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Verification of f(R)-gravity in binary pulsars

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Abstract. We develop the parameterized post-Keplerian approach for class of analytic $f(R)$-gravity models. Using the double binary pulsar system PSR J0737-3039 data we obtain restrictions on the parameters of this class of $f(R)$-models and show that $f(R)$-gravity is not ruled out by the observations in strong field regime.

1 Introduction

The theory of general relativity (GR) greatly improved the understanding of our Universe. It allows going beyond the Newtonian picture of the world. Anyway our present understanding has several shortcomings. A set of phenomena exists that we can not directly explain in the framework of GR. For explanation these phenomena many extended gravity models are created. One of the most perspective way to expand GR is to include additional corrections in terms of the Ricci scalar into the Lagrangian. This method underlies $f(R)$-gravity [1], [2]. This theory of gravity could allow to explain Dark Energy (DE) and Dark Matter (DM) by changing the geometry of a space-time and each scale demonstrates its specific geometry. This fact fully agrees with the early spirit of GR that could not act in the same way at all scales [3], [4].

However any theory of gravity should be verifiable. In this work we use the data for the binary pulsar PSR J0737-3039 for test $f(R)$-gravity in the strong field regime[6], [7]. PSR J0737-3039 is the only known double binary pulsar. The extraordinary closeness of the system components, small orbital period and the fact that we see almost edge-on system allow to investigate the manifestation of relativistic effects with the highest available precision.

The structure of the paper is the following. In Section 2 we develop the post-Keplerian formalism for analytic $f(R)$-gravity models and obtain observational constraints arising from PSR J0737-3039. The Section 3 contains our conclusions.

2 $f(R)$-Gravity and Restriction Constraints

2.1 $f(R)$-gravity background

The $f(R)$-gravity models are based on different expansions and enlargements of GR. Higher-order curvature invariants and minimally or non-minimally coupled scalar fields are added into the gravity

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action. In $f(R)$-gravity one considers a general function $f(R)$ instead of the Hilbert-Einstein Lagrangian R that is linear with respect to the Ricci scalar R. The only assumption that we make is that $f(R)$ is an analytic function. The gravitational action now looks like[3], [4], [8], [9]:

$$S = \int d^4x \sqrt{-g}\,[f(R) + \kappa L_{\rm m}]\,, \tag{1}$$

where $\kappa = 16\pi G/c^4$ is the coupling, g is the determinant of the metric and $L_{\rm m}$ is the standard matter contribution.

We assume that the $f(R)$ Lagrangian is analytic (i.e. Taylor expandable) in terms of the Ricci scalar, therefore[8], [9]:

$$f(R) = \sum_n \frac{f^n(R_0)}{n!}(R - R_0)^n \simeq f_0 + f_0' R + \frac{1}{2} f_0'' R^2 + \ldots, \tag{2}$$

where

$$f_0 = const, \quad f_0' = \left.\frac{df(R)}{dR}\right|_{\rm R=0}, \quad f_0'' = \left.\frac{d^2 f(R)}{dR^2}\right|_{\rm R=0}. \tag{3}$$

The flat background is recovered for $R = R_0 \simeq 0$. GR is recovered in the limit $f_0 = 0, f_0' = 4/3, f_0'' = 0$[3], [4]. Hereafter we assume $f_0 = 0$ [3], [4].

2.2 The parameterized post-Keplerian formalism

The parameterized post-Keplerian formalism (PPK) was developed by Damour and Deruelle[10] and further improved by Damour and Taylor [11]. It was created to link the arrival time of pulses and their radiation time in the frame of the pulsar. Damour et al. have showed that all the independent $O(v^2/c^2)$ timing effects can be represented in a simple mathematical way that is generic for a wide class of extended gravities. PPK is a strong-field analogue of the PPN formalism [5]. It includes such effects as the Einstein time delay, Römer time delay, Shapiro time delay and the effects of aberration. The general form of these corrections is model-independent, therefore all possible manifestations of the extended gravity model can be expressed through the 8 PPK parameters $\dot{\omega}$, γ, $\dot{P}_{\rm b}$, r, s, δ_θ, $\dot{e}$, $\dot{x}$. However, we consider only the most accurately measured parameters, so we do not take into account the last three of them. In this work we obtain the analytical form of four PPK parameters:

$$\dot{\omega} = \left(\frac{2\pi}{P_b}\right)^{5/3} \frac{G^{2/3} M_\odot^{2/3} (m_1 + m_2)^{2/3}}{c^2(1-e^2)} \times \left(\frac{(f_0'')^2}{2(f_0') + 2f_0'R + 3(f_0'')^2}\right)^{2/3}$$
$$\times\left(\frac{7}{2} - \frac{2(f_0'')^2}{f_0' + f_0'R + 2(f_0'')^2} - \frac{1}{16}\frac{(2f_0' + 2f_0''R + 3(f_0'')^2)}{(f_0' + f_0''R + 2(f_0'')^2)^4}\right.$$
$$\times\left[\frac{(4(f_0')^3 + 12(f_0')^2 f_0''R + 22(f_0')^2(f_0'')^2 + 12f_0'(f_0'')^2R^2}{(f_0' + f_0''R + 2(f_0'')^2)^4} +\right.$$
$$\left.\left. + \frac{44f_0'(f_0'')^3R + 39f_0'(f_0'')^4 + 4(f_0'')^3R^3 + 22(f_0'')^4R^2 + 39(f_0'')^5R + 24(f_0'')^6)}{(f_0' + f_0''R + 2(f_0'')^2)^4}\right]\right),$$
$$\gamma = e\left(\frac{2\pi}{P_b}\right)^{-1/3} \frac{G^{2/3} M_\odot^{2/3} m_2}{c^2(m_1 + m_2)^{4/3}} \times$$
$$\times\left(m_1 + m_2\left[2 + \frac{(f_0'')^2}{2f_0' + 2f_0'R + 3(f_0'')^2}\right]\right) \times$$
$$\times\left(1 + \frac{(f_0'')^2}{2f_0' + 2f_0'R + 3(f_0'')^2}\right)^{-1/3},$$
$$r = \frac{1}{4c^3} GM_\odot m_2,$$
$$s = \left(\frac{2\pi}{P_b}\right)^{2/3} \frac{cx(m_1 + m_2)^{2/3}}{(GM_\odot)^{1/3}\, m_2} \left(1 + \frac{(f_0'')^2}{2f_0' + 2f_0'R + 3(f_0'')^2}\right)^{-1/3}. \tag{4}$$

The expression for the last parameter $\dot{P}_b$ was obtained by Capozziello and De Laurentis [3], [4] in the form:

$$\dot{P}_b = -\frac{6\pi}{20}\left(\frac{2\pi}{P_b}\right)^{5/3} \frac{(GM_\odot)^{5/3}}{c^5(1-e^2)^{7/2}} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}$$
$$\times\left((f_0'(37e^4 + 292e^2 + 96) - \frac{f_0''\pi^2}{2P_b(1+e^2)^3}\right.$$
$$\left.\times\left(891e^8 + 28016e^6 + 82736e^4 + 43520e^2 + 3072\right)\right). \tag{5}$$

2.3 Restrictions from PSR J0737-3039

Now we can proceed directly to the method of testing models of gravity [11] and our additions. We plot the curves representing post-Keplerian parameters on the plane, in the Y-axis we put the allowed values of the companion masses m_2 and in the X-axis we put possible values of the pulsar m_1 masses. The region of all curves intersection within the measured accuracy displays the possible range of the pulsar's mass and its companion. For the appropriate gravity model all the curves of post-Keplerian parameters should "meet" at one point (taking into account the accuracy). The measurements of n post-Keplerian parameters give $n-2$ tests. In our case there are $6-2=4$ ones.

Here it is importantly to point out that all the curves for PSR J0737-3039 intersect within the measurement accuracy in the GR case (see Fig. 1).

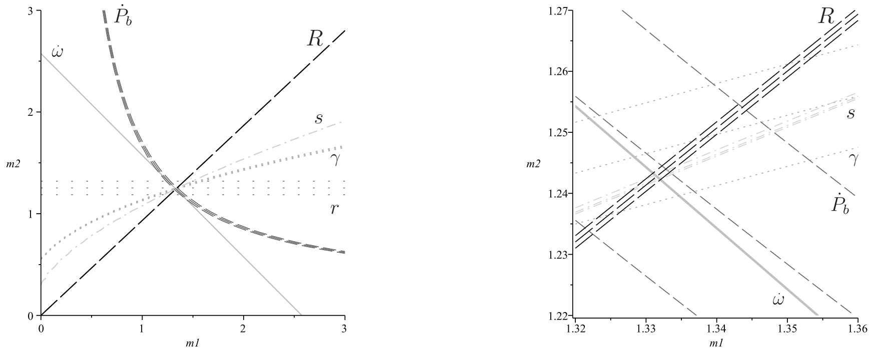

Figure 1. Dependence of the companion mass upon the pulsar one at different scales for GR, $f_0' = 4/3$ and $f_0'' = 0$.

Table 1. Parameters PSR J0737-3039

Parameter	Physical meaning	Experimental value
$P_b(day)$	orbital period	0.10225156248(5)
e	eccentricity	0.0877775(9)
$x(s)$	projected semimajor axis of the pulsar orbit	1.415032(1)
$\dot{\omega}(deg/yr)$	secular advance of the periastron	16.89947(68)
$\tilde{\gamma}(ms)$	time dilation parameter	0.3856(26)
$\dot{P}_b$	secular change of the orbital period	$-1.252(17) \times 10^{-12}$
s	Shapiro delay parameter	0.99974(−39, +16)
$r(\mu s)$	Shapiro delay parameter	6.21(33)
$R = \dfrac{m_1}{m_2} = \dfrac{x_2}{x_1}$	mass ratio	1.0714(11)

To obtain restrictions for $f(R)$-gravity we solve the system (4-5) considering observational data within the measurement accuracy for PSR J0737-3039 (see 1). We fix $f_0' = 4/3$ and obtain that the curves intersect only when f_0'' takes the following values (see Fig. 2) :

$$-0.0623 \leq f_0'' \leq 0. \tag{6}$$

This is the maximum limit that we can get for f_0'' when $R << 1$. In this case masses of pulsar and its component take the following values:

$$\begin{aligned} m_{p1} &= 1.3331, \\ m_{p2} &= 1.2429. \end{aligned} \tag{7}$$

Hence the difference between GR and $f(R)$-gravity is 0.5%.

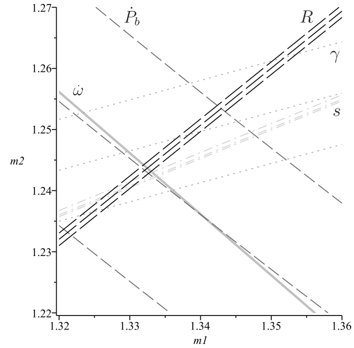

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Figure 2. Dependence of the companion mass upon the pulsar one,$f_0' = 4/3$ and $f_0'' = -0.0623$. The last point of intersection $s(m_1, m_2)$, $R(m_1, m_2)$ and $\dot{\omega}(m_1, m_2)$ within the measurement accuracy, when f_0' is fixed.

3 Conclusions

We consider the analytical $f(R)$-gravity in the system of binary pulsars. To find the observational limitations on these models the PPK parameters and other orbital data of the double binary pulsar PSR J0737-3039 [6], [7] are applied. We find the analytical form of the four PPK parameters for the analytic $f(R)$-gravity. It is important that we use the general form of $f(R)$-gravity. The only assumption is that $f(R)$ is supposed to be an analytical function.

In this work we obtained strict limits for free parameter f_0'' (assuming that $f_0' = 4/3$ and $R << 1$):

$$-0.0623 \leq f_0'' \leq 0. \tag{8}$$

This test proves that Extended Theories of Gravity namely the analytic $f(R)$-gravity models are not excluded in strong field regimes. This result gives rise for further studies of $f(R)$-gravity and other models based of such type of GR extension: Palatini $f(R)$-gravity, [13] hybrid metric-Palatini $f(R)$-gravity [14] and also scalar-tensor theories, for example Horndeski gravity [15].

Acknowledgments

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