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Holographic dictionary and defects in the bulk

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Abstract. We study the holographic dual of the AdS_3 spacetime with a conical defect. We calculate the boundary two-point correlator using the holographic Gubser-Klebanov-Polyakov/Witten dictionary for a scalar field in the bulk. We consider the general case, when the conical defect breaks conformal symmetry at the boundary. The results are compared with previous studies based on the geodesic approximation. They are in good agreement for short correlators, and main discrepancy comes in the region of long correlations. It is shown that in the case when the spacetime is the $AdS_3/\mathbb{Z}_N$ orbifold, both methods give the same result which also produces the result expected from the orbifold CFT.

1 Introduction and summary

One of the key features of the AdS_3/CFT_2-correspondence [1, 2] is the fact that the $3D$ gravity is much simpler than its higher-dimensional counterparts, primarily because of its topological nature. There are no propagating gravitational degrees of freedom, and thus solutions of Einstein equations at least asymptotically look like topological quotients of the form AdS'_3/Γ, where Γ is a representation of a topological identification in the AdS isometry group, and AdS'_3 is the part of AdS_3 on which Γ which acts discretely [3]. The dynamics of the bulk objects described by these solutions is almost trivial in a sense that it can only be innately realized in the action of the identification Γ, or it can be induced by acting with isometries (such e. g. as Lorentzian symmetry) on the basic static configurations. Yet this underwhelming at first sight picture is still highly useful in the holographic approach for study of problems such as the thermalization problem [4–9], entanglement physics [10–13], chaos in QFT [14]. Moreover, there are also many powerful analytic method suitable for studying the 2D CFT on the boundary. Recent developments in techniques based on the extensive utilization of the Virasoro symmetry allow to study some properties of 3D gravity as quantum field theory [15, 16].

One of the simplest non-vacuum solutions of the three-dimensional gravity, which we consider in the present paper, is a point particle. The point particles [17–20] produce conical singularities in the spacetime. In the context of the asymptotically AdS solution [18], the point particle can be represented as a wedge cut out from the spacetime with an endpoint running along the worldline of the particle. Such spacetime is still a locally AdS spacetime, however it has a non-trivial topology. The latter leads to the fact that two given points in the bulk (or at the boundary) can be connected, generally speaking, by several geodesics. This has a crucial impact on the holographic dual of the AdS_3 with a point

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particle, as was shown in the recent work [21–25] based on the geodesic approximation [26]. The main object of study therein were two-point correlation functions and the entanglement entropy in the boundary dual to the AdS_3-deficit spacetime in the framework of geodesic approximation.

The goal of the present investigation is to study two-point boundary correlators in the framework of the holographic GKPW prescription [27, 28] on AdS_3 with a conical defect, and clarify the applicability of the geodesic approximation in the AdS-deficit spacetime by comparison of GKPW correlators to the geodesic correlators obtained in the earlier work. We find that in a certain special case, namely when the space is an orbifold $AdS_3/\mathbb{Z}_N$ both GKPW and geodesic correlators are given by a finite sum over images which precisely matches the expected result from the orbifold CFT. In the general case we illustrate that the discontinuities in the geodesic result correspond to the non-conformal regime. In this regime the most substantial discrepancy of two computations comes from the region of long-range correlations. Similar picture is observed in the time dependence of the correlators.

2 Scalar field in the AdS-deficit spacetime

2.1 The background

In $3D$ gravity the geometry with a conical singularity at the origin arises as a solution of the Einstein equations with a point-like source, which was obtained by Deser, Jackiw and t'Hooft originally in the flat space [17] and generalized to the case of constant curvature in [18]. In the AdS_3 case, when the negative cosmological constant is added, for a static source the solution exists only when the source is located at the origin of AdS spatial slice. A static particle in the AdS_3 spacetime is described by AdS_3 metric in global coordinates which can be written as follows (we set AdS radius to 1):

$$ds^2 = \frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho d\vartheta^2\right), \tag{1}$$

where we have the holographic coordinate $\rho \in [0, \frac{\pi}{2})$, and $\vartheta \in [0, 2\pi A)$ is the angular coordinate. The size of the angle deficit is related to the mass of the particle μ by the relation

$$A = 1 - 4G\mu, \tag{2}$$

where G is the Newtonian constant. Then the angle deficit equals to $\gamma = 2\pi(1 - A) = 8\pi G\mu$. When $A = 1$, the spacetime is the pure AdS, and the case of $A = 0$ is the BTZ black hole threshold.

2.2 The GKPW prescription for boundary correlators in AdS-deficit space

To calculate correlation functions in the boundary dual of the AdS-deficit spacetime, we use the Gubser-Klebanov-Polyakov/Witten (GKPW) holographic prescription [27, 28]. We use the version of the prescription for the Lorentzian signature proposed by Skenderis and van Rees [29]. We require that the partition functions of bulk and boundary theories are equal in the semiclassical limit (after the renormalization):

$$\left\langle e^{i\int dtd\vartheta\, \varphi_0 O}\right\rangle_{\mathrm{CFT}} = e^{iS_{on-shell}[\phi]}|_{\phi|_{\mathrm{bd}}=\varphi_0}\,; \tag{3}$$

where in the right hand side we have the classical action of the field evaluated on the solution of the Dirichlet boundary value problem with the boundary value of the field given by φ_0. The correlation functions in the boundary theory are then obtained by varying the right hand side with respect to φ_0. We are interested in the two-point Lorentzian correlators in the boundary theory, and therefore we need to make sure that those functional derivatives of the generating functional give a specific two-point correlator with correct $i\epsilon$-prescription. To be more concrete, let us specify the contour C. Suppose it has three segments:

$$C_- \ : \ \ t = -T + i\kappa\, ; \ \ \kappa \in [0, \infty)\, , \quad C_0 \ : \ \ t \in [-T,\ T]\, , \quad C_+ \ : \ \ t = T - i\kappa\, .$$

The CFT path integral then equals to

$$\mathcal{Z}_{\text{boundary}}[\phi_0] = \int \mathcal{D}\phi_- \mathcal{D}\phi_+ \, \langle 0|\phi_-, -T\rangle \, \langle \phi_-, -T| \mathrm{e}^{i\int_C d^2x \varphi_0 O} |\phi_+, T\rangle \, \langle \phi_+, T|0\rangle \ , \tag{4}$$

where the source ϕ_0 satisfies the condition $\varphi_0(\pm T) = 0$.

Wave function insertions can be interpreted as Euclidean path integrals with no sources present and with specified boundary conditions. It is natural then to construct a write a holographic dual to this quantity as $\mathcal{Z}_{\text{bulk}} = \mathrm{e}^{-S_- + iS_0 - S_+}$, where

$$S_0 = \frac{1}{2} \int_{-T}^{T} dt \int d^{d-1}x \sqrt{-g} \left(-(\partial\phi)^2 - m^2\phi^2 \right)|_{\phi=\Phi_0} \, , \tag{5}$$

$$S_- = \frac{1}{2} \int_{-\infty}^{0} d\tau \int d^{d-1}x \sqrt{g} \left((\partial\phi)^2 + m^2\phi^2 \right)|_{\phi=\Phi_-} \, , \tag{6}$$

$$S_+ = \frac{1}{2} \int_{0}^{\infty} d\xi \int d^{d-1}x \sqrt{-g} \left((\partial\phi)^2 + m^2\phi^2 \right)|_{\phi=\Phi_+} \, . \tag{7}$$

The action S_0 is that of a scalar field theory on a AdS-deficit spacetime with Lorentzian signature, computed on a solution Φ_0 of the field equation, which satisfies the Dirichlet boundary conditions with $\phi(\text{boundary}) = \varphi_0$. The solutions of the equations of motion $\Phi_\pm$ for the Euclidean actions $S_\pm$ are not supposed to generate any boundary sources, so they are required to simply decay when approaching to the boundary of the Euclidean AdS-deficit bulk spacetime. In addition, all solutions Φ are required to be regular at the origin of the spacetime.

The discussed on-shell actions have by definition vanishing bulk terms, but they also have generally non-trivial surface terms [29]. Those can be of two kinds: AdS boundary terms and corner terms from integration along the time segments. While the terms of the first kind are exactly what we need to study boundary theory, the terms of the second kind should vanish, since we do not have a consistent analytic continuation into the complex time plane of the usual relation (3) otherwise. The cancellation of the corner terms happens is we impose the matching conditions on the solutions Φ:

$$\Phi_-|_{\tau=0} = \Phi_0|_{t=-T} \, , \qquad \Phi_0|_{t=T} = \Phi_+|_{\xi=0} \, ; \tag{8}$$

$$i\partial_t \Phi_0|_{t=-T} + \partial_\tau \Phi_-|_{\tau=0} = 0 \, , \qquad i\partial_t \Phi_0|_{t=T} + \partial_\tau \Phi_+|_{\xi=0} = 0 \, . \tag{9}$$

These conditions basically mean C^1-smoothness of Φ as a solution of the EOM on the entire contour. The next step is to find the solutions Φ themselves.

To construct the bulk solution, we define the bulk-boundary propagator as a function $R_{\omega,l}(\rho)$, through which the Lorentzian solution Φ_0 can be expressed in following way:

$$\Phi_0(\rho, t, \vartheta) = \frac{1}{(2\pi)^2} \sum_{l \in \mathbb{Z}} \int d\omega \, \mathrm{e}^{-i\omega t + i\frac{l}{A}\vartheta} \varphi_0(\omega, l) R_{\omega,l}(\rho) \, , \tag{10}$$

where $\varphi_0(\omega, l)$ is Fourier image of $\varphi_0(t, \vartheta) = \Phi_0(t, \rho, \vartheta)|_{\rho=\pi/2}$. We also introduce the notations $h_\pm = \frac{1}{2}(1 \pm \sqrt{1 + m^2})$, $\nu = h_+ - h_-$.

For the boundary condition to be satisfied, we normalize R in such way that $R(\rho = \frac{\pi}{2} - \epsilon) = \epsilon^{2h_-} + \ldots$. It is clear that in our case R is defined by the fundamental mode solution which is regular at the origin. The non-normalizable (growing) part of the solution has the unit as a coefficient in front of the leading power of ρ, providing the saturation of the boundary condition, and also has analytic behaviour in the frequency space in all (growing) orders, since all poles from gamma-functions compensate each other.

However, the normalizable (decaying) piece is not analytic in the complex frequency plane. The leading coefficient can be read off from the representation of the mode solution as a sum of normalizable and non-normalizable solutions[1]. It equals to $\alpha(\omega, l)\beta(\omega, l)$, where

$$\alpha(\omega, l) := \frac{1}{\nu!(\nu-1)!}\frac{\Gamma(\left(h_+ + \frac{1}{2}\left(\frac{|l|}{A}+\omega\right)\right)\Gamma\left(h_+ + \frac{1}{2}\left(\frac{|l|}{A}-\omega\right)\right)}{\Gamma\left(h_- + \frac{1}{2}\left(\frac{|l|}{A}+\omega\right)\right)\Gamma\left(h_- + \frac{1}{2}\left(\frac{|l|}{A}-\omega\right)\right)}, \tag{11}$$

$$\beta(\omega, l) := -\left(\psi\left(h_+ + \frac{1}{2}\left(\frac{|l|}{A}+\omega\right)\right) + \psi\left(h_+ + \frac{1}{2}\left(\frac{|l|}{A}-\omega\right)\right)\right) + \dots ; \tag{12}$$

Here ψ is the digamma function - the main source of singularities. Those digamma functions which are independent of ω are denoted as $\dots$ and will be ignored in future since they do not contribute to singularities. The poles of β are located at

$$\omega^{\pm}_{nl} = \pm(2h_+ + 2n + \frac{|l|}{A}), \qquad n \in \mathbb{Z}_+ ; \tag{13}$$

In order for (10) to be well-defined, the contour of integration around the frequency poles needs to be specified. We rewrite (10) as

$$\Phi_0(\rho, t, \vartheta) = \sum_{l\in\mathbb{Z}} \int_C \frac{d\omega}{(2\pi)^2}\, \mathrm{e}^{-i\omega t + i\frac{l}{A}\vartheta}\varphi_0(\omega, l) R_{\omega,l}(\rho) + \sum_{\pm}\sum_{l\in\mathbb{Z}}\sum_{n=0}^{\infty} c^{\pm}_{nl}\mathrm{e}^{-i\omega^{\pm}_{nl}t + i\frac{l}{A}\vartheta}\mathrm{Res}(R_{\omega,l}(\rho))|_{\omega=\omega_{nl}}, \tag{14}$$

We have here integration over some reference contour C, and arbitrary coefficients $c^{\pm}_{nl}$ deform the contour by adding or removing extra poles to the sum over residues defined by C. One can show, that if one takes C to be of Feynman form, than from the condition $\varphi_0(\pm T, \vartheta) = 0$, and matching conditions (8,9) it follows that all $c^{\pm}_{nl}$ are zero. The shape of the time contour C through matching conditions for the bulk field defines the shape of the contour C in the frequency space, resulting in any desirable behaviour of correlators.

Now we can use the bulk-boundary propagator to calculate the bulk on-shell action and compute the correlation functions of a scalar operator O of conformal dimension $\Delta = 2h_+ = 1 + \nu$. Varying over φ_0 twice, we obtain the Feynman propagator:

$$\begin{aligned}\langle TO_\Delta(t_1, \vartheta_1) O_\Delta(t_2, \vartheta_2)\rangle = {} & \frac{2(\Delta-1)\theta(t_1-t_2)}{\pi}\sum_{l\in\mathbb{Z}}\sum_{n=0}^{\infty}\alpha(\omega_{nl}, |l|)\mathrm{e}^{-i\omega^{+}_{nl}(t_1-t_2)+i\frac{l}{A}(\vartheta_1-\vartheta_2)} + \\ & + \frac{2(\Delta-1)\theta(t_2-t_1)}{\pi}\sum_{l\in\mathbb{Z}}\sum_{n=0}^{\infty}\alpha(\omega_{nl}, |l|)\mathrm{e}^{-i\omega^{-}_{nl}(t_1-t_2)+i\frac{l}{A}(\vartheta_1-\vartheta_2)}.\end{aligned} \tag{15}$$

It is easy to extract Wightman correlators:

$$\langle O_{1+\nu}(t, \vartheta) O_{1+\nu}(0,0)\rangle = \frac{2}{\pi(\nu-1)!^2}\sum_{l\in\mathbb{Z}}\sum_{n=0}^{\infty}\frac{(n+\nu)!}{n!}\frac{\left(n+\frac{|l|}{A}+\nu\right)!}{\left(n+\frac{|l|}{A}\right)!}\times \mathrm{e}^{-i(1+\nu+2n)t - i\frac{|l|}{A}t + i\frac{l}{A}\vartheta}; \tag{16}$$

[1] See [30] for thorough discussion of (non)-normalizable mode solutions in the pure AdS case

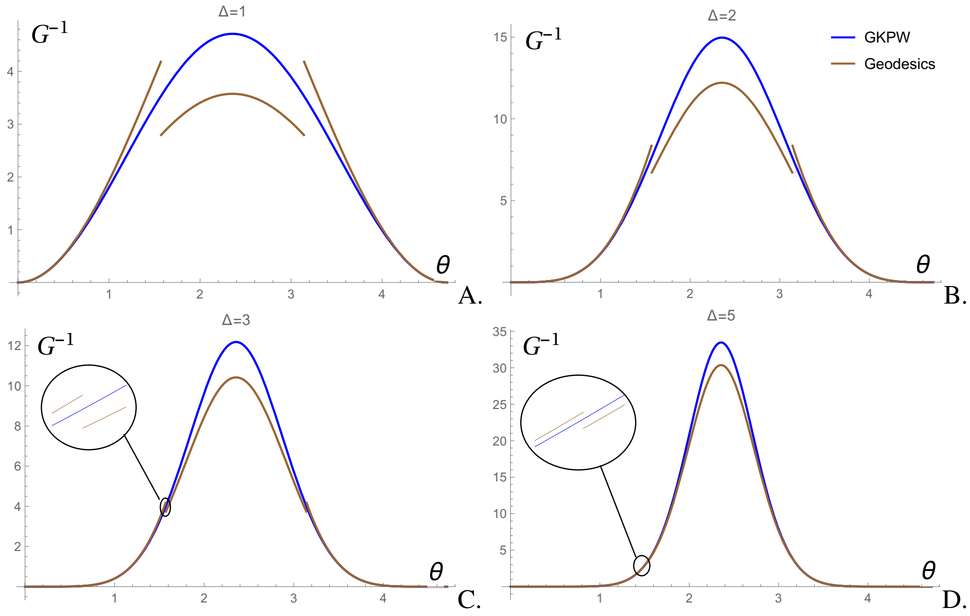

Figure 1: Inverse equal time correlators obtained via GKPW prescription and the geodesic image method for $A = \frac{3}{4}$ for different conformal weights. Contributions of discontinuities in the geodesic result (represented by the brown line) diminish as Δ increases.

where we have used (11) to write the result more explicitly and omitted the ϵ-prescription. In case of $\nu = 0$ the summation can be performed explicitly:

$$\langle \mathcal{O}_1(t,\vartheta)\mathcal{O}_1(0,0)\rangle = \frac{1}{\pi}\frac{\sin\frac{t}{A}}{\sin t}\frac{1}{\cos\frac{t}{A}-\cos\frac{\vartheta}{A}}. \tag{17}$$

Then the result for arbitrary integer $\nu = \Delta - 1$ by differentiating the sum and formally written as

$$\langle \mathcal{O}_{1+\nu}(t,\vartheta)\mathcal{O}_{1+\nu}(0,0)\rangle = \frac{\nu}{2^\nu(\nu-1)!\pi}(-1)^\nu\frac{\partial^\nu}{\partial(\cos t)^\nu}\left(\frac{\sin\frac{t}{A}}{\sin t}\frac{1}{\cos\frac{t}{A}-\cos\frac{\vartheta}{A}}\right). \tag{18}$$

3 Comparison with geodesic approximation

The geodesic prescription for correlation functions [26] dual to particles (both static and moving) in AdS_3 has been constructed in [21–25]. It takes into account geodesics which wind around conical defects functions by including the contributions of image geodesics generated by the identification isometry. As a result, one gets an image method for the two-point correlation function. However, by construction, the number of geodesic images is always finite, unlike in case of image method for a Green's function problem. Also, the correlator given by the geodesic image method sum has discontinuities in its dependence on the angular coordinate. This is because the number of possible windings of a geodesic between two given points in general case depends on their position. As we see, this fact contributes to the deviation from the GKPW correlator, see [32] for more details. In this section we compare the GKPW correlators given by (17,18) with correlators given by the geodesic approximation.

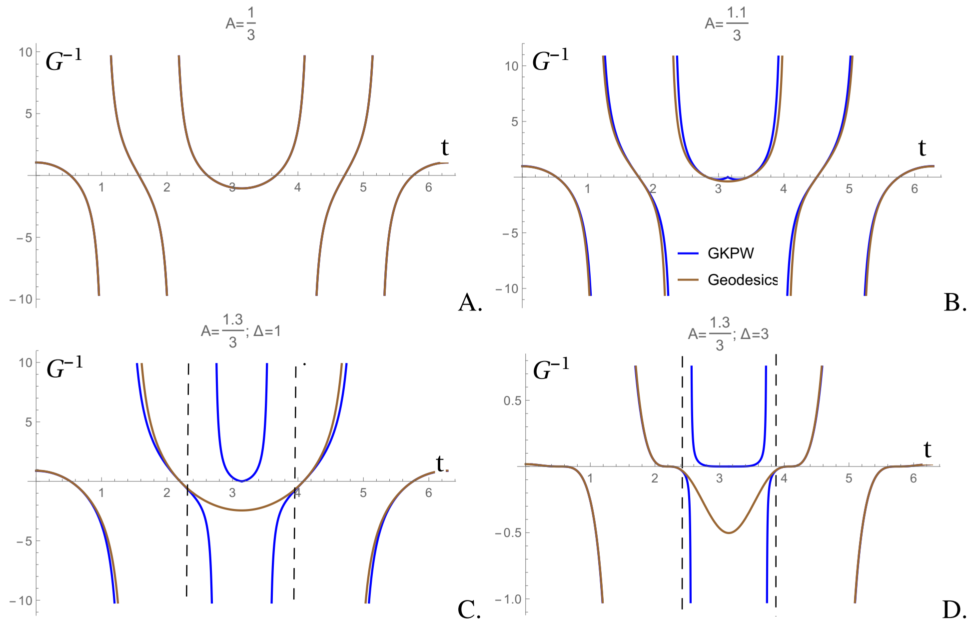

Figure 2: Time dependence of the inverse correlators obtained by GKPW and geodesic prescriptions. Plots A-C show the increase of discrepancy between two prescriptions in case of $\Delta = 1$ when the deficit parameter is close to the orbifold value $A = \frac{1}{3}$. Plot D shows the discrepancy for $\Delta = 3$. The value of angular variable is fixed $\vartheta = \frac{\pi}{6}$.

3.1 Special case: $\mathrm{AdS}_3/\mathbb{Z}_N$-orbifold

First, we consider a special case when $A = 1/N$, where $N = 1, 2, 3, \dots$. In this case the spacetime is a conical orbifold $\mathrm{AdS}_3/\mathbb{Z}_N$. It is important to note that the boundary of this spacetime preserves asymptotic Virasoro symmetry [13, 31]. In our case, one can show that in this case a sum over images emerges naturally:

$$\frac{2}{(\nu-1)!^2}\sum_{l\in\mathbb{Z}}\sum_{n=0}^{\infty}\frac{(n+\nu)!}{n!}\frac{(n+N|l|+\nu)!}{(n+N|l|)!}\mathrm{e}^{-i(1+\nu+2n)t-i|l|Nt+ilN\vartheta} = \frac{\nu^2}{2^\nu N}\sum_{k=0}^{N-1}\left(\frac{1}{\cos t-\cos\left(\vartheta+2\pi\frac{k}{N}\right)}\right)^{\nu+1} \tag{19}$$

and thus we get for arbitrary integer $\Delta > 1$ ($\Delta = 1$ case yields a different constant prefactor):

$$\langle O_\Delta(t,\vartheta)O_\Delta(0,0)\rangle = \frac{2(\Delta-1)^2}{\pi N}\sum_{k=0}^{N-1}\left(\frac{1}{2(\cos t-\cos\left(\vartheta+2\pi\frac{k}{N}\right))}\right)^{\Delta}. \tag{20}$$

The geodesic approximation [23–25] gives the same result up to a constant prefactor. Moreover, this formula coincides with the two-point correlator in the orbifold CFT obtained by usual image method [13, 32]. Thus, in the orbifold case we have a full agreement of GKPW, geodesic and CFT computations.

3.2 General case: temporal and angular dependences

In the general case the angle deficit breaks the asymptotic Virasoro symmetry at the boundary. This effect is sharply manifest in correlators given by the geodesic image method [32]. The geodesic two-point correlator acquires discontinuities in its angular dependence, which is reflected in the fig. 1. We see that there is a discrepancy between two holographic prescriptions, and GKPW prescriptions has the largest difference from the geodesic prescription in the region of long-range correlations, where the longest geodesics contribute. This difference, however, diminishes for large Δ.

The correlators calculated via GKPW and geodesic prescriptions also start exhibit slight differences in their time dependence if one moves the deficit parameter from the orbifold value. As illustrated in fig. 2A-C, once one deforms the orbifold, the GKPW inverse correlator obtains an additional zero. For general non-orbifold angle deficits geodesic and GKPW correlators have different number of poles. This effect does not depend on the value of angle and, in general, on the number of geodesic images: we see in Fig.2C that the number of images has decreased, and GKPW expression reflects this as well, but still keeps its extra zero. The comparison of plots C and D in Fig.2 shows that while increasing the conformal dimension does not insure that the analytic structure of correlators in the region between the dashed lines is not completely in agreement for increasing Δ, geodesic correlator still approaches the GKPW expression quantitatively.

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