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Transition formfactors $\gamma^* \to \gamma f_2(1270)$ and $\gamma^* \to \gamma a_2(1320)$ in the e^+e^- collisions

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Abstract. Transition form factors $\gamma^* \to \gamma f_2(1270)$ and $\gamma^* \to \gamma a_2(1320)$ and corresponding $e^+e^- \to \gamma^* \to f_2\gamma$ and $e^+e^- \to \gamma^* \to a_2\gamma$ cross sections are considered up to high energies. It is shown that the QCD asymptotics of the amplitudes of the $e^+e^- \to \gamma^* \to f_2\gamma$ and $e^+e^- \to \gamma^* \to a_2\gamma$ reactions can be reached only by taking into account a compensation of contributions of $\rho(770)$, $\omega(782)$ with contributions of their radial excitations. Recent BABAR measurement of the $e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma$ cross section at 10.58 GeV gives hope for detailed investigation of the $\gamma^*(s) \to f_2\gamma$ and $\gamma^*(s) \to a_2\gamma$ transition form factors at high energy region. Recent Belle data on the $\gamma^*(Q^2)\gamma \to f_2$ transition up to $Q^2 = 30$ GeV^2 are taken into account.

It is accepted that $f_2(1270)$ and $a_2(1320)$ tensor mesons are two-quark states, $a_2 = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $f_2 = (u\bar{u} + d\bar{d})/\sqrt{2}$.

The $\gamma^* \to f_2(1270)\gamma$ and $\gamma^* \to a_2(1320)\gamma$ transition form factors manifest themselves in $e^+e^- \to \gamma^* \to T\gamma$ (time-like region of γ^*) and $\gamma^*(Q^2)\gamma \to T$ (space-like region) reactions, where $T = f_2(1270)$ or $a_2(1320)$. These reactions are connected with each other, so progress in investigation of one reactions can shed light on the other reactions.

Recently BABAR presented a measurement $\sigma(e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma)$ at hight-energy point [1], $\sigma(e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma, 10.58\ \text{GeV}) = 37^{+24}_{-18}$, and Belle measured the $\gamma^*(Q^2)\gamma \to f_2$ transition form factor in the space-like region for Q^2 up to 30 GeV^2 [2].

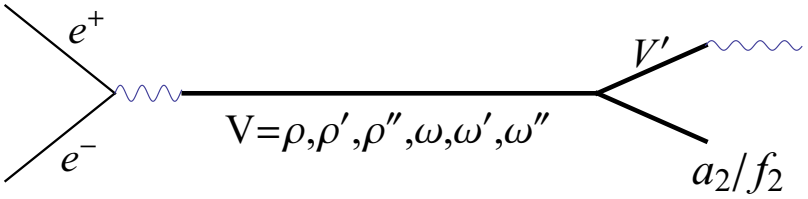

Figure 1. Diagram of $e^+e^- \to a_2(f_2)\gamma$ process.

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The diagram of the $e^+e^- \to \gamma^* \to T\gamma$ process is shown in Fig. 1. V and V' are appropriate vector mesons, ρ, ω and their radial excitations.

It is known that in $\gamma\gamma \to f_2 \to \pi\pi$ reaction tensor mesons are produced mainly by the photons with the opposite helicity states. This is naturally described by the Lagrangian [3]

$$L = g_{T\gamma\gamma} T_{\mu\nu} F_{\mu\sigma} F_{\nu\sigma} \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $T_{\mu\nu}$ is a tensor meson T field.

In the frame of GVDM it is assumed that the effective Lagrangian of the reaction $f_2 \to VV$ is

$$L = g_{f_2VV} T_{\mu\nu} F^V_{\mu\sigma} F^V_{\nu\sigma} \tag{2}$$

where $F^V_{\mu\sigma} = \partial_\mu V_\sigma - \partial_\sigma V_\mu$ and $V = \rho_1, \rho_2, \rho_3, \omega_1, \omega_2, \omega_3$, where $\rho_1 = \rho$, $\rho_2 = \rho'$, $\rho_2 = \rho''$ and $\omega_1 = \omega$, $\omega_2 = \omega'$, $\omega_3 = \omega''$.

Assuming the GVDM mechanism $e^+e^- \to \gamma^* \to (\rho + \rho' + \rho'' + \omega + \omega' + \omega'') \to f_2(\rho + \rho' + \rho'' + \omega + \omega' + \omega'') \to f_2\gamma$ one obtains [4]

$$\sigma_{e^+e^- \to f_2\gamma}(s) = \frac{4\pi^2}{9}\,\alpha^3 \Big(1 - \frac{m^2_{f_2}}{s}\Big)^3 \Big(\frac{s^2}{m^4_{f_2}} + 3\frac{s}{m^2_{f_2}} + 6\Big) \times$$

$$\left| \frac{m^2_\rho g_{f_2\rho\rho}}{f^2_\rho D_\rho(s)} + \frac{m^2_{\rho'} g_{f_2\rho'\rho'}}{f^2_{\rho'} D_{\rho'}(s)} + \frac{m^2_{\rho''} g_{f_2\rho''\rho''}}{f^2_{\rho''} D_{\rho''}(s)} + \frac{m^2_\omega g_{f_2\omega\omega}}{f^2_\omega D_\omega(s)} + \frac{m^2_{\omega'} g_{f_2\omega'\omega'}}{f^2_{\omega'} D_{\omega'}(s)} + \frac{m^2_{\omega''} g_{f_2\omega''\omega''}}{f^2_{\omega''} D_{\omega''}(s)} \right|^2. \tag{3}$$

Here $D_V(s)$ are inverse propagators

$$D_V(s) = m^2_V - s - i\sqrt{s}\Gamma_V(s), \tag{4}$$

the forms of widths $\Gamma_V(s)$ are described below. The constants f_V are related to e^+e^- widths as usual:

$$\Gamma(V \to e^+e^-) = \frac{4\pi\alpha^2}{3 f^2_V} m_V. \tag{5}$$

In $q\bar{q}$ model $g_{f_2\rho_i\rho_i} = g_{f_2\omega_i\omega_i}$, $m_{\omega_i} = m_{\rho_i}$, $f_{\omega_i} = 3f_{\rho_i}$, this gives

$$\Gamma_{f_2\to\gamma\gamma} = \Big(\frac{10}{9}\Big)^2 \frac{\pi\alpha^2}{5} \left| \frac{g_{f_2\rho\rho}}{f^2_\rho} + \frac{g_{f_2\rho'\rho'}}{f^2_{\rho'}} + \frac{g_{f_2\rho''\rho''}}{f^2_{\rho''}} \right|^2 m^3_{f_2} = 3.03 \pm 0.35 \text{ keV [5]}, \tag{6}$$

$$\sigma_{e^+e^- \to f_2\gamma}(s) = \frac{20\,\pi}{9}\,\alpha\,\frac{\Gamma_{f_2\to\gamma\gamma}}{m^3_{f_2}} \big|F_{\gamma^*\to f_2\gamma}(s)\big|^2 \Big(1 - \frac{m^2_{f_2}}{s}\Big)^3 \Big(\frac{s^2}{m^4_{f_2}} + 3\frac{s}{m^2_{f_2}} + 6\Big). \tag{7}$$

Here $F_{\gamma^*\to f_2\gamma}(s)$ is a transition form factor. Since ω_i contributions are suppressed, we neglect the difference between Γ_{ω_i} and Γ_{ρ_i} in $D_{\omega_i}(s)$ to simplify the formula.

Let's denote $a_{f_2} = g_{f_2\rho'\rho'} f^2_\rho / g_{f_2\rho\rho} f^2_{\rho'}$ and $b_{f_2} = g_{f_2\rho''\rho''} f^2_\rho / g_{f_2\rho\rho} f^2_{\rho''}$. One has

$$F_{\gamma^*\to f_2\gamma}(s) = \frac{1}{1 + a_{f_2} + b_{f_2}} \Big(\frac{m^2_\rho}{D_\rho(s)} + a_{f_2} \frac{m^2_{\rho'}}{D_{\rho'}(s)} + b_{f_2} \frac{m^2_{\rho''}}{D_{\rho''}(s)} \Big). \tag{8}$$

The QCD asymptotics of $\sigma(e^+e^- \to \textit{light hadrons})$ at $s \to \infty$ is equal to $\sum_f Q^2_f 4\pi\alpha^2/s$ to within logarithms (Q_f are charges of u, d, and s quarks). Due to asymptotic freedom at $s \to \infty$ imaginary part of $D_V(s)$ vanishes

$$Im(D_V(s)) = -\sqrt{s}\Gamma_V(s) \to 0; \;\; \frac{1}{D_V(s)} \to \frac{1}{m^2_V - s}\,. \tag{9}$$

It means that $F_{\gamma^*\to f_2\gamma}(s) \sim 1/s^2$, i.e.

$$m_\rho^2 + a_{f_2} m_{\rho'}^2 + b_{f_2} m_{\rho''}^2 = 0 \tag{10}$$

to suppress asymptotics $\sim 1/s$ in Eq. 8.

Note the analogous condition was obtained in Refs. [6, 7] after taking into account the QCD based asymptotics of the $\gamma^*(Q^2)\gamma \to a_2$ amplitude.

The Belle Collaboration presented 3 data sets for each helicity difference $\lambda \equiv |\lambda_{\gamma^*} - \lambda_\gamma| = 0, 1, 2$. These data were described in Ref. [7] with $b_{f_2} = 0$. To take ρ'' into account it is enough to modify $\gamma^*(Q^2)\gamma \to f_2$ transition form factor (Eq. (10) of that paper):

$$F_T(Q) \to F_T(Q) = \frac{1}{1 + a_{f_2} + b_{f_2}} \left(\frac{1}{1 + \frac{Q^2}{m_\rho^2}} + \frac{a_{f_2}}{1 + \frac{Q^2}{m_{\rho'}^2}} + \frac{b_{f_2}}{1 + \frac{Q^2}{m_{\rho''}^2}} \right). \tag{11}$$

This gives two minimums of the χ^2 function, constructed for the Belle data: $b_{f_2} = 0.082 \pm 0.025$ and $b_{f_2} = 0.740 \pm 0.011$, see Fig. 2.

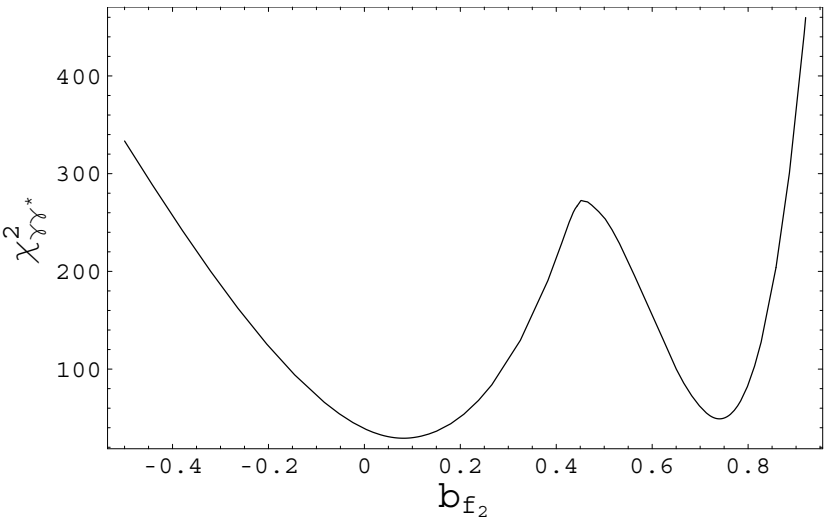


Figure 2. χ^2 function for Belle data on $\gamma^*\gamma \to f_2$ transition form factor.

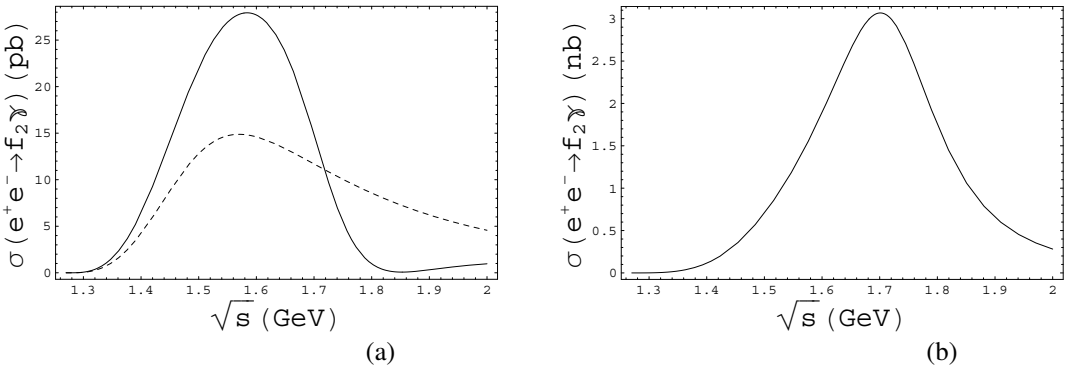
Let's consider three cases:
1) $(a_{f_2}, b_{f_2}) = (-0.393, 0.082)$ - best minimum of the χ^2 function ($\chi^2/n.d.f. = 29.3/28$);

2) $(a_{f_2}, b_{f_2}) = (-1.3, 0.74)$ - second minimum of the χ^2 function ($\chi^2/n.d.f. = 49.1/28$);

3): $(a_{f_2}, b_{f_2}) = (-0.28, 0)$ - no ρ'' contribution ($\chi^2/n.d.f. = 39.1/29$).

Recently the $e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma$ cross section was measured at BABAR at 10.58 GeV: $\sigma(\pi^+\pi^-\gamma)_{10.58} \equiv \sigma(e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma,\ 10.58\ \text{GeV}) = 37^{+24}_{-18}$ fb.

Taking into account $Br(f_2 \to \pi^+\pi^-) \approx 56.5\%$ and $D_V(s) \to m_V^2 - s$ at $s \to \infty$, we find $\sigma(\pi^+\pi^-\gamma)_{10.58} = 1.0$ fb for $b_{f_2} = 0.082$, $\sigma(\pi^+\pi^-\gamma)_{10.58} = 3.8$ fb for $b_{f_2} = 0.74$ and $\sigma(\pi^+\pi^-\gamma)_{10.58} = 1.5$



(a) (b)

Figure 3. The $e^+e^- \to f_2\gamma$ cross section up to 2 GeV. (a) Solid line is for $(a_{f_2}, b_{f_2}) = (-0.393, 0.082)$, dashed line is for $(a_{f_2}, b_{f_2}) = (-0.28, 0)$ (ρ'' is absent). (b) The plot for $(a_{f_2}, b_{f_2}) = (-1.3, 0.74)$.

fb for $b_{f_2} = 0$. The elaboration of the pioneer measurement of $\sigma(\pi^+\pi^-\gamma)_{10.58}$ is important for the $\gamma^* \to \gamma f_2/a_2$ transition form factor investigations.

The $e^+e^- \to f_2\gamma$ cross section Eq. (7) for all three cases under consideration is shown in Figs. 3 and 4. In vector propagators Eq. (4) we use constant widths taken from Ref. [5] below 2 GeV and zero widths above 5 GeV. Dashed lines show continuation of these curves to the intermediate region.

The treatment of the $e^+e^- \to \gamma^* \to a_2\gamma$ cross section is similar:

$$\sigma_{e^+e^-\to a_2\gamma}(s) = \frac{\pi^2}{9}\,\alpha^3\Big(1-\frac{m_{a_2}^2}{s}\Big)^3\Big(\frac{s^2}{m_{a_2}^4}+3\frac{s}{m_{a_2}^2}+6\Big)\times$$
$$\Big|\frac{m_\rho^2 g_{a_2\rho\omega}}{f_\rho f_\omega D_\rho(s)}+\frac{m_\omega^2 g_{a_2\rho\omega}}{f_\rho f_\omega D_\omega(s)}+\frac{m_{\rho'}^2 g_{a_2\rho'\omega'}}{f_{\rho'} f_{\omega'} D_{\rho'}(s)}+\frac{m_{\omega'}^2 g_{a_2\rho'\omega'}}{f_{\rho'} f_{\omega'} D_{\omega'}(s)}+$$
$$\frac{m_{\rho''}^2 g_{a_2\rho''\omega''}}{f_{\rho''} f_{\omega''} D_{\rho''}(s)}+\frac{m_{\omega''}^2 g_{a_2\rho''\omega''}}{f_{\rho''} f_{\omega''} D_{\omega''}(s)}\Big|^2, \tag{12}$$

$$\Gamma_{a_2\to\gamma\gamma} = \frac{4\pi\alpha^2}{5}\Big|\frac{g_{a_2\rho\omega}}{f_\rho f_\omega}+\frac{g_{a_2\rho'\omega'}}{f_{\rho'} f_{\omega'}}+\frac{g_{a_2\rho''\omega''}}{f_{\rho''} f_{\omega''}}\Big|^2 m_{a_2}^3 = 1.00 \pm 0.06 \text{ keV [5]}\ . \tag{13}$$

Denoting $a_{a_2} = g_{a_2\rho'\omega'} f_\rho f_\omega / g_{a_2\rho\omega} f_{\rho'} f_{\omega'}$ and $b_{a_2} = g_{a_2\rho''\omega''} f_\rho f_\omega / g_{a_2\rho\omega} f_{\rho''} f_{\omega''}$ one gets

$$\sigma_{e^+e^-\to a_2\gamma}(s) = \frac{20\pi}{9}\,\alpha\,\frac{\Gamma_{a_2\to\gamma\gamma}}{m_{a_2}^3}\Big|F_{\gamma^*\to a_2\gamma}(s)\Big|^2\Big(1-\frac{m_{a_2}^2}{s}\Big)^3\Big(\frac{s^2}{m_{a_2}^4}+3\frac{s}{m_{a_2}^2}+6\Big) \tag{14}$$

with transition form factor

$$F_{\gamma^*\to a_2\gamma}(s) = \frac{1}{2(1+a_{a_2}+b_{a_2})}\times$$
$$\Big(\frac{m_\rho^2}{D_\rho(s)}+\frac{m_\rho^2}{D_\omega(s)}+a_{a_2}\frac{m_{\rho'}^2}{D_{\rho'}(s)}+a_{a_2}\frac{m_{\rho'}^2}{D_{\omega'}(s)}+b_{a_2}\frac{m_{\rho''}^2}{D_{\rho''}(s)}+b_{a_2}\frac{m_{\rho''}^2}{D_{\omega''}(s)}\Big) \tag{15}$$

and the asymptotic condition $m_\rho^2 + a_{a_2} m_{\rho'}^2 + b_{a_2} m_{\rho''}^2 = 0$.

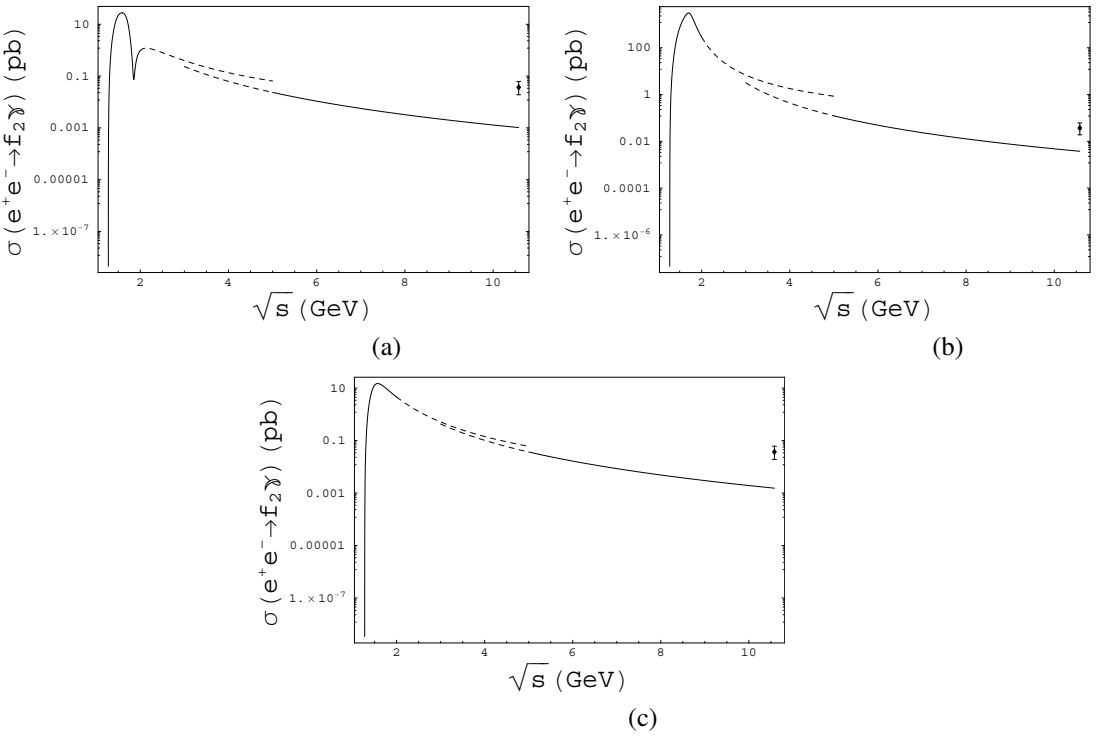


Figure 4. The $e^+e^- \to f_2\gamma$ cross section up to 10.58 GeV. In each logarithmic plot solid line below 2 GeV is drawn with constant vector widths taken from Ref. [5], solid line above 5 GeV is drawn with zero vector widths, dashed lines are their continuations. The experimental BaBar point [1] is shown also. (a) $(a_{f_2}, b_{f_2}) = (-0.393, 0.082)$, (b) $(a_{f_2}, b_{f_2}) = (-1.3, 0.74)$, (c) $(a_{f_2}, b_{f_2}) = (-0.28, 0)$ (ρ'' is absent).

In the naive $q\bar{q}$ model for $i = 1, 2, 3$ $g_{a_2\rho_i\omega_i} = g_{f_2\rho_i\rho_i}$. It means that in our approach $a_{a_2} = a_{f_2}$, $b_{a_2} = b_{f_2}$.

The resulting cross-section for the same sets of (a_{a_2}, b_{a_2}) as for $e^+e^- \to f_2(1270)\gamma$ reaction is shown in Fig. 5.

It is natural that our $q\bar{q}$ consideration agrees with QCD requirement for high s

$$\frac{\sigma(e^+e^- \to f_2\gamma, s)}{\sigma(e^+e^- \to a_2\gamma, s)} = \frac{25}{9} \tag{16}$$

and the well-known relation being in perfect agreement with the data

$$\frac{\Gamma(f_2 \to \gamma\gamma)}{\Gamma(a_2 \to \gamma\gamma)} = \frac{25}{9}\frac{m_{f_2}^3}{m_{a_2}^3} \approx \frac{25}{9}\,. \tag{17}$$

The above-mentioned BABAR measurement $\sigma(e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma,\ 10.58\ \text{GeV}) = 37^{+24}_{-18}$ fb gives

$$\sigma(e^+e^- \to a_2\gamma,\ 10.58\ \text{GeV}) = 24^{+15}_{-11}\ \text{fb}\,. \tag{18}$$

Brief summary and conclusions:

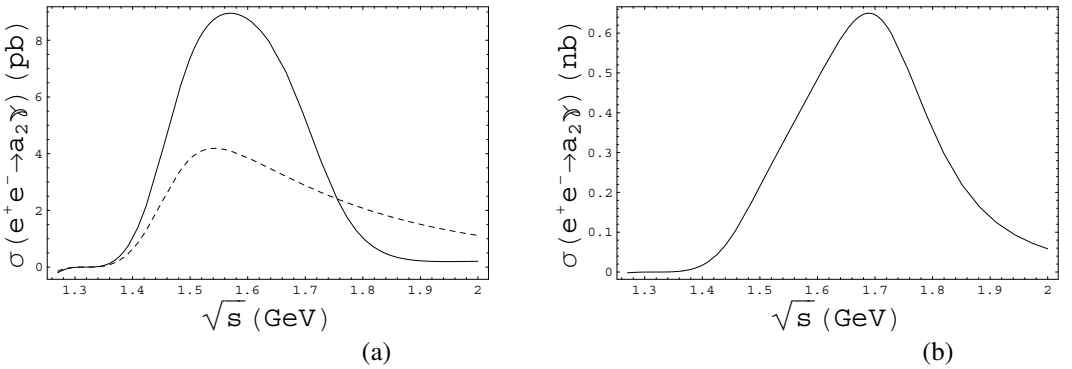


Figure 5. The $e^+e^- \to a_2\gamma$ cross section. (a) Solid line is for $(a_{a_2}, b_{a_2}) = (-0.393, 0.082)$, dashed line is for $(a_{a_2}, b_{a_2}) = (-0.28, 0)$. (b) The plot for $(a_{a_2}, b_{a_2}) = (-1.3, 0.74)$.

1. It is shown that the QCD asymptotics of the amplitudes of the reactions $e^+e^- \to \gamma^* \to f_2\gamma$ and $e^+e^- \to \gamma^* \to a_2\gamma$ can be reached only by taking into account a compensation of contributions of $\rho(770)$, $\omega(782)$ with contributions of their radial excitations.

2. The ρ and ω excitation contribution to the $e^+e^- \to f_2\gamma$ and $e^+e^- \to a_2\gamma$ cross sections is essential and allows the experimental investigation of $e^+e^- \to \eta\pi^0\gamma$ and $e^+e^- \to \pi^0\pi^0\gamma$ processes, for example, at VEPP-2000 collider. Note that the best channel to study the a_2 production is the $e^+e^- \to a_2\gamma \to \rho\pi\gamma$ because $Br(a_2 \to \rho\pi) = 70\%$ and the background is expected to be small.

3. At the BABAR energy (10.58 GeV) our estimation of the $e^+e^- \to f_2\gamma \to \pi^+\pi^-\gamma$ cross section, based on the Belle data about the $\gamma * (Q^2)\gamma \to f_2 \to \pi^+\pi^-$ cross section, is much less than the average BABAR value, but the experimental error is half of the average value, so no conclusions concerning this disagreement could be done now. Anyway, the pioneer BABAR measurement gives hope that the detailed investigation of the $\gamma^*(s) \to f_2\gamma$ and $\gamma^*(s) \to a_2\gamma$ transition form factors at high energy region is possible.

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