jk}}{M} \bar{Q}_{Li} H \Phi d_{Rk} + H.c. ,
$$

(36)

$$
L_{Q_u\Phi} = -\frac{h_{Q_u\Phi,jk}}{M} \bar{Q}_{Li} H \Phi^* u_{Rk} + H.c. ,
$$

(37)

$$
L_{L_e\Phi} = -\frac{h_{L_e\Phi,jk}}{M} \bar{L}_{Li} H \Phi e_{Rk} + H.c. ,
$$

(38)

$$
L_{L_\nu\Phi} = -\frac{h_{L_\nu\Phi,jk}}{M} \bar{L}_{Li} H \Phi^* \nu_{Rk} + H.c. ,
$$

(39)

\(^3\)Roughly speaking we have to improve the discovery potential by 3-4 orders of magnitude.

\(^4\)In Refs. [12] new light vector boson interacting with the $L_\mu - L_\tau$ current has been proposed for $(g-2)_\mu$ anomaly explanation, see also Ref.[13] where the model with new light gauge boson interacting with the SM electromagnetic current has been proposed for the $(g-2)_\mu$ anomaly explanation and Ref.[14] where the interaction of light gauge boson with $(B-L) + xY$ current has been considered.
Note that proposed model is free from $\gamma_5$-anomalies and we can consider the origin of the $U_R(1)$ gauge group as a result of the gauge symmetry breaking $SU_L(2) \otimes SU_R(2) \otimes U(1) \to SU_L(2) \otimes U_R(1) \otimes U(1)$. We assume that in the considered model $<\Phi> \neq 0$ that leads to nonzero $X$ gauge boson mass and nonzero fermion masses for the first and second generations. In the unitaire gauge $\Phi = \phi + <\Phi>$, where $\phi = \phi^*$ is real scalar field as in previous section plus we have massive vector boson $X$. So in this model after $U_R(1)$ gauge symmetry breaking in addition to the $\nu$MSM spectrum we have both scalar and vector particles. The one loop contribution to the anomalous muon(electron) magnetic moment due to the $\phi$ and $X$ exchanges is

$$\Delta a_\mu = \Delta a_\mu(\phi) + \Delta a_\mu(X),$$  \hspace{1cm} (40)$$

where the $\Delta a_\mu(\phi)$ contribution is given by the formulae (24,25,31) and the vector $X$ boson contribution for $(V + A)$ right-handed coupling\(^5\) with fermions is [8]

$$\Delta a_\mu = -\frac{g_X^2}{8\pi^2} \frac{m_\mu^2}{M_X^2} \int_0^1 \frac{2x^2(2 - x) + 2x(1 - x)(x - 4) - 4\lambda x^3}{(1 - x)(1 - \lambda^2 x) + \lambda^2 x},$$ \hspace{1cm} (41)$$

$$\Delta a_\mu = -\frac{g_X^2}{8\pi^2} \frac{m_\mu^2}{M_X^2} (\text{for } M_X \gg m_\mu).$$ \hspace{1cm} (42)$$

The $X$ boson contribution (41) to the $(g - 2)$ is negative. For instance, for $\alpha_X = \frac{g_X^2}{4\pi} = 10^{-8}$ and $M_X = 500 \text{ MeV}$ the $X$-boson contribution to $\Delta a_\mu$ is $\Delta a_\mu(X) = -1.8 \cdot 10^{-10}$. The positive contribution due to $\phi$ boson exchange is positive and cancels the negative contribution from the $X$ boson exchange. The most perspective experiments for the search for light vector $X$-boson with the electron coupling constant $\alpha_X = O(10^{-8})$ are the electron fixed target experiments or $e^+e^-$ experiments. The $X$ boson can decay into electron-positron or muon-antimuon pairs, also invisible decays of the $X$ boson into light sterile neutrino are possible. The experiment NA64 [15] at CERN will be able to search for both invisible and visible $X$ boson decay modes with the $\alpha_X \geq O(10^{-12})$ [16].

4 Conclusion

The $\nu$MSM with additional scalar field and nonrenormalizable interaction for the first two generations can explain the observed muon $(g - 2)$ anomaly. The model predicts the existence of flavour violating

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\(^5\)The interaction Lagrangian of the vector $X$ field with muons is $L_{X\mu} = g_X X^\mu \gamma_\nu (1 + \gamma_5) \mu$. 

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quark and lepton decays like $\tau \rightarrow \mu\mu\mu, \mu\mu, \mu e, \mu e$. Besides the $U(1)$ gauge generalization of the model with real isosinglet scalar field is also able to explain muon $(g - 2)$ anomaly.

We are indebted to colleagues from INR theoretical department for discussions and comments.

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