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Electromagnetic properties of neutrinos: three new phenomena in neutrino spin oscillations

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Abstract. In studies of neutrino electromagnetic properties we discuss three very interesting aspects related to neutrino spin oscillations. First we consider neutrino mixing and oscillations in the mass and flavour bases under the influence of a constant magnetic field with nonzero transversal and longitudinal components. Then we discuss the effect of neutrino spin oscillations induced by electroweak interactions of neutrino with moving matter in case there is matter transversal current or polarization. In the final part of the paper we discuss recently developed approach to description of neutrino spin and spin-flavour oscillations in a constant magnetic field that is based on the use of the exact neutrino stationary states in the magnetic field.

1 Introduction

It is well known that massive neutrinos have nontrivial electromagnetic properties, and at least the magnetic moment is not zero [1]. Thus, neutrinos do participate also in the electromagnetic interaction (see [2] for a review). The best terrestrial laboratory upper bound on neutrino magnetic moments is obtained by the GEMMA reactor neutrino experiment [3]. The best astrophysical upper bound was derived from considering stars cooling [4]. The neutrino magnetic moment procession in the transversal magnetic field $\mathbf{B}_{\perp}$ was first considered in [5], then spin-flavor precession in vacuum was discussed in [6], the importance of the matter effect was emphasized in [7]. The effect of resonant amplification of neutrino spin oscillations in $\mathbf{B}_{\perp}$ in the presence of matter was proposed in [8, 9], the impact of the longitudinal magnetic field $\mathbf{B}_{||}$ was discussed in [10].

Here below we discuss three very interesting aspects related to the neutrino spin and spin-flavour oscillations:

1) we consider in details [11] neutrino mixing and oscillations in arbitrary constant magnetic field that have $\mathbf{B}_{\perp}$ and $\mathbf{B}_{||}$ nonzero components in mass and flavour bases ,

2) we show that neutrino spin and spin-flavour oscillations can be induced not only by the neutrino interaction with a magnetic field but also by neutrino interactions with matter in the case when there is a transversal matter current or matter polarization (see [12] and references therein),

3) we develop a new (and more precise than the usual one) approach to description of neutrino spin and spin-flavor oscillations in the presence of an arbitrary magnetic field; our approach [13] is

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based on the use of the stationary states in the magnetic field for classification of neutrino spin states, contrary to the customary approach when the neutrino helicity states are used for this purpose.

2 Neutrino spin oscillations in mass and flavour bases

2.1 Oscillations in mass basis

We start [11] with two neutrino physical states ν_1 and ν_2 having masses m_1 and m_2 and introduce neutrino electromagnetic interaction via magnetic moment matrix $\mu_{\alpha\beta}$, $\alpha,\beta = 1,2$:

$$H_{EM} = \frac{1}{2}\mu_{\alpha\beta}\overline{\nu}_\beta\sigma_{\mu\nu}\nu_\alpha F^{\mu\nu} + h.c. \,, \tag{1}$$

where $F^{\mu\nu}$ is the electromagnetic field tensor and $\sigma_{\mu\nu} = i/2(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. In a uniform magnetic field the Hamiltonian (1) becomes

$$H_{EM} = -\mu_{\alpha\alpha'}\bar{\nu}_{\alpha'}\mathbf{\Sigma B}\nu_\alpha + h.c., \quad \Sigma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \tag{2}$$

where σ_i are the Pauli matrices.

In the neutrino oscillation framework, one is interested in evolution of chiral neutrino components within the common neutrino beam state. Since in the ultrarelativistic limit the latter are approximated by free neutrino states with definite helicity $s = \pm 1$ the 4-component basis $(\nu_{1,s=1}, \nu_{1,s=-1}, \nu_{2,s=1}, \nu_{2,s=-1})$ is adopted to describe neutrino beam. With the standard column vector notation, $\nu_m \equiv (\nu_{1,s=1}, \nu_{1,s=-1}, \nu_{2,s=1}, \nu_{2,s=-1})^\tau$ the neutrino evolution equation relevant to electromagnetic interaction has the Schrödinger-like form

$$i\frac{d}{dt}\nu_m(t) = H_{eff}\nu_m(t). \tag{3}$$

The effective Hamiltonian consists of the vacuum and interaction parts

$$H_{eff} = H_{vac} + H_B \tag{4}$$

where the interaction part is composed with matrix elements of the field interaction Hamiltonian taken over the helicity neutrino states: $H_B = \langle\nu_{\alpha,s}|H_{EM}|\nu_{\alpha',s'}\rangle$.

Let us calculate the effective interaction Hamiltonian under assumption that neutrino moves along the z-axis. From the magnetic field interaction Hamiltonian (1) we have:

$$H^B_{\alpha,s;\alpha',s'} = \langle\nu_{\alpha,s}|H_{EM}|\nu_{\alpha',s'}\rangle = -\frac{\mu_{\alpha,\alpha'}}{2}\int d^3x\nu^\dagger_\alpha\gamma_0\begin{pmatrix}\mathbf{\Sigma B} & 0\\ 0 & \mathbf{\Sigma B}\end{pmatrix}\nu_{\alpha'}. \tag{5}$$

In the spinor representation the free neutrino states are given by

$$\nu_{\alpha,s} = C_\alpha\sqrt{\frac{E_\alpha + m_\alpha}{2E_\alpha}}\begin{pmatrix} u_s \\ \frac{\mathbf{\Sigma p}_\alpha}{E_\alpha+m_\alpha}u_s\end{pmatrix}e^{i\mathbf{p}_\alpha x}, \tag{6}$$

where $\mathbf{p}_\alpha$ is the neutrino ν_α momentum. The two-component spinors u_s define neutrino helicity states, and are given by

$$u_{s=1} = \begin{pmatrix}1\\0\end{pmatrix}, \quad u_{s=-1} = \begin{pmatrix}0\\1\end{pmatrix}. \tag{7}$$

Recall that in the ultrarelativistic limit these are correspondent to the right-handed ν_R and left-handed ν_L chiral neutrinos, respectively.

Substituting (6) into the effective Hamiltonian formula (5) we get

$$H^B_{\alpha,s;\alpha',s'} = -\frac{1}{2}\mu_{\alpha\alpha'} C_\alpha C_{\alpha'} \int d^3x B \left(u_s^\dagger \quad \tfrac{\Sigma p_\alpha}{E_\alpha + m_\alpha} u_s^\dagger\right) \begin{pmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{pmatrix} \begin{pmatrix} u_{s'} \\ \tfrac{\Sigma p_{\alpha'}}{E_{\alpha'} + m_{\alpha'}} u_{s'} \end{pmatrix}$$

$$\times \frac{\sqrt{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}}{2\sqrt{E_\alpha E_{\alpha'}}} \exp(i\Delta p x) . \quad (8)$$

Decomposing the magnetic field vector into longitudinal and transversal with respect to neutrino motion components $\mathbf{B} = \mathbf{B}_{\|} + \mathbf{B}_{\perp}$ it is possible to show that

$$\mathbf{B} \left(u_s^\dagger \quad \tfrac{\Sigma p_\alpha}{E_\alpha + m_\alpha} u_s^\dagger\right) \begin{pmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{pmatrix} \begin{pmatrix} u_{s'} \\ \tfrac{\Sigma p_{\alpha'}}{E_{\alpha'} + m_{\alpha'}} u_{s'} \end{pmatrix} =$$

$$u_s^\dagger \left(\Sigma B_{\|} \left(1 - \frac{p_\alpha p_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}\right) + \Sigma B_\perp \left(1 + \frac{p_\alpha p_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}\right)\right) u_{s'} . \quad (9)$$

Let us apply the ultrarelativistic condition $\frac{m_\alpha}{E_\alpha} \ll 1$ to the part of the integrand in (8):

$$\left(1 \mp \frac{p_\alpha p_{\alpha'}}{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}\right) \frac{\sqrt{(E_\alpha + m_\alpha)(E_{\alpha'} + m_{\alpha'})}}{2\sqrt{E_\alpha E_{\alpha'}}} \approx \left\{ \begin{array}{c} \gamma^{-1}_{\alpha\alpha'} \\ 1 \end{array} \right. \quad (10)$$

where $\gamma_{\alpha\alpha'} = \frac{1}{2}\left(\frac{m_\alpha}{E_\alpha} + \frac{m_{\alpha'}}{E_{\alpha'}}\right)$ is the transition gamma-factor.

Introducing an angle β between $\mathbf{B}$ and $\mathbf{p}_\alpha$ vectors and assuming that $\mathbf{B}_\perp$ is aligned along the x-axis we further obtain:

$$u^\dagger_{s=1} \Sigma \mathbf{B} u_{s=1} = B\cos\beta, \quad u^\dagger_{s=1} \Sigma \mathbf{B} u_{s=-1} = B\sin\beta, \quad (11)$$

$$u^\dagger_{s=-1} \Sigma \mathbf{B} u_{s=1} = B\sin\beta, \quad u^\dagger_{s=-1} \Sigma \mathbf{B} u_{s=-1} = -B\cos\beta. \quad (12)$$

As it was expected, in neutrino transitions without change of helicity only the $B_{\|} = B\cos\beta$ component of the magnetic field contribute to the effective potential, whereas in transitions with change of the neutrino helicity the transversal component $B_\perp = B\sin\beta$ matters.

Performing remaining some simple algebra one can readily write out the H_B matrix. For the effective Hamiltonian with the diagonal vacuum part H_{vac} we get

$$i\frac{d}{dt}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} = \begin{pmatrix} E_1 + \mu_{11}\frac{B_{\|}}{\gamma_{11}} & \mu_{11}B_\perp & \mu_{12}\frac{B_{\|}}{\gamma_{12}} & \mu_{12}B_\perp \\ \mu_{11}B_\perp & E_1 - \mu_{11}\frac{B_{\|}}{\gamma_{11}} & \mu_{12}B_\perp & -\mu_{12}\frac{B_{\|}}{\gamma_{12}} \\ \mu_{12}\frac{B_{\|}}{\gamma_{12}} & \mu_{12}B_\perp & E_2 + \mu_{22}\frac{B_{\|}}{\gamma_{22}} & \mu_{22}B_\perp \\ \mu_{12}B_\perp & -\mu_{12}\frac{B_{\|}}{\gamma_{12}} & \mu_{22}B_\perp & E_2 - \mu_{22}\frac{B_{\|}}{\gamma_{22}} \end{pmatrix} \begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix}. \quad (13)$$

This equation governs all possible oscillations of the four neutrino mass states determined by the masses m_1 and m_2 and helicities $s = 1$ and $s = -1$ in the presence of a magnetic field. Thus, it follows that: 1) the change of helicity is due to the magnetic (or transition) moment interaction with $\mathbf{B}_\perp$, 2) the longitudinal field $\mathbf{B}_{\|}$, coupled to the magnetic moment, shifts the neutrino energy, 3) an additional mixing between neutrino states with different masses is induced by the magnetic moment interaction with $\mathbf{B}_{\|}$.

2.2 Oscillations in flavour basis

Once having physics in the mass basis in hands, our next step is to bring it to observational terms [11]. This means that we must elaborate a generalization of the mixing matrix for transitions between neutrino vector written in two four-component bases ν_m and $\nu_f = (\nu_e^R, \nu_e^L, \nu_\mu^R, \nu_\mu^L)^\tau$ so that

$$\nu_f = U\nu_m. \tag{14}$$

This procedure appears to be not quite direct since we should hold the condition that the polarization of the fields must preserve under transformation of the bases elements. That is why we put (still keeping in mind that chiral components are almost helicity ones):

$$\nu_e^{R,L} = \nu_{1,s=\pm 1}\cos\theta + \nu_{2,s=\pm 1}\sin\theta, \quad \nu_\mu^{R,L} = -\nu_{1,s=\pm 1}\sin\theta + \nu_{2,s=\pm 1}\cos\theta. \tag{15}$$

Then, using Eqs. (14) and (15), it is easy to obtain that

$$U = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{16}$$

Given the transition matrix (16), derivation of the evolution equation in the flavor basis is straightforward:

$$i\frac{d}{dt}\nu_f = UHU^\dagger \nu_f, \tag{17}$$

so that the effective interaction Hamiltonian $\tilde{H}_B^f \equiv UH_BU^\dagger$ has the following structure,

$$\tilde{H}_B^f = \begin{pmatrix} (\frac{\mu}{\gamma})_{ee}B_\parallel & \mu_{ee}B_\perp & (\frac{\mu}{\gamma})_{e\mu}B_\parallel & \mu_{e\mu}B_\perp \\ \mu_{ee}B_\perp & -(\frac{\mu}{\gamma})_{ee}B_\parallel & \mu_{e\mu}B_\perp & -(\frac{\mu}{\gamma})_{e\mu}B_\parallel \\ (\frac{\mu}{\gamma})_{e\mu}B_\parallel & \mu_{e\mu}B_\perp & (\frac{\mu}{\gamma})_{\mu\mu}B_\parallel & \mu_{\mu\mu}B_\perp \\ \mu_{e\mu}B_\perp & -(\frac{\mu}{\gamma})_{e\mu}B_\parallel & \mu_{\mu\mu}B_\perp & -(\frac{\mu}{\gamma})_{\mu\mu}B_\parallel \end{pmatrix}. \tag{18}$$

Here we introduce the following formal notations intended to manifest an analogy with the standard spin and spin-flavor oscillation formalism,

$$\begin{aligned} \Big(\frac{\mu}{\gamma}\Big)_{ee} &= \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta \\ \Big(\frac{\mu}{\gamma}\Big)_{e\mu} &= \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\Big(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\Big)\sin 2\theta \\ \Big(\frac{\mu}{\gamma}\Big)_{\mu\mu} &= \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta \end{aligned} \tag{19}$$

$$\begin{aligned} \mu_{ee} &= \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta \\ \mu_{e\mu} &= \mu_{12}\cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11})\sin 2\theta \\ \mu_{\mu\mu} &= \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta - \mu_{12}\sin 2\theta \end{aligned} \tag{20}$$

It should be noted that equations (20) follows from the general expression that settles relations between two neutrino bases [2].

It is interesting to consider a particular case when only the longitudinal magnetic field $\mathbf{B}_{\parallel}$ is present in an environment ($\mathbf{B}_{\perp} = 0$). Then the Hamiltonian (18) is reduced to the following one,

$$\tilde{H}_B^f = \begin{pmatrix} (\frac{\mu}{\gamma})_{ee}B_{\parallel} & 0 & (\frac{\mu}{\gamma})_{e\mu}B_{\parallel} & 0 \\ 0 & -(\frac{\mu}{\gamma})_{ee}B_{\parallel} & 0 & -(\frac{\mu}{\gamma})_{e\mu}B_{\parallel} \\ (\frac{\mu}{\gamma})_{e\mu}B_{\parallel} & 0 & (\frac{\mu}{\gamma})_{\mu\mu}B_{\parallel} & 0 \\ 0 & -(\frac{\mu}{\gamma})_{e\mu}B_{\parallel} & 0 & -(\frac{\mu}{\gamma})_{\mu\mu}B_{\parallel} \end{pmatrix}. \tag{21}$$

Obviously, the neutrino states with different flavor and same chirality decouple and form subsystems independently mixed by the magnetic field. For example, one would have two states (ν_e^L, ν_μ^L) mixed in accordance with the equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L \\ \nu_\mu^L\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta - (\frac{\mu}{\gamma})_{ee}B_{\parallel} & \frac{\Delta m^2}{4E}\sin 2\theta - (\frac{\mu}{\gamma})_{e\mu}B_{\parallel} \\ \frac{\Delta m^2}{4E}\sin 2\theta - (\frac{\mu}{\gamma})_{e\mu}B_{\parallel} & \frac{\Delta m^2}{4E}\cos 2\theta - (\frac{\mu}{\gamma})_{\mu\mu}B_{\parallel} \end{pmatrix}\begin{pmatrix}\nu_e^L \\ \nu_\mu^L\end{pmatrix}. \tag{22}$$

From this it follows that the neutrino magnetic moment interactions with the longitudinal magnetic field can generate the neutrino flavour mixing (an additional mixing to the usual effect due to neutrino mixing angle θ) without changing neutrino chirality. For the flavour neutrino oscillation probability in the adiabtic case we get

$$P_{\nu_e^L\to\nu_\mu^L} = \frac{\left(\frac{\Delta m^2}{2E}\sin 2\theta - 2(\frac{\mu}{\gamma})_{e\mu}B_{\parallel}\right)^2}{\left(\frac{\Delta m^2}{2E}\sin 2\theta - 2(\frac{\mu}{\gamma})_{e\mu}B_{\parallel}\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta + 2\frac{\mu_{12}}{\gamma_{12}}B_{\parallel}\sin 2\theta\right)^2}\sin^2\left(\frac{1}{2}\sqrt{D}x\right), \tag{23}$$

where $D = \left(\frac{\Delta m^2}{2E}\sin 2\theta - 2(\frac{\mu}{\gamma})_{e\mu}B_{\parallel}\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta + 2\frac{\mu_{12}}{\gamma_{12}}B_{\parallel}\sin 2\theta\right)^2$. It follows that $B_{\parallel}$ not only generates flavour neutrino mixing but also can produce the resonance amplification of the corresponding oscillations.

3 Neutrino spin precession and oscillations due to matter transversal motion

Consider, as an example, an electron neutrino spin procession in the case when neutrinos with the Standard Model interaction are propagating through moving and polarized matter composed of electrons (electron gas) in the presence of an electromagnetic field given by the electromagnetic-field tensor $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$. As discussed in [14, 15] (see also [16–18]), the the generalized Bargmann-Michel-Telegdi equation describes the evolution of the three-dimensional neutrino spin vector $\vec{S}$,

$$\frac{d\mathbf{S}}{dt} = \frac{2}{\gamma}\Big[\mathbf{S}\times(\mathbf{B}_0 + \mathbf{M}_0)\Big], \tag{24}$$

where the magnetic field $\mathbf{B_0}$ in the neutrino rest frame is determined by the transversal and longitudinal (with respect to the neutrino motion) magnetic and electric field components in the laboratory frame,

$$\mathbf{B}_0 = \gamma\Big(\mathbf{B}_{\perp} + \frac{1}{\gamma}\mathbf{B}_{\parallel} + \sqrt{1-\gamma^{-2}}\Big[\mathbf{E}_{\perp}\times\frac{\beta}{\beta}\Big]\Big), \tag{25}$$

$\gamma = (1-\beta^2)^{-\frac{1}{2}}$, β is the neutrino velocity.

The matter term $\mathbf{M}_0$ in Eq. (24) is also composed of the transversal $\mathbf{M}_{0_\parallel}$ and longitudinal $\mathbf{M}_{0_\perp}$ parts,

$$\mathbf{M}_0 = \mathbf{M}_{\mathbf{0}_\parallel} + \mathbf{M}_{\mathbf{0}_\perp}, \tag{26}$$

$$\mathbf{M}_{0_\parallel} = \gamma\beta \frac{n_0}{\sqrt{1-v_e^2}} \left\{ \rho_e^{(1)} \left(1 - \frac{\mathbf{v}_e \beta}{1-\gamma^{-2}} \right) \right\} - \frac{\rho_e^{(2)}}{1-\gamma^{-2}} \left\{ \zeta_e \beta \sqrt{1-v_e^2} + \left(\zeta_e \mathbf{v}_e \frac{(\beta \mathbf{v}_e)}{1+\sqrt{1-v_e^2}} \right) \right\}, \tag{27}$$

$$\mathbf{M}_{0_\perp} = -\frac{n_0}{\sqrt{1-v_e^2}} \left\{ \mathbf{v}_{e_\perp} \left(\rho_e^{(1)} + \rho_e^{(2)} \frac{(\zeta_e \mathbf{v}_e)}{1+\sqrt{1-v_e^2}} \right) + \zeta_{e_\perp} \rho_e^{(2)} \sqrt{1-v_e^2} \right\}. \tag{28}$$

Here $n_0 = n_e \sqrt{1-v_e^2}$ is the invariant number density of matter given in the reference frame for which the total speed of matter is zero. The vectors $\mathbf{v}_e$, and ζ_e $(0 \le |\zeta_e|^2 \le 1)$ denote, respectively, the speed of the reference frame in which the mean momentum of matter (electrons) is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The coefficients $\rho_e^{(1,2)}$ calculated within the extended Standard Model supplied with $SU(2)$-singlet right-handed neutrino ν_R are respectively, $\rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}$, $\rho_e^{(2)} = -\frac{G_F}{2\sqrt{2}\mu}$, where $\tilde{G}_F = G_F(1+4\sin^2\theta_W)$. For neutrino evolution between two neutrino states $\nu_e^L \Leftrightarrow \nu_e^R$ in presence of the magnetic field and moving matter we get the following equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} = \begin{pmatrix} \frac{1}{\gamma}\left|\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}\right| & \left|\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{M}_{0\perp}\right| \\ \left|\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{M}_{0\perp}\right| & -\frac{1}{\gamma}\mid \mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}\right| \end{pmatrix} \begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix}. \tag{29}$$

Thus, the probability of the neutrino spin oscillations in the adiabatic approximation is given by (see [14, 15])

$$P_{\nu_L\to\nu_R}(x) = \sin^2 2\theta_{\text{eff}} \sin^2 \frac{\pi x}{L_{\text{eff}}}, \ \sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \quad L_{\text{eff}} = \frac{2\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}}, \tag{30}$$

where $E_{\text{eff}} = \left|\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{M}_{0\perp}\right|$, $\Delta_{\text{eff}} = \frac{}{\gamma}\left|\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}\right|$. It follows that even without presence of an electromagnetic field, $\mathbf{B}_\perp = \mathbf{B}_{0\parallel} = 0$, neutrino spin oscillations can be induced in the presence of matter when the transverse matter term $\mathbf{M}_{0\perp}$ is not zero. This possibility is realized in the case when the transverse component of the background matter velocity or its transverse polarization is not zero. It is obvious that for neutrinos with nonzero transition magnetic moments a similar effect of spin-flavour oscillations exists under the same background conditions. A possibility of neutrino spin procession and oscillations induced by the transversal matter current or polarization was first discussed in [14, 15]. The existence of this effect has been recently confirmed in [19–21] where neutrinos propagation in anisotropic media is studied.

4 Neutrino spin oscillations and stationary spin states in magnetic field

We develop a new approach [13], that is more precise than the usual one, to description of neutrino spin and spin-flavor oscillations in the presence of an arbitrary magnetic field. The derived probability of neutrino oscillations does not coincide with the usual one and the difference might have important phenomenological consequences. Within this customary approach the helicity operator is used for classification of a neutrino spin states in a magnetic field. However, the helicity operator does not commute with the neutrino evolution Hamiltonian in a magnetic field. This case resembles situation of the flavour neutrino oscillations in the nonadiabatic case when the neutrino mass states are not stationary. The proposed alternative approach to neutrino spin oscillations is based on the exact solutions of the corresponding Dirac equation for a massive neutrino wave function in the presence of a magnetic field that stipulates the description of the neutrino spin states with the corresponding spin operator that commutes with the neutrino dynamic Hamiltonian in the magnetic field.

Here we again consider a simple model with two generations of flavour neutrinos ν_e and ν_μ that are the orthogonal superpositions of mass states ν_1 and ν_2 $\nu_f = \sum_i U_{fi}\nu_i$, where U_{fi} are elements the mixing matrix given by (15) and $f = e, \mu$, $i = 1, 2$. We start with consideration of a massive ν_i with the magnetic moment μ_i that propagates along $\mathbf{n_z}$ direction in presence of constant homogeneous arbitrary orientated magnetic field $\mathbf{B} = (B_\perp, 0, B_\parallel)$. The neutrino wave function in the momentum representation is given by a plane wave solution of the modified Dirac equation

$$(\gamma p - m_i - \mu_i \Sigma \mathbf{B})\nu_i(p) = 0. \tag{31}$$

For the neutrino energy spectrum we obtain

$$E_i^{\pm} = \sqrt{m_i^2 + p^2 + \mu_i{}^2\mathbf{B}^2 \pm 2\mu_i \sqrt{m_i^2\mathbf{B}^2 + p^2 B_\perp^2}}, \tag{32}$$

where "±" denotes two different eigenvalues of the Hamiltonian $H_i = \gamma_0(m_i + \gamma\mathbf{p} + \mu_i\Sigma\mathbf{B})$, which describes dynamics of the neutrino system under consideration.

We define different neutrino spin states in the mass basis as eigenstates of the spin operator

$$S_i = \frac{1}{|\mathbf{B}|}(\mathbf{\Sigma B} - \frac{i}{m_i}\gamma_0\gamma_5\,[\mathbf{\Sigma} \times \mathbf{p}]\,\mathbf{B}), \tag{33}$$

which commutes with the Hamiltonian H_i. Hence, we specify the neutrino spin states as the stationary states for the Hamiltonian, contrary to the case when the helicity operator is used.

Consider the mass state ν_i as a superposition of neutrinos ν_i^+ and ν_i^- in a definite spin state, $\nu_i = c_i^+\nu_i^+ + c_i^-\nu_i^-$. The complex coefficients denote two different eigenstates of the spin operator S_i and $|c_i^+|^2 + |c_i^-|^2 = 1$. Thus, the neutrino mass states evolve following to

$$\nu_i(x) = \left[c_i^+ e^{-iE_i^+ t}\xi_i^+ + c_i^- e^{-iE_i^- t}\xi_i^-\right] e^{ipx}, \tag{34}$$

where the neutrino initial state at $t = 0$ is given by $\nu_i^{\pm}(t = 0) = \xi_i^{\pm} e^{ipx}$. In the following calculations the term e^{ipx} is neglected because it is irrelevant for the neutrino oscillation probability.

Next we assume that the initial neutrino state $\nu(t = 0)$ is a pure electron state which is defined as the superposition of the mass states,

$$\nu(t = 0) = [c_1^+\xi_1^+ + c_1^-\xi_1^-]\cos\theta + [c_2^+\xi_2^+ + c_2^-\xi_2^-]\sin\theta. \tag{35}$$

Using (34) we see that this state depends on time as

$$\nu(t) = \left[c_1^+ e^{-iE_1^+ t}\xi_1^+ + c_1^- e^{-iE_1^- t}\xi_1^-\right]\cos\theta + \left[c_2^+ e^{-iE_2^+ t}\xi_2^+ + c_2^- e^{-iE_2^- t}\xi_2^-\right]\sin\theta. \tag{36}$$

Therefore, the probability to observe the muon neutrino state ν_μ at time t is given by $P_{\nu_e\to\nu_\mu}(t) = \left|\left\langle \nu_\mu | \nu(t)\right\rangle\right|^2$, where $\nu_\mu = -\left[c_1^+\xi_1^+ + c_1^-\xi_1^-\right]\sin\theta + \left[c_2^+\xi_2^+ + c_2^-\xi_2^-\right]\cos\theta$. It is clear that $\xi_i^{s\dagger}\xi_i^{s'} =$

$\delta^{ss'}$ and $\xi_2^{s\dagger}\xi_1^{s'} = \xi_1^{s\dagger}\xi_2^{s'} = 0$, where $s, s' = \pm$. Using (35) and (36) we get for the oscillation probability

$$P_{\nu_e\to\nu_\mu}(t) = \Big\{ -|c_2^+|^2|c_2^-|^2 \sin^2\frac{E_2^+ - E_2^-}{2}t + |c_2^+|^2|c_1^+|^2 \sin^2\frac{E_2^+ - E_1^+}{2}t + + |c_2^+|^2|c_1^-|^2 \sin^2\frac{E_2^+ - E_1^-}{2}t + |c_2^-|^2|c_1^+|^2 \sin^2\frac{E_2^- - E_1^+}{2}t + +|c_2^-|^2|c_1^-|^2 \sin^2\frac{E_2^- - E_1^-}{2}t - |c_1^+|^2|c_1^-|^2 \sin^2\frac{E_1^+ - E_1^-}{2}t\Big\} \sin^2 2\theta. \quad (37)$$

It is usually assumed that the initial state of relativistic neutrino is a negative-helicity state, which means that

$$\frac{\mathbf{\Sigma p}}{|\mathbf{p}|}\xi_i = -\xi_i. \quad (38)$$

Next we consider the left-handed spinors because only the left-handed fermions participate in the production and detection processes and we suppose that each of the mass states of the initial electron neutrino are left-handed. In our case the helicity operator is equal to $\mathbf{\Sigma p}/|\mathbf{p}| = \sigma_3$, therefore the initial neutrino state is given by $\psi_L = (0, 1, 0, 0)^T$. Let us write the initial neutrino state ψ_L as a superposition of the eigenvectors of the spin operator S_i. From (33) we get

$$S_i = \begin{pmatrix} \cos\phi & \sin\phi & 0 & -\frac{p}{m_i}\sin\phi \\ \sin\phi & -\cos\phi & \frac{p}{m_i}\sin\phi & 0 \\ 0 & \frac{p}{m_i}\sin\phi & \cos\phi & \sin\phi \\ -\frac{p}{m_i}\sin\phi & 0 & \sin\phi & -\cos\phi \end{pmatrix}, \quad (39)$$

where ϕ is the angle between $\mathbf{B}$ and $\mathbf{p}$. It is obvious that $S_i^2 = \left(1 + \frac{p^2}{m_i^2}\sin^2\phi\right)\hat{I}_{4\times4}$. In order to define the spin projector operators we introduce the normalized spin operator following to

$$\tilde{S}_i = \sqrt{\frac{1}{1 + \frac{p^2}{m_i^2}\sin^2\phi}}S_i \equiv N_i S_i, \quad \tilde{S}_i^2 = 1, \quad N_i = \sqrt{\frac{1}{1 + \frac{p^2}{m_i^2}\sin^2\phi}}. \quad (40)$$

The spin projector operators are $P_i^\pm = \frac{1\pm\tilde{S}_i}{2}$, and we use them to split the initial neutrino state ψ_L in two neutrino states with definite spin quantum numbers

$$\psi_i^+ = P_i^+\psi_L = \frac{1}{2}\begin{pmatrix} N_i\sin\phi \\ 1 - N_i\cos\phi \\ \frac{p}{m_i}N_i\sin\phi \\ 0 \end{pmatrix}, \quad \psi_i^- = P_i^-\psi_L = \frac{1}{2}\begin{pmatrix} -N_i\sin\phi \\ 1 + N_i\cos\phi \\ -\frac{p}{m_i}N_i\sin\phi \\ 0 \end{pmatrix}. \quad (41)$$

Note that $\psi_i = \psi_i^+ + \psi_i^- = c_i^+\eta_i^+ + c_i^-\eta_i^-$, where $\eta_i^\pm$ is a basis in the spin operator S_i eigenspace. From the condition $\psi_i^{\dagger\pm}\psi_L^\pm = |c_i^\pm|^2$ we get that $|c_i^+|^2 = \frac{1-N_i\cos\phi}{2}, \quad |c_i^-|^2 = \frac{1+N_i\cos\phi}{2}$.

Now we can insert the obtained expressions for $|c_i^\pm|^2$ in (37). In the forthcoming evaluation of the probability $P_{\nu_e\to\nu_\mu}(t)$ we consider the case when the magnetic field $\mathbf{B}$ is nearly a transversal one and $B_\perp \gg B_\parallel$, therefore $\sin\phi \approx 1, \quad \cos\phi \approx 0$. Then we get (it is also supposed that $\frac{p^2}{m_i^2} \gg 1$)

$$N_i \approx 1 - \frac{\frac{p^2}{2m_i^2}\sin^2\phi}{1 + \frac{p^2}{m_i^2}\sin^2\phi} \approx \frac{m_i^2}{p\sin^2\phi}. \quad (42)$$

Thus, for typical combinations of the coefficients $|c_i^{\pm}|^2$ of eq.(37) in the linear approximation over $\cos\phi \ll 1$ we get

$$|c_i^+|^2|c_i^-|^2 = \frac{1-N_i^2\cos^2\phi}{4} \approx \frac{1}{4}, \tag{43}$$

$$|c_2^s|^2|c_1^s|^2 \approx \frac{1}{4}\left(1-s(N_1+N_2)\cos\phi\right) \approx \frac{1}{4}\left(1-s\frac{m_1^2+m_2^2}{p^2}\cos\phi\right), \tag{44}$$

$$|c_2^+|^2|c_1^-|^2 \approx \frac{1}{4}\left(1-(N_2-N_1)\cos\phi\right) \approx \frac{1}{4}\left(1-\frac{m_2^2-m_1^2}{p^2}\cos\phi\right), \tag{45}$$

$$|c_2^-|^2|c_1^+|^2 \approx \frac{1}{4}\left(1-(N_1-N_2)\cos\phi\right) \approx \frac{1}{4}\left(1-\frac{m_1^2-m_2^2}{p^2}\cos\phi\right). \tag{46}$$

Using these expressions finally from (37) we get

$$\begin{aligned} P_{\nu_{eL}\to\nu_\mu}(t) = \frac{1}{4}\sin^2 2\theta\Big\{&-\sin^2\frac{E_2^+-E_2^-}{2}t+\sin^2\frac{E_2^+-E_1^+}{2}t+\sin^2\frac{E_2^+-E_1^-}{2}t+ \\ &+\sin^2\frac{E_2^--E_1^+}{2}t+\sin^2\frac{E_2^--E_1^-}{2}t-\sin^2\frac{E_1^+-E_1^-}{2}t\Big\}+ \\ &+\frac{m_1^2+m_2^2}{p^2}\sin^2 2\theta\cos\phi\left\{\sin^2\frac{E_2^--E_1^-}{2}t-\sin^2\frac{E_2^+-E_1^+}{2}t\right\}+ \\ &+\frac{m_2^2-m_1^2}{p^2}\sin^2 2\theta\cos\phi\left\{\sin^2\frac{E_2^--E_1^+}{2}t-\sin^2\frac{E_2^+-E_1^-}{2}t\right\}. \end{aligned} \tag{47}$$

Note that two last terms here are suppressed by the presence of $\cos\phi \ll 1$. If we also account for a rather general condition $2\mu B_\perp \ll p$ then for the neutrino energies we get

$$E_i^{\pm} \approx \sqrt{m_i^2+p^2\pm 2\mu_i p B_\perp\left(1+\frac{m_i^2}{2p^2}\right)} \approx p\sqrt{1+\frac{m_i^2}{p^2}\pm\frac{2\mu_i B_\perp}{p}} \approx p+\frac{m_i^2}{2p}\pm\mu_i B_\perp. \tag{48}$$

Finally, for the neutrino oscillations probability in the flavour basis we get (here $\Delta m^2 \equiv m_2^2 - m_1^2$)

$$\begin{aligned} P_{\nu_{eL}\to\nu_\mu}(t) \approx \sin^2 2\theta\sin^2\frac{\Delta m^2}{4p}t+\frac{1}{2}\left(\sin^2\frac{\mu_2-\mu_1}{2}B_\perp t+\sin^2\frac{\mu_2+\mu_1}{2}B_\perp t\right)\sin^2 2\theta\cos\frac{\Delta m^2}{2p}t \\ -\frac{1}{4}\sin^2 2\theta\left(\sin^2\mu_1 B_\perp t+\sin^2\mu_2 B_\perp t\right). \end{aligned} \tag{49}$$

It should be emphasized that, as it follows from the above derivations, the obtained probability $P_{\nu_{eL}\to\nu_\mu}(t)$ accounts for the transitions from the initial left-handed electron neutrino to the final muon neutrino that can be in both left- and right-handed states. Note that in the mass basis the transition $\nu_{1L} \to \nu_{2R}$ is not possible when $\mu_{12} = 0$ (see (29)).

Consider the difference $\Delta P(t) = P_{\nu_{eL}\to\nu_\mu} - P_{\nu_{eL}\to\nu_{\mu L}}$ of the probability $P_{\nu_{eL}\to\nu_\mu}(t)$ given by (50) and the usual result $P_{\nu_{eL}\to\nu_{\mu L}}(t) = \sin^2 2\theta\sin^2\frac{\Delta m^2}{4E_\nu}t$ for $\nu_{eL} \to \nu_{\mu L}$. Obviously, this difference is just a probability for the transition $P_{\nu_{eL}\to\nu_{\mu R}}$, and we get (here $\Delta\mu_\pm = \mu_2 \pm \mu_1$)

$$\begin{aligned} P_{\nu_{eL}\to\nu_{\mu R}} = \frac{1}{2}\left(\sin^2\frac{\Delta\mu_-}{2}B_\perp t+\sin^2\frac{\Delta\mu_+}{2}B_\perp t\right)\sin^2 2\theta\cos\frac{\Delta m^2}{2p}t \\ -\frac{1}{4}\sin^2 2\theta\left(\sin^2\mu_1 B_\perp t+\sin^2\mu_2 B_\perp t\right). \end{aligned} \tag{50}$$

Note that in the usual approach (see, for instance, in [2]) transitions $\nu_{eL} \to \nu_{\mu R}$ in $B_{\perp}$ are not possible if the transition magnetic moments are zero. However, within the developed approach it is shown that the previous statement is not correct.

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