

# The SM extensions with additional light scalar singlet, nonrenormalizable Yukawa interactions and $(g - 2)_\mu$

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**Abstract.** We consider the SM extension with additional light real singlet scalar, right-handed neutrino and nonrenormalizable Yukawa interaction for the first two generations. We show that the proposed model can explain the observed  $(g - 2)_\mu$  anomaly. Phenomenological consequences as flavour violating decays  $\tau \rightarrow \mu\mu\mu, \mu\mu e, \mu e e$  are briefly discussed. We also propose the  $U_R(1)$  gauge generalization of the SM with complex scalar singlet and nonzero right-handed charges for the first two generations.

## 1 Introduction

The discovery of the neutrino oscillations [1, 2] means that at least two neutrinos have nonzero masses. The minimal extension of the SM with nonzero neutrino masses is the  $\nu$ MSM [3, 4]. In this model one adds to the SM three additional massive Majorana(right-handed) fermions  $\nu_{Ri}$ ,  $i = 1, 2, 3$ . Due to seesaw mechanism [3, 5] after the spontaneous  $SU_L(2) \otimes U(1)$  electroweak symmetry breaking the neutrinos acquire masses  $m_{\nu_i} = \frac{m_{Di}^2}{M_{Ri}}$ . Here  $m_{Di}$  are the Dirac neutrino masses and  $M_{Ri}$  are the masses of the  $\nu_{Ri}$  neutrinos. The  $\nu$ MSM has a candidate - the lightest Majorana neutrino with a mass  $M_{\nu_R} \leq O(50) \text{ KeV}$  - for dark matter. Besides, the model with light Majorana neutrino can solve the problem of the baryon asymmetry in our Universe [4].

In this report which is based mainly on Refs.[6] we consider the extension of the  $\nu$ MSM with additional scalar field and nonrenormalizable Yukawa interaction for the first two generations. We show that the SM extension with additional light real singlet field can explain the  $(g - 2)$  muon anomaly. Phenomenological consequences of the proposed model as flavour violating decays  $\tau \rightarrow \mu\mu\mu, \mu ee$  are briefly discussed. We also propose the  $U_R(1)$  gauge generalization of the SM with complex scalar singlet and nonzero righthanded charges for the first two generations.

## 2 The $\nu$ MSM extension with additional real scalar isosinglet and nonrenormalizable Yukawa interaction

In this section we consider the extension of the  $\nu$ MSM with additional scalar field and nonrenormalizable Yukawa interaction for the first two generations [6]. The Lagrangian of the model has the form

$$L_{tot} = L_{SM} + L_{Qd\phi} + L_{Qu\phi} + L_{Le\phi} + L_{\phi} + L_{\nu R}. \quad (1)$$

Here

$$L_{Qd\phi} = -\frac{h_{Qd\phi,ik}}{M} \bar{Q}_{Li} \bar{H} \phi d_{Rk} + H.c. , \quad (2)$$

$$L_{Qu\phi} = -\frac{h_{Qu\phi,ik}}{M} \bar{Q}_{Li} H \phi u_{Rk} + H.c. , \quad (3)$$

$$L_{Le\phi} = -\frac{h_{Le\phi,ik}}{M} \bar{L}_{Li} \bar{H} \phi e_{Rk} + H.c. , \quad (4)$$

$$L_{\phi} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - \frac{M_{\phi}^2 \phi^2}{2} - \lambda \phi^4 , \quad (5)$$

$$L_{\nu R} = i \bar{\nu}_{Rj} \hat{\partial} \nu_{Rj} - \left( \frac{M_{\nu Rj}}{2} \nu_{Rj} \nu_{Rj} + h_{Lij} \bar{L}_i H \nu_{Rj} + H.c. \right), \quad (6)$$

where  $L_{SM}$  is the SM Lagrangian,  $\nu_{Rj}$  - are the Majorana neutrinos and  $L_1 = (\nu_{eL}, e_L)$ ,  $L_2 = (\nu_{\mu L}, \mu_L)$ ,  $L_3 = (\nu_{\tau L}, \tau_L)$ ,  $e_{R1} = e_R$ ,  $e_{R2} = \mu_R$ ,  $e_{R3} = \tau_R$ ,  $Q_{L1} = (u_L, d_L)$ ,  $Q_{L2} = (c_L, s_L)$ ,  $Q_{L3} = (t_L, b_L)$ ,  $d_{R1} = d_R$ ,  $d_{R2} = s_R$ ,  $d_{R3} = b_R$ ,  $u_{R1} = u_R$ ,  $u_{R2} = c_R$ ,  $u_{R3} = t_R$ ,  $\bar{H} = (-(H^-)^*, (H^0)^*)^1$ . The main peculiarity of the model (1-6) is the use of nonrenormalizable Yukawa interactions (2-4)<sup>2</sup>. Here we consider the particular case of the general model (1 - 6) with nonzero renormalizable Yukawa interaction only for

<sup>1</sup>Here  $H = (H^0, H^-)$  is the SM Higgs doublet and  $M$  is some high energy scale .

<sup>2</sup>The nonrenormalizable Yukawa interactions have been considered in Refs.[7].

the third fermion generation. We assume that the masses of the first two light generations arise due to nonrenormalizable interactions (2-4). We impose the discrete symmetry

$$\phi \rightarrow -\phi, \quad (7)$$

$$e_{Rk} \rightarrow -e_{Rk} \quad (k = 1, 2), \quad (8)$$

$$d_{Rk} \rightarrow -d_{Rk} \quad (k = 1, 2). \quad (9)$$

The discrete symmetry (7 - 9) restricts the form of nonrenormalizable interactions (2-4), namely

$$h_{Qd\phi,i3} = h_{Qu\phi,i3} = 0, \quad (10)$$

$$h_{Le\phi,i3} = 0, \quad (11)$$

As a consequence of the (7-9) the renormalizable SM Yukawa interaction with the first two quark and lepton generations vanishes and the fermions of the first two generations acquire masses only due to nonrenormalizable interactions (2-4).

We can consider the nonrenormalizable interactions (2 - 4) as some effective interactions arising from renormalizable interactions. For instance, the interaction (2) can be realized in renormalizable extension of the SM with additional scalar field  $\phi$  and new massive quark  $SU(2)_L$  singlet fields  $D_R$ ,  $D_L$  with a mass  $M_D$  and  $U(1)$  hypercharges  $Y_{D_L} = Y_{D_R} = -\frac{1}{3}$ . The interaction of new quark fields  $D_R$ ,  $D_L$  with ordinary quarks and the neutral scalar field  $\phi$  is

$$L_{qD\phi} = -c_i \bar{Q}_{Li} \bar{H} D_R - k_j \bar{D}_L d_{Rj} \phi + H.c.. \quad (12)$$

In the heavy  $D$ -quark mass limit  $M_D \rightarrow \infty$  we obtain the effective interaction (2) with

$$\frac{h_{Qd\phi,ij}}{M} = \frac{c_i k_j}{M_D}. \quad (13)$$

Analogously we can consider nonrenormalizable interaction (4) as an effective interaction which arises in renormalizable extension of the SM with additional scalar field and new massive lepton  $SU(2)_L$  singlet fields  $E_R$ ,  $E_L$  with  $Y_{E_L} = Y_{E_R} = 1$ . The interaction of new lepton fields  $E_R$ ,  $E_L$  with ordinary quarks and the neutral scalar field  $\phi$  is

$$L_{eE\phi} = -d_i \bar{L}_{Li} \bar{H} E_R - f_j \bar{E}_L e_{Rj} \phi + H.c.. \quad (14)$$

In the heavy  $E$ -lepton mass limit  $M_E \rightarrow \infty$  we obtain the effective interaction (4) with

$$\frac{h_{Le\phi,ij}}{M} = \frac{d_i f_j}{M_E}. \quad (15)$$

After the spontaneous  $SU(2)_L \otimes U(1)$  electroweak symmetry breaking the Yukawa interaction of the scalar field with charged leptons takes the form

$$L_{ll\phi} = -\bar{h}_{Le,ik} \bar{e}_{Li} \phi e_{Rk} + H.c., \quad (16)$$

where

$$\bar{h}_{Le,ik} = h_{Le\phi,ik} \frac{\langle H \rangle}{M}, \quad (17)$$

$i = 1, 2, 3$ ,  $k = 1, 2$  and  $e_{L1} = e_L$ ,  $e_{L2} = \mu_L$ ,  $e_{L3} = \tau_L$ ,  $\langle H \rangle = 174 \text{ GeV}$ . Nonzero vacuum expectation value for the real field  $\phi$  generates nonzero lepton masses for electrons and muons while the mass of  $\tau$ -lepton arises due to renormalizable Yukawa coupling. The lepton mass matrix and the Yukawa lepton  $\phi^\dagger = \phi - \langle \phi \rangle$  interactions are

$$L_{ll} = -\bar{h}_{Le,ik} \langle \phi \rangle \bar{e}_{Li} e_{Rk} - m_\tau \bar{e}_{L3} e_{R3} + H.c., \quad (18)$$

$$L_{ll\phi} = -\bar{h}_{Le,ik} \bar{e}_{Li} \phi^\dagger e_{Rk} + H.c., \quad (19)$$

where  $i = 1, 2, 3$  and  $k = 1, 2$ . The mass terms (18) and the interaction (19) have different flavour structure that leads to the tree level flavour changing transitions like  $\tau \rightarrow \mu + \phi$ ,  $\tau \rightarrow e + \phi$  and as a consequence to the flavour violating decays like

$$\tau^- \rightarrow \mu^- + \phi^* \rightarrow \mu^- \mu^+ \mu^-, \quad (20)$$

$$\tau^- \rightarrow \mu^- + \phi^* \rightarrow \mu^- e^+ e^-, \quad (21)$$

$$\tau \rightarrow e^- + \phi^* \rightarrow e^- \mu^+ \mu^-, \quad (22)$$

$$\tau \rightarrow e^- + \phi^* \rightarrow e^- e^+ e^-. \quad (23)$$

At present state of art we can't predict the value of flavour violating Yukawa couplings  $\bar{h}_{Le31}$ ,  $\bar{h}_{Le32}$ .

The interaction (19) leads, in particular, to the additional one loop contribution to muon magnetic moment due to  $\phi^\dagger$  scalar exchange, namely [8]

$$\Delta a_\mu = \frac{h_{Le,22}^2 m_\mu^2}{8\pi^2 M_\phi^2} \int_0^1 \frac{x^2(2-x)}{(1-x)(1-\lambda^2 x) + \lambda^2 x} dx, \quad (24)$$

where  $\lambda = \frac{m_\mu}{m_\phi}$ . In the limit  $M_\phi \gg m_\mu$

$$\Delta a_\mu = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_\phi^2} \bar{h}_{Le,22}^2 \left[ \ln\left(\frac{M_\phi}{m_\mu}\right) - \frac{7}{12} \right]. \quad (25)$$

The precise measurement of the anomalous magnetic moment of the positive muon from the Brookhaven AGS experiment [9] gives a result which is  $3.6\sigma$  higher than the Standard Model (SM) prediction

$$a_\mu^{exp} - a_\mu^{SM} = (288 \pm 80) \cdot 10^{-11}, \quad (26)$$

where  $a_\mu \equiv \frac{g_\mu - 2}{2}$ . Using the formulae (25, 26) we find that for  $m_\phi = (100, 10, 1, 0.5) \text{ GeV}$  the muon  $g - 2$  anomaly can be explained for

$$\bar{h}_{Le,22}^2 = (1.6 \pm 0.5) \cdot 10^{-2} \quad \text{for } m_\phi = 100 \text{ GeV}, \quad (27)$$

$$\bar{h}_{Le,22}^2 = (2.6 \pm 0.8) \cdot 10^{-4} \quad \text{for } m_\phi = 10 \text{ GeV}, \quad (28)$$

$$\bar{h}_{Le,22}^2 = (6.2 \pm 1.9) \cdot 10^{-6} \quad \text{for } m_\phi = 1 \text{ GeV}. \quad (29)$$

$$\bar{h}_{Le,22}^2 = (2.6 \pm 0.8) \cdot 10^{-6} \quad \text{for } m_\phi = 0.5 \text{ GeV}. \quad (30)$$

For the opposite limit  $m_\mu \gg m_\phi$

$$\Delta a_\mu = \frac{3h_{Le,22}^2}{16\pi^2} \quad (31)$$

and as a consequence of (26, 31) we find that

$$\bar{h}_{Le,22}^2 = (1.5 \pm 0.5) \cdot 10^{-7}. \quad (32)$$

As in the SM the Yukawa couplings  $\bar{h}_{Le,ii}$  are proportional to the lepton masses. It means that the interaction of the  $\phi$  scalar with electrons is weaker than the interaction of the  $\phi$  scalar with muons by factor  $m_\mu/m_e \approx 200$  and the contribution of the  $\phi$  scalar to the electron magnetic moment is suppressed at least by factor  $(m_e/m_\mu)^2$  in comparison with the muon magnetic moment even for superlight  $m_\phi \ll m_e$  scalar. For instance, for  $m_\phi = 1 \text{ GeV}$  the contribution of the  $\phi$  scalar to the electron magnetic moment is

$$(\Delta a_e)_\phi = (0.16 \pm 0.05) \cdot 10^{-17} \quad (33)$$

that is much smaller the bound from  $a_e$  [10]

$$\Delta a_e = a_e^{exp} - a_e^{SM} = (-1.06 \pm 0.82) \cdot 10^{-12} \quad (34)$$

Due to the suppression factor  $\frac{m_e}{m_\mu}$  for electron Yukawa coupling in comparison with muon Yukawa coupling the search for light  $\phi$  scalar in electron fixed target experiments or  $e^+e^-$  experiments is very problematic but not hopeless.<sup>3</sup> The search for very light  $\phi$  scalar in  $\pi \rightarrow (\phi \rightarrow e^+e^-)\gamma$  decay is possible but again we have additional suppression factor  $(\frac{m_u - m_d}{m_\mu})^2 \sim O(10^{-2})$ . Light scalar particle  $\phi$  with a mass  $m_\phi \lesssim 1 \text{ GeV}$  decaying into muon pair can be searched for at CERN SPS secondary muon beam in full analogy with the search for new light vector boson  $Z'$  [11].

### 3 The $U(1)$ gauge generalization of the model with real scalar field

Here we outline one of possible generalizations of the model (1-6). In the proposed generalization instead of real scalar  $\phi$  we use complex scalar  $\Phi$  and new abelian gauge group  $U_R(1)$ <sup>4</sup> with nonzero charges for right handed fermions of the first and second generations, namely

$$Q_X(u_R) = Q_X(c_R) = Q_X(\nu_{eR}) = Q_X(\nu_{\mu R}) = -Q_X(d_R) = -Q_X(s_R) = -Q_X(e_R) = -Q_X(\mu_R). \quad (35)$$

The nonrenormalizable Yukawa interactions of the first and second generation fermions in full analogy with the (2-4) interactions take the form

$$L_{Qd\Phi} = -\frac{h_{Qd\Phi,ik}}{M} \bar{Q}_{Li} \bar{H} \Phi d_{Rk} + H.c. , \quad (36)$$

$$L_{Qu\Phi} = -\frac{h_{Qu\Phi,ik}}{M} \bar{Q}_{Li} H \Phi^* u_{Rk} + H.c. , \quad (37)$$

$$L_{Le\Phi} = -\frac{h_{Le\Phi,ik}}{M} \bar{L}_{Li} \bar{H} \Phi e_{Rk} + H.c. , \quad (38)$$

$$L_{L\nu\Phi} = -\frac{h_{L\nu\Phi,ik}}{M} \bar{L}_{Li} \bar{H} \Phi^* \nu_{Rk} + H.c. , \quad (39)$$

<sup>3</sup>Roughly speaking we have to improve the discovery potential by 3-4 orders of magnitude.

<sup>4</sup>In Refs. [12] new light vector boson interacting with the  $L_\mu - L_\tau$  current has been proposed for  $(g-2)_\mu$  anomaly explanation, see also Ref.[13] where the model with new light gauge boson interacting with the SM electromagnetic current has been proposed for the  $(g-2)_\mu$  anomaly explanation and Ref.[14] where the interaction of light gauge boson with  $(B-L) + xY$  current has been considered.

Note that proposed model is free from  $\gamma_5$ -anomalies and we can consider the origin of the  $U_R(1)$  gauge group as a result of the gauge symmetry breaking  $SU_L(2) \otimes SU_R(2) \otimes U(1) \rightarrow SU_L(2) \otimes U_R(1) \otimes U(1)$ . We assume that in the considered model  $\langle \Phi \rangle \neq 0$  that leads to nonzero  $X$  gauge boson mass and nonzero fermion masses for the first and second generations. In the unitaire gauge  $\Phi = \phi + \langle \Phi \rangle$ , where  $\phi = \phi^*$  is real scalar field as in previous section plus we have massive vector boson  $X$ . So in this model after  $U_R(1)$  gauge symmetry breaking in addition to the  $\nu$ MSM spectrum we have both scalar and vector particles. The one loop contribution to the anomalous muon(electron) magnetic moment due to the  $\phi$  and  $X$  exchanges is

$$\Delta a_\mu = \Delta a_\mu(\phi) + \Delta a_\mu(X), \quad (40)$$

where the  $\Delta a_\mu(\phi)$  contribution is given by the formulae (24,25,31) and the vector  $X$  boson contribution for  $(V + A)$  right-handed coupling<sup>5</sup> with fermions is [8]

$$\Delta a_\mu = \frac{g_X^2}{8\pi^2} \frac{m_\mu^2}{M_X^2} \int_0^1 \frac{2x^2(2-x) + 2x(1-x)(x-4) - 4\lambda^2 x^3}{(1-x)(1-\lambda^2 x) + \lambda^2 x}, \quad (41)$$

$$\Delta a_\mu = -\frac{g_X^2}{3\pi^2} \frac{m_\mu^2}{M_X^2} \quad (\text{for } M_X \gg m_\mu). \quad (42)$$

The  $X$  boson contribution (41) to the  $(g - 2)$  is negative. For instance, for  $\alpha_X = \frac{g_X^2}{4\pi} = 10^{-8}$  and  $M_X = 500 \text{ MeV}$  the  $X$ -boson contribution to  $\Delta a_\mu$  is  $\Delta a_\mu(X) = -1.8 \cdot 10^{-10}$ . The positive contribution due to  $\phi$  boson exchange is positive and cancels the negative contribution from the  $X$  boson exchange. The most perspective experiments for the search for light vector  $X$ -boson with the electron coupling constant  $\alpha_X = O(10^{-8})$  are the electron fixed target experiments or  $e^+e^-$  experiments. The  $X$  boson can decay into electron-positron or muon-antimuon pairs, also invisible decays of the  $X$  boson into light sterile neutrino are possible. The experiment NA64 [15] at CERN will be able to search for both invisible and visible  $X$  boson decay modes with the  $\alpha_X \geq O(10^{-12})$  [16].

## 4 Conclusion

The  $\nu$ MSM with additional scalar field and nonrenormalizable interaction for the first two generations can explain the observed muon  $(g - 2)$  anomaly. The model predicts the existence of flavour violating

<sup>5</sup>The interaction Lagrangian of the vector  $X$  field with muons is  $L_{X\bar{\mu}\mu} = g_X X^\nu \bar{\mu} \gamma_\nu (1 + \gamma_5) \mu$ .

quark and lepton decays like  $\tau \rightarrow \mu\mu\mu, \mu\mu e, \mu ee$ . Besides the  $U(1)$  gauge generalization of the model with real isosinglet scalar field is also able to explain muon ( $g - 2$ ) anomaly.

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