Squeezed and Entangled Gluon States in QCD Jet

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Abstract. Theoretical justification for the occurrence of multimode squeezed and entangled colour states in QCD is given. We show that gluon entangled states which are closely related with corresponding squeezed states can appear by the four-gluon self-interaction. Correlations for the collinear gluons are revealed two groups of the colour correlations which is significant at consider of the quark-antiquark pair productions.

1 Introduction

Many experiments at e^+e^- , $p\bar{p}$, ep colliders are devoted to hadronic jet physics, since detailed studies of jets are important for better understanding and testing both perturbative and non-perturbative QCD and also for finding manifestations of new physics. Although the nature of jets is of a universal character, e^+e^- annihilation stands out among hard processes, since jet events admit a straightforward and clear-cut separation in this process. In the reaction $e^+e^- \rightarrow \text{hadron}$ four evolution phases are recognized by various time and space scales (Fig.1). These are (I) the production of a quark-antiquark pair: $e^+e^- \rightarrow q\bar{q}$; (II) the emissions of gluons and quarks from primary partons — perturbative evolution of the quark-gluon cascade; (III) the non-perturbative evolution and the hadronization of quarks and gluons; (IV) the decays of unstable particles.

Figure 1. Jet evolution

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The second phase of e^+e^- - annihilation has been well understood and sufficiently accurate predictions for it have been obtained within the perturbative QCD (PQCD) [1]. But predictions of the PQCD are limited by small effective coupling $\alpha(Q^2) < 1$ and third phase is usually taken into account either through a constant factor which relates partonic features with hadronic ones (within local parton-hadron duality) or through the application of various phenomenological models of hadronization. As a consequence, theoretical predictions both for intrajet and for interjet characteristics remain unsatisfactory. For example, the width of the multiplicity distribution (MD) according to the predictions of PQCD is larger than the experimental one. The discrepancies between theoretical calculations and experimental data, for example the width of MD, suggest that the non-perturbative evolution of the quark-gluon cascade plays important role. New gluon states, generated at the non-perturbative stage, contribute to various features of jets. In particular, such a contribution to MD can be in the form of the sub-Poissonian distribution [2, 3].

It is known that such property is inherent for the squeezed states (SS), which are well studied in quantum optics (QO) [4]-[6]. Squeezed states possess uncommon properties: they display a specific behaviour of the factorial and cumulant moments [7] and can have both sub-Poissonian and super-Poissonian statistics corresponding to antibunching and bunching of photons.

Therefore we believe that the non-perturbative stage of gluon evolution can be one of sources of the gluon SS in QCD by analogy with nonlinear medium for photon SS in QO. Gluon MD in the range of the small transverse momenta (thin ring of jet) is Poissonian [8]. Quark-gluon MD in the whole jet at the end of the perturbative cascade can be represented as a combination of Poissonian distributions each of which corresponds to a coherent state. Studying a further evolution of gluon states at the non-perturbative stage of jet evolution we obtain new gluon states. These states are formed as a result of non-perturbative self-interaction of the gluons expressed by nonlinearities of Hamiltonian. Using the Local parton hadron duality it is easy to show that in this case behaviour of hadron multiplicity distribution in jet events is differentiated from the negative binomial one that is confirmed by experiments for pp , $p\bar{p}$ -collisions [9]-[11].

At finite squeezed parameter r a continuous variables entangled state is known from quantum optics as a two-mode squeezed state [4, 14]

$$|f\rangle = \hat{S}_{12}(r)|0\rangle_1|0\rangle_2 = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n |n\rangle_1|n\rangle_2, \quad (1)$$

where $\hat{S}_{12}(r) = \exp\{r(\hat{a}_1^+\hat{a}_2^+ - \hat{a}_1\hat{a}_2)\}$ is operator of two-mode squeezing. It is not difficult to demonstrate that the state vector $|f\rangle$ describes the entangled state. Each of these entangled states has a uncommon property: a measurement over one particle have an instantaneous effect on the other, possibly located at a large distance.

The dimensionless coefficient

$$y = \left[\frac{|\overline{\hat{a}_1\hat{a}_2^+}|^2 + |\overline{\hat{a}_1\hat{a}_2}|^2}{2(\overline{\hat{a}_1^+\hat{a}_1} + 1/2)(\overline{\hat{a}_2^+\hat{a}_2} + 1/2)} \right]^{1/2} \quad (2)$$

is the measure of entanglement for two-mode states [15], $0 \leq y < 1$ (entanglement is not observed when $y = 0$). Here $\overline{\hat{a}_i\hat{a}_j^+} = \langle \hat{a}_i\hat{a}_j^+ \rangle - \langle \hat{a}_i \rangle \langle \hat{a}_j^+ \rangle$, $\hat{a}_i, \hat{a}_j^+$ are the annihilation and creation operators correspondingly. Averaging the annihilation and creation operators in the expression (2) over the vector $|f\rangle$ (1) at small squeeze factor we have

$$y \simeq \sqrt{2}r. \quad (3)$$

Two-mode gluon states with two different colours can lead to $q\bar{q}$ -entangled states. Interaction of the quark entangled states with stochastic vacuum (quantum measurement) has a remarkable property,

namely, as soon as some measurement projects one quark onto a state with definite colour, the other quark also immediately obtains opposite colour that leads to coupling of quark-antiquark pair, string tension inside $q\bar{q}$ -pairs and free propagation of colourless hadrons. Therefore the investigation of the gluon entangled states connected with the corresponding squeezed ones is issue of the day.

2 Multimode squeezed states of the gluons

By analogy with QO [6] a multimode squeezing condition for gluons with different colors $i_1, \dots, i_p$ is written as

$$\hat{N} \left(\Delta(\hat{X}_\lambda^{i_1, \dots, i_p})_1 \right)^2 \rangle < 0, \quad (4)$$

where $\Delta(\hat{X}_\lambda^{i_1, \dots, i_p}) = \hat{X}_\lambda^{i_1, \dots, i_p} - \langle \hat{X}_\lambda^{i_1, \dots, i_p} \rangle$, $\hat{N}$ is a normal ordering operator, the phase-sensitive Hermitian operators $(\hat{X}_\lambda^{i_1, \dots, i_p})_1 = \frac{1}{2\sqrt{p}} \sum_{j=1}^p [\hat{b}_\lambda^{i_j} + (\hat{b}_\lambda^{i_j})^+]$ and $(\hat{X}_\lambda^{i_1, \dots, i_p})_2 = \frac{1}{2i\sqrt{p}} \sum_{j=1}^p [\hat{b}_\lambda^{i_j} - (\hat{b}_\lambda^{i_j})^+]$ are linear combination of the annihilating (creating) operators $\hat{b}_\lambda^{i_j}$ ($\hat{b}_\lambda^{i_j+}$), $i_1, \dots, i_p = \overline{1, 8}$ are gluon color charges, λ is a polarization index. Averaging in (4) is performed over final state vector

$$|f\rangle \simeq |in\rangle - it \left(\hat{H}_1^{(3)}(0) + \hat{H}_1^{(4)}(0) \right) |in\rangle \quad (5)$$

which describes gluon system later small time t . Operators $\hat{H}_1^{(3)}(t)$ and $\hat{H}_1^{(4)}(t)$ describing the three- and four-gluon selfinteractions include combinations of three and four annihilating and creating operators [16]:

$$\begin{aligned} \hat{H}_1^{(3)}(t) = & i \frac{g}{(2\pi)^{3/2}} f_{abc} \sum_{\lambda_1, \lambda_2, \lambda_3} \int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \times \left\{ \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon_{\lambda_1}^\nu(k_1) \varepsilon_{\lambda_3}^{\lambda_3}(k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \right. \\ & \times [\hat{b}_{\lambda_1}^a(k_1) \hat{b}_{\lambda_2}^b(k_2) \hat{b}_{\lambda_3}^c(k_3) e^{-2i(k_{01}+k_{02}+k_{03})t} - \hat{b}_{\lambda_1}^{a+}(k_1) \hat{b}_{\lambda_2}^{b+}(k_2) \hat{b}_{\lambda_3}^{c+}(k_3) e^{2i(k_{01}+k_{02}+k_{03})t}] \\ & + \hat{b}_{\lambda_1}^{a+}(k_1) \hat{b}_{\lambda_2}^b(k_2) \hat{b}_{\lambda_3}^c(k_3) e^{2i(k_{01}-k_{02}-k_{03})t} \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3) \\ & \times [\vec{k}_3 \vec{\varepsilon}_{\lambda_1}(k_1) \varepsilon_{\lambda_2}^\nu(k_2) \varepsilon_{\lambda_3}^{\lambda_3}(k_3) + \vec{k}_2 \vec{\varepsilon}_{\lambda_3}(k_3) \varepsilon_{\lambda_1}^\nu(k_1) \varepsilon_{\lambda_2}^{\lambda_2}(k_2) - \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon_{\lambda_1}^\nu(k_1) \varepsilon_{\lambda_3}^{\lambda_3}(k_3)] \\ & + \hat{b}_{\lambda_1}^{a+}(k_1) \hat{b}_{\lambda_2}^{b+}(k_2) \hat{b}_{\lambda_3}^c(k_3) e^{2i(k_{01}+k_{02}-k_{03})t} \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3) \\ & \left. \times [\vec{k}_1 \vec{\varepsilon}_{\lambda_3}(k_3) \varepsilon_{\lambda_1}^\nu(k_1) \varepsilon_{\lambda_2}^{\lambda_2}(k_2) + \vec{k}_3 \vec{\varepsilon}_{\lambda_1}(k_1) \varepsilon_{\lambda_2}^\nu(k_2) \varepsilon_{\lambda_3}^{\lambda_3}(k_3) - \vec{k}_1 \vec{\varepsilon}_{\lambda_2}(k_2) \varepsilon_{\lambda_1}^\nu(k_1) \varepsilon_{\lambda_3}^{\lambda_3}(k_3)] \right\}, \end{aligned} \quad (6)$$

$$\begin{aligned} \hat{H}_1^{(4)}(t) = & \frac{g^2}{4(2\pi)^3} \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} \int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 \left\{ \varepsilon_{\lambda_1}^\mu(k_1) \varepsilon_{\lambda_2}^{\lambda_3}(k_3) \varepsilon_{\lambda_3}^{\lambda_4}(k_4) \varepsilon_{\lambda_4}^{\lambda_4}(k_4) f_{abc} f_{ade} \right. \\ & \times [\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) (\hat{b}_{\lambda_1}^b(k_1) \hat{b}_{\lambda_2}^c(k_2) \hat{b}_{\lambda_3}^d(k_3) \hat{b}_{\lambda_4}^e(k_4) e^{-2i(k_{01}+k_{02}+k_{03}+k_{04})t} \\ & + \hat{b}_{\lambda_1}^{b+}(k_1) \hat{b}_{\lambda_2}^{c+}(k_2) \hat{b}_{\lambda_3}^{d+}(k_3) \hat{b}_{\lambda_4}^{e+}(k_4) e^{2i(k_{01}+k_{02}+k_{03}+k_{04})t}) \\ & + 4\hat{b}_{\lambda_1}^{b+}(k_1) \hat{b}_{\lambda_2}^c(k_2) \hat{b}_{\lambda_3}^d(k_3) \hat{b}_{\lambda_4}^e(k_4) e^{2i(k_{01}-k_{02}-k_{03}-k_{04})t} \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \\ & + 4\hat{b}_{\lambda_1}^{b+}(k_1) \hat{b}_{\lambda_2}^{c+}(k_2) \hat{b}_{\lambda_3}^d(k_3) \hat{b}_{\lambda_4}^e(k_4) e^{2i(k_{01}+k_{02}+k_{03}-k_{04})t} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{k}_4)] \\ & + 2\hat{b}_{\lambda_1}^{b+}(k_1) \hat{b}_{\lambda_2}^{c+}(k_2) \hat{b}_{\lambda_3}^d(k_3) \hat{b}_{\lambda_4}^e(k_4) e^{2i(k_{01}+k_{02}-k_{03}-k_{04})t} \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \\ & \left. \times [\varepsilon_{\lambda_1}^\mu(k_1) \varepsilon_{\lambda_2}^{\lambda_3}(k_3) \varepsilon_{\lambda_3}^{\lambda_4}(k_4) (f_{abc} f_{ade} + f_{abe} f_{adc}) \right. \\ & \left. + \varepsilon_{\lambda_1}^\mu(k_1) \varepsilon_{\lambda_2}^{\lambda_3}(k_2) \varepsilon_{\lambda_3}^{\lambda_4}(k_3) \varepsilon_{\lambda_4}^{\lambda_4}(k_4) f_{abd} f_{ace}] \right\}. \end{aligned} \quad (7)$$

Here g is a self-interaction constant, $d\vec{k} = \frac{d^3k}{\sqrt{2k_0}}$, k_0 is a gluon energy, ε_λ^μ is a polarization vector, f_{ahb} are structure constants of $SU_c(3)$ group.

Initial state vector $|\text{in}\rangle$ describe gluon system at end of perturbative stage [8] and is product of the coherent states of the gluons with different colours and polarization indexes

$$|\text{in}\rangle \equiv |\alpha\rangle = \prod_{\lambda=1}^3 \prod_{b=1}^8 |\alpha_\lambda^b\rangle. \quad (8)$$

By analogy with QO gluon coherent state vector $|\alpha_\lambda^b\rangle$ is the eigenvector of the corresponding annihilation operator $\hat{b}_\lambda^b$ with the eigenvalue α_λ^b which can be written in terms of the gluon coherent field amplitude $|\alpha_\lambda^b|$ and phase γ_λ^b of the given gluon field $\alpha_\lambda^b = |\alpha_\lambda^b| e^{i\gamma_\lambda^b}$. In each gluon coherent state $|\alpha_\lambda^b\rangle$ the gluon number with fixed colour b and polarization λ is arbitrary (the average multiplicity of given gluon is equal to square of the gluon coherent field amplitude $\langle n_\lambda^b \rangle = |\alpha_\lambda^b|^2$) and phase of considering state γ_λ^b is fixed.

Averaging the annihilation and creation operators $\hat{b}_\lambda^{ij}, \hat{b}_\lambda^{j+}$ in (4) over the evolved vector $|\text{f}\rangle$ which is defined according to (5) and taking into account of chosen initial state vector we write the multimode squeezing condition in the form

$$\langle \hat{N} (\Delta(\hat{X}_\lambda^{i_1, \dots, i_p})_2)^2 \rangle = \mp \frac{it}{4p} \sum_{j,k=1}^p \left\{ \langle \alpha | [\hat{b}_\lambda^{ik}, [\hat{b}_\lambda^{ij}, \hat{H}_1(0)]] | \alpha \rangle - \langle \alpha | [[\hat{H}_1(0), \hat{b}_\lambda^{j+}], \hat{b}_\lambda^{ik+}] | \alpha \rangle \right\} < 0, \quad (9)$$

where $\hat{H}_1(0) = \hat{H}_1^{(3)}(0) + \hat{H}_1^{(4)}(0)$.

It can be shown that only the four-gluon self-interaction can yield the multimode squeezing effect since

$$[[\hat{H}_1^{(3)}(0), \hat{b}_\lambda^{j+}], \hat{b}_\lambda^{ik+}] = 0, \quad [\hat{b}_\lambda^{ik}, [\hat{b}_\lambda^{ij}, \hat{H}_1^{(3)}(0)]] = 0. \quad (10)$$

Indeed, the multimode squeezing condition can be written in the explicit form as

$$\begin{aligned} \langle \hat{N} (\Delta(\hat{X}_\lambda^{i_1, \dots, i_p})_2)^2 \rangle = & \pm \frac{it}{8p k_0} \frac{g^2}{(2\pi)^3} \sum_{j,k=1}^p \sum_{\lambda_1, \lambda_2=1}^3 \int \prod_{i=1}^2 \frac{d^3k_i}{\sqrt{2k_{0i}}} \\ & \times \left[(\varepsilon_\mu^{\lambda_1}(k_1) \varepsilon_\mu^{\lambda_2}(k_2)) (\varepsilon_\nu^{\lambda_1}(k) \varepsilon_\nu^{\lambda_2}(k)) - (\varepsilon_\mu^{\lambda_1}(k_1) \varepsilon_\mu^{\lambda_2}(k)) (\varepsilon_\nu^{\lambda_1}(k_2) \varepsilon_\nu^{\lambda_2}(k)) \right] \\ & \times \left\{ f_{aij b} f_{aik c} [\langle \alpha | \hat{b}_{\lambda_1}^{b+}(k_1) \hat{b}_{\lambda_2}^{c+}(k_2) | \alpha \rangle - \langle \alpha | \hat{b}_{\lambda_1}^b(k_1) \hat{b}_{\lambda_2}^c(k_2) | \alpha \rangle] \right. \\ & \times [\delta(2\vec{k} - \vec{k}_1 - \vec{k}_2) - \delta(2\vec{k} + \vec{k}_1 + \vec{k}_2)] + (f_{aij b} f_{aik c} + f_{aik b} f_{aij c}) \\ & \times \langle \alpha | \hat{b}_{\lambda_1}^{b+}(k_1) \hat{b}_{\lambda_2}^c(k_2) | \alpha \rangle [\delta(2\vec{k} - \vec{k}_1 + \vec{k}_2) - \delta(2\vec{k} + \vec{k}_1 - \vec{k}_2)] \left. \right\} < 0. \quad (11) \end{aligned}$$

In particular for the collinear gluons we have corresponding squeezing condition

$$\langle \hat{N} (\Delta(X_\lambda^{i_1, \dots, i_p})_2)^2 \rangle = \pm \frac{g^2 t}{8p (2\pi)^3 k_0^2} \sum_{j,k=1}^p \sum_{\lambda_1 \neq \lambda} f_{aik b} f_{aij c} |\alpha_{\lambda_1}^b| |\alpha_{\lambda_1}^c| \sin(\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c) < 0, \quad (12)$$

Here $|\alpha_{\lambda_1}^b|$ and $\gamma_{\lambda_1}^b$ are an amplitude and a phase of the initial gluon coherent field, f_{ahb} is a structure constant of the color group $S U_c(3)$. The multimode squeezing condition (12) is fulfilled for any cases

apart from if all initial gluon coherent fields are real or imaginary. Obviously, the larger are both the amplitudes of the initial gluon coherent fields with different colour and polarization indexes and coupling constant, the larger is multimode squeezing effect.

By analogy with two-mode photon state (1) we can make corresponding gluon states with squeezed operator

$$\hat{S}_{12}(r) = \exp \{ r (\hat{b}_\lambda^h \hat{b}_\lambda^g - \hat{b}_\lambda^{h+} \hat{b}_\lambda^{g+}) \}. \quad (13)$$

In case of the evolved gluon system during small time t we have

$$\langle \hat{N} \left((\widehat{\Delta X}_\lambda^{h,g})_1 \right)^2 \rangle = \frac{r(r \mp 1)}{2}. \quad (14)$$

Two-mode squeezing condition is

$$-\frac{1}{8} < \langle \hat{N} \left((\widehat{\Delta X}_\lambda^{h,g})_1 \right)^2 \rangle < 0 \quad \text{or} \quad 0 < \langle \hat{N} \left((\widehat{\Delta X}_\lambda^{h,g})_2 \right)^2 \rangle < 1. \quad (15)$$

In particular for the collinear gluons we have corresponding two-mode squeezing condition

$$-\frac{1}{8} < \frac{g^2 t}{16 (2\pi)^3 k_0^2} (f_{ahb} f_{ahc} + f_{agb} f_{agc} + f_{ahb} f_{agc} + f_{agb} f_{ahc}) \sum_{\lambda_1 \neq \lambda} |\alpha_{\lambda_1}^b| |\alpha_{\lambda_1}^c| \sin(\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c) < 0. \quad (16)$$

Thus non-perturbative four-gluon selfinteraction is source of the multimode squeezing effect.

3 Gluon entangled states

One of entangled condition is

$$0 < y < 1, \quad (17)$$

where entangled measure is defined by dimensionless value y (2) which in our case is

$$y = \left[\frac{|\overline{\hat{b}_\lambda^h \hat{b}_\lambda^{g+}}|^2 + |\overline{\hat{b}_\lambda^h \hat{b}_\lambda^g}|^2}{2(\overline{\hat{b}_\lambda^{h+} \hat{b}_\lambda^h} + 1/2)(\overline{\hat{b}_\lambda^{g+} \hat{b}_\lambda^g} + 1/2)} \right]^{1/2} \quad (18)$$

Entangled condition is

$$-\frac{1}{8} < \langle \hat{N} \left((\widehat{\Delta X}_\lambda^{h,g})_1 \right)^2 \rangle < \frac{1 - \sqrt{2}}{4} \quad \text{or} \quad 0 < \langle \hat{N} \left((\widehat{\Delta X}_\lambda^{h,g})_2 \right)^2 \rangle < \frac{1 + \sqrt{2}}{4}, \quad (19)$$

Obviously the entangled condition (19) imposes greater restrictions than the squeezing condition (15).

Indeed, corresponding entangled condition for collinear gluons is

$$-\frac{1}{8} < \frac{g^2 t}{16 (2\pi)^3 k_0^2} (f_{ahb} f_{ahc} + f_{agb} f_{agc} + f_{ahb} f_{agc} + f_{agb} f_{ahc}) \sum_{\lambda_1 \neq \lambda} |\alpha_{\lambda_1}^b| |\alpha_{\lambda_1}^c| \sin(\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c) < \frac{1 - \sqrt{2}}{4}. \quad (20)$$

Obviously squeezed gluon states are simultaneously entangled if the amplitudes of the initial gluon coherent fields are small enough.

Thus by analogy with quantum optics as a result of four-gluon self-interaction we obtain two-mode squeezed gluon states which are also entangled.

4 Colour correlations of the collinear gluons

Second-order colour correlation function is defined as

$$C_{\lambda(2)}^{h,g}(k) = \rho_{\lambda(2)}^{h,g}(k) - \rho_{\lambda(1)}^h(k)\rho_{\lambda(1)}^g(k) = it \left[\alpha_{\lambda}^g \alpha_{\lambda}^h \langle \alpha | [\hat{H}_I(0), \hat{b}_{\lambda}^{h+}], \hat{b}_{\lambda}^{g+} | \alpha \rangle - \alpha_{\lambda}^{g*} \alpha_{\lambda}^{h*} \langle \alpha | [\hat{b}_{\lambda}^g, [\hat{b}_{\lambda}^h, \hat{H}_I(0)]] | \alpha \rangle \right] \quad (21)$$

Corresponding function for the collinear gluons is

$$C_{\lambda(2)}^{h,g}(k) = -\frac{g^2 t}{16\pi^3 k_0^2} \sum_{\substack{\lambda_1=1 \\ \lambda_1 \neq \lambda}}^3 f_{hab} f_{gac} |\alpha_{\lambda}^h| |\alpha_{\lambda}^g| |\alpha_{\lambda_1}^b| |\alpha_{\lambda_1}^c| \sin(\gamma_{\lambda}^h + \gamma_{\lambda}^g - \gamma_{\lambda_1}^b - \gamma_{\lambda_1}^c) \quad (22)$$

It is obviously this function is sin-like function of the difference between the phases of the investigated and other colour phases of the coherent gluon states. Indeed, the colour correlation is positive at $0 < (\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c) - (\gamma_{\lambda}^h + \gamma_{\lambda}^g) < \pi$ (the gluon bunching) and one is negative at $0 < (\gamma_{\lambda}^h + \gamma_{\lambda}^g) - (\gamma_{\lambda_1}^b + \gamma_{\lambda_1}^c) < \pi$ (the gluon antibunching). Moreover correlation of colours h and g is defined by the structure constants f_{hab}, f_{gac} of $SU_c(3)$ colour group.

The study of color correlations for collinear gluons revealed two groups of colour correlations:

1) First group: $h = 1, 2, 3$ $g = 1, 2, 3$; $h = 4, g = 5$; $h = 6, g = 7$ is determined only by gluons with different polarizations. For example at $h = 1$ or 2 and $g = 2$ or 1 we have

$$C_{\lambda(2)}^{1,2}(k) = C_{\lambda(2)}^{2,1}(k) = \frac{g^2 t}{16\pi^3 k_0^2} \sum_{\substack{\lambda_1=1 \\ \lambda_1 \neq \lambda}}^3 |\alpha_{\lambda}^1| |\alpha_{\lambda}^2| |\alpha_{\lambda_1}^1| |\alpha_{\lambda_1}^2| \sin(\gamma_{\lambda}^1 + \gamma_{\lambda}^2 - \gamma_{\lambda_1}^1 - \gamma_{\lambda_1}^2); \quad (23)$$

2) Second group: $h = 1, 2, 3$ and $g = 4, 5, 6, 7$; $h, g = 4, 5, 6, 7, 8$ is defined in addition by gluons with other colours. For example at $h = 1$ or 4 and $g = 4$ or 1 we have

$$\begin{aligned} C_{\lambda(2)}^{1,4}(k) = \frac{g^2 t}{(4\pi)^3 k_0^2} \sum_{\substack{\lambda_1=1 \\ \lambda_1 \neq \lambda}}^3 |\alpha_{\lambda}^1| |\alpha_{\lambda}^4| \{ & |\alpha_{\lambda_1}^1| |\alpha_{\lambda_1}^4| \sin(\gamma_{\lambda}^1 + \gamma_{\lambda}^4 - \gamma_{\lambda_1}^1 - \gamma_{\lambda_1}^4) \\ & - 3|\alpha_{\lambda_1}^2| |\alpha_{\lambda_1}^5| \sin(\gamma_{\lambda}^1 + \gamma_{\lambda}^4 - \gamma_{\lambda_1}^2 - \gamma_{\lambda_1}^5) + 3|\alpha_{\lambda_1}^3| |\alpha_{\lambda_1}^6| \sin(\gamma_{\lambda}^1 + \gamma_{\lambda}^4 - \gamma_{\lambda_1}^3 - \gamma_{\lambda_1}^6) \\ & + \sqrt{3}|\alpha_{\lambda_1}^6| |\alpha_{\lambda_1}^8| \sin(\gamma_{\lambda}^1 + \gamma_{\lambda}^4 - \gamma_{\lambda_1}^6 - \gamma_{\lambda_1}^8) \} \end{aligned} \quad (24)$$

5 Conclusion

Investigating of the gluon fluctuations we have proved theoretically the possibility of existence of the multimode gluon squeezed states. The emergence of such remarkable states becomes possible owing to the four-gluon self-interaction. The three-gluon self-interaction does not lead to squeezing effect.

We have shown that QCD jet non-perturbative evolution leads both to squeezing and entanglement of gluons. It should be noted that the greater are both the amplitudes of the initial gluon coherent fields with different colour and polarization indexes and coupling constant, the greater is multimode squeezing effect of the colour gluons. We have demonstrated that entanglement condition of the gluon states with two colours imposes greater restrictions than the squeezing condition and is also defined by the amplitudes and phases of initial coherent gluon fields.

Because two-mode gluon states with two different colours can lead to $q\bar{q}$ -entangled states role of colour correlations could be very significant for explanation of the confinement phenomenon. The

study of color correlations for collinear gluons revealed two groups of colour correlations: first group of the correlations ($h = 1, 2, 3$ and $g = 1, 2, 3$; $h = 4, g = 5$; $h = 6, g = 7$) depends only on distinguished of the gluon polarization, in the second group ($h = 1, 2, 3$ and $g = 4, 5, 6, 7$; $h, g = 4, 5, 6, 7, 8$) correlation behaviour is defined in addition by gluons with other colours.

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