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Some aspects of $\mathcal{N} = 1$ SYM renormalization

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Abstract. Using the BRST invariant version of the higher covariant derivative regularization, we demonstrate that in $\mathcal{N} = 1$ supersymmetric gauge theories the three-point vertices with two ghost legs and a single leg of the quantum gauge superfield are finite in all orders. This theorem is proved by the help of the Slavnov–Taylor identities and the supergraph technique. Its correctness is verified by explicit one-loop calculation. Using finiteness of the considered vertices we express the NSVZ relation in terms of the anomalous dimensions of the gauge superfield, of the Faddeev–Popov ghosts, and of the matter superfields.

1 Introduction

The ultraviolet behaviour of supersymmetric theories is better than in the non-supersymmetric case due to non-renormalization theorems. For $\mathcal{N} = 1$ supersymmetric theories the statement that the superpotential has no divergent quantum corrections [1] is usually called the non-renormalization theorem. However, it is also possible to consider the relation between the β-function and the anomalous dimension of the matter superfields as a non-renormalization theorem. This relation is usually called the exact NSVZ β-function [2–5], because for the pure $\mathcal{N} = 1$ SYM theory it gives the exact expression for the β-function in the form of the geometric series.

In this paper, following Ref. [6], we argue that in $\mathcal{N} = 1$ SYM theories the three-point ghost-gauge vertices are finite in all orders. This statement can be also considered as a non-renormalization theorem.

The non-renormalization theorem originate from large symmetry of the theory. That is why deriving them it is necessary to quantize the theory in such a way that most symmetries of the theory remain unbroken. In particular, it is desirable to use the invariant regularization. However, the dimensional regularization breaks supersymmetry [7], while the dimensional reduction [8] is not self-consistent [9] and due to this can also lead to supersymmetry breaking [10]. The consistent regularization applicable for supersymmetric theories is the higher covariant derivative regularization [11, 12]. It can be formulated in explicitly $\mathcal{N} = 1$ supersymmetric way [13, 14] and reveals some interesting features of quantum corrections [15–18]. In particular, it allows deriving the NSVZ β-function at least in the Abelian case by direct summation of superdiagrams [19, 20]. That is why in proving the above mentioned non-renormalization theorem we will use the higher covariant derivative regularization.

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2 BRST invariant version of the higher covariant derivative regularization for $\mathcal{N}=1$ supersymmetric gauge theories

We consider the $\mathcal{N}=1$ SYM theory, which is described by the action

$$S=\frac{1}{2e_0^2}\,\mathrm{Re}\,\mathrm{tr}\int d^4x\,d^2\theta\,W^aW_a+\frac{1}{4}\int d^4x\,d^4\theta\,\phi^{*i}(e^{2V})_i{}^j\phi_j$$
$$+\Big\{\int d^4x\,d^2\theta\Big(\frac{1}{4}m_0^{ij}\phi_i\phi_j+\frac{1}{6}\lambda_0^{ijk}\phi_i\phi_j\phi_k\Big)+\mathrm{c.c.}\Big\},\qquad(1)$$

assuming that the theory is invariant under the gauge transformations $\phi\to e^{A}\phi$; $e^{2V}\to e^{-A^+}e^{2V}e^{-A}$, where $A=ie_0A^BT^B$ is an arbitrary chiral superfield. We will also use the background field method. The quantum-background splitting in the supersymmetric case is made by the substitution $e^{2V}\to e^{\mathbf{\Omega}^+}e^{2V}e^{\mathbf{\Omega}}$. Then, the background gauge superfield $\boldsymbol{V}$ is defined as $e^{2\boldsymbol{V}}=e^{\mathbf{\Omega}^+}e^{\mathbf{\Omega}}$.

We will regularize this theory by higher covariant derivatives. For this purpose it is necessary to add a term with higher degrees of the covariant derivatives to the classical action. Then divergences remain only in the one-loop approximation. These remaining divergences are regularized by the Pauli–Villars determinants [21]. We choose the higher derivative term

$$S_\Lambda=\frac{1}{2e_0^2}\,\mathrm{Re}\,\mathrm{tr}\int d^4x\,d^2\theta\,e^{\Omega}e^{\mathbf{\Omega}}W^ae^{-\mathbf{\Omega}}e^{-\Omega}\Big[R\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big)-1\Big]_{Adj}e^{\Omega}e^{\mathbf{\Omega}}W_ae^{-\mathbf{\Omega}}e^{-\Omega}$$
$$+\frac{1}{4}\int d^4x\,d^4\theta\,\phi^+e^{\mathbf{\Omega}^+}e^{\Omega^+}\Big[F\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big)-1\Big]e^{\Omega}e^{\mathbf{\Omega}}\phi,\qquad(2)$$

and the gauge fixing term

$$S_{\mathrm{gf}}=\frac{1}{e_0^2}\mathrm{tr}\int d^4x\,d^4\theta\Big(16\xi_0 f^+\Big[e^{\mathbf{\Omega}^+}K^{-1}\Big(-\frac{\boldsymbol{\nabla}^2\boldsymbol{\nabla}^2}{16\Lambda^2}\Big)e^{\mathbf{\Omega}}\Big]_{Adj}f+e^{\mathbf{\Omega}}fe^{-\mathbf{\Omega}}\boldsymbol{\nabla}^2V+e^{-\mathbf{\Omega}^+}f^+e^{\mathbf{\Omega}^+}\boldsymbol{\nabla}^2V\Big),\qquad(3)$$

where the regulators R, F, and K rapidly grow at infinity. The corresponding action for the Faddeev–Popov is

$$S_{\mathrm{FP}}=\frac{1}{e_0^2}\mathrm{tr}\int d^4x\,d^4\theta\,\Big(e^{\mathbf{\Omega}}\bar c e^{-\mathbf{\Omega}}+e^{-\mathbf{\Omega}^+}\bar c^+e^{\mathbf{\Omega}^+}\Big)\Big\{\Big(\frac{V}{1-e^{2V}}\Big)_{Adj}\Big(e^{-\mathbf{\Omega}^+}c^+e^{\mathbf{\Omega}^+}\Big)+\Big(\frac{V}{1-e^{-2V}}\Big)_{Adj}\Big(e^{\mathbf{\Omega}}ce^{-\mathbf{\Omega}}\Big)\Big\}.\qquad(4)$$

(Also it is necessary to add the Nielsen–Kallsodh ghosts.) The total action of the gauge fixed theory is invariant under the BRST transformations [22, 23]

$$\delta V=-\varepsilon\Big\{\Big(\frac{V}{1-e^{2V}}\Big)_{Adj}\Big(e^{-\mathbf{\Omega}^+}c^+e^{\mathbf{\Omega}^+}\Big)+\Big(\frac{V}{1-e^{-2V}}\Big)_{Adj}\Big(e^{\mathbf{\Omega}}ce^{-\mathbf{\Omega}}\Big)\Big\};\qquad\delta\phi=\varepsilon c\phi;\qquad\delta f=0;$$
$$\delta\bar c=\varepsilon\bar D^2(e^{-2\boldsymbol{V}}f^+e^{2\boldsymbol{V}});\qquad\delta\bar c^+=\varepsilon D^2(e^{2\boldsymbol{V}}fe^{-2\boldsymbol{V}});\qquad\delta c=\varepsilon c^2;\qquad\delta c^+=\varepsilon(c^+)^2,\qquad(5)$$

where ε is an anticommuting scalar parameter.

We will defined the renormalization constants by the following equations

$$\frac{1}{\alpha_0} = \frac{Z_\alpha}{\alpha}; \qquad \frac{1}{\xi_0} = \frac{Z_\xi}{\xi}; \qquad \boldsymbol{V} = \boldsymbol{V}_R; \qquad V = Z_V Z_\alpha^{-1/2} V_R; \qquad \phi_i = (\sqrt{Z_\phi})_i{}^j (\phi_R)_j;$$
$$\bar{c}c = Z_c Z_\alpha^{-1} \bar{c}_R c_R; \qquad m^{ij} = m_0^{mn} (Z_m)_m{}^i (Z_m)_n{}^j; \qquad \lambda^{ijk} = \lambda_0^{mnp} (Z_\lambda)_m{}^i (Z_\lambda)_n{}^j (Z_\lambda)_p{}^k. \tag{6}$$

The renormalized superfields are denoted by the subscript R; α, λ, m, and ξ are also the renormalized parameters. One can also impose the conditions $(Z_m)_i{}^j = (Z_\lambda)_i{}^j = (\sqrt{Z_\phi})_i{}^j$; $Z_\xi = Z_V^{-2}$ to these renormalization constants.

3 Non-renormalization of the three-point vertices with two ghost legs and a single leg of the quantum gauge superfield

We will prove that the three-point vertices with 2 ghost legs and 1 leg of the quantum gauge superfield are finite in all orders. There are 4 such vertices: $\bar{c}\,Vc$, $\bar{c}^+ Vc$, $\bar{c}\,Vc^+$, and $\bar{c}^+ Vc^+$. They have the same renomalization constant $Z_\alpha^{-1/2} Z_c Z_V$. Therefore, the above statement can be written as

$$\frac{d}{d\ln\Lambda}(Z_\alpha^{-1/2} Z_c Z_V) = 0. \tag{7}$$

(In the one-loop approximation this has first been noted in [24].) Thus, there is a subtraction scheme in which $Z_\alpha^{-1/2} Z_c Z_V = 1$. It is important to note that Z_c is divergent. (This is demonstrated by an explicit calculation below.) Therefore, the Green functions of the type $\bar{c}\,V^n c$ are divergent for $n \neq 1$.

The finiteness of the considered vertices will be proved using the Slavnov–Taylor identities [25, 26]. The Slavnov–Taylor identity can be obtained by making the substitution coinciding with the BRST transformations in the generating functional and has the form

$$0 = \int d^4x\, d^4\theta_x\, \frac{\delta\Gamma}{\delta V_x^A} \left\langle \delta V_x^A \right\rangle + \int d^4x\, d^2\theta_x \Big(\left\langle \delta\bar{c}_x^A \right\rangle \frac{\delta\Gamma}{\delta\bar{c}_x^A} + \left\langle \delta c_x^A \right\rangle \frac{\delta\Gamma}{\delta c_x^A} + \left\langle \delta\phi_i \right\rangle \frac{\delta\Gamma}{\delta\phi_i} \Big)$$
$$+ \int d^4x\, d^2\bar{\theta}_x \Big(\left\langle \delta\bar{c}_x^{*A} \right\rangle \frac{\delta\Gamma}{\delta\bar{c}_x^{*A}} + \left\langle \delta c_x^{*A} \right\rangle \frac{\delta\Gamma}{\delta c_x^{*A}} + \left\langle \delta\phi^{*i} \right\rangle \frac{\delta\Gamma}{\delta\phi^{*i}} \Big), \tag{8}$$

where we did not omit the ε-dependence. We will also use one more identity that is obtained by making the substitution $\bar{c} \to \bar{c} + a$, where a is an arbitrary chiral superfield:

$$\varepsilon \frac{\delta\Gamma}{\delta\bar{c}_x^A} = \frac{1}{4}\bar{D}^2 \left\langle \delta V_x^A \right\rangle; \qquad \varepsilon \frac{\delta\Gamma}{\delta\bar{c}_x^{*A}} = \frac{1}{4} D^2 \left\langle \delta V_x^A \right\rangle, \tag{9}$$

where the background gauge superfield is set to 0, for simplicity.

To derive the Slavnov–Taylor identity for the ghost-gauge three-point functions, we differentiate the Slavnov–Taylor identity (8) with respect to $\bar{c}_y^{*B}$, c_z^C, and c_w^D using the equations

$$\frac{\delta^2\Gamma}{\delta\bar{c}_y^{*B}\delta c_x^A} = -\frac{D_y^2 \bar{D}_x^2}{16} G_c \delta_{xy}^8 \delta_{AB}; \qquad \frac{\delta}{\delta c_x^A} \left\langle \delta V_y^B \right\rangle = -\varepsilon \cdot \frac{1}{4} G_c \bar{D}^2 \delta_{xy}^8 \delta_{AB}. \tag{10}$$

Then we obtain the identity relating the three-point Green functions

$$\varepsilon G_c(\partial_w^2/\Lambda^2)\bar{D}_w^2 \frac{\delta^3\Gamma}{\delta\bar{c}_y^{*B}\delta V_w^D \delta c_z^C} - \varepsilon G_c(\partial_z^2/\Lambda^2)\bar{D}_z^2 \frac{\delta^3\Gamma}{\delta\bar{c}_y^{*B}\delta V_z^C \delta c_w^D} + \frac{1}{2} G_c \left(\partial_y^2/\Lambda^2\right) D_y^2 \frac{\delta^2}{\delta c_z^C \delta c_w^D} \left\langle \delta c_y^B \right\rangle = 0. \tag{11}$$

To simplify this identity, we will use explicit expressions for the Green functions which can be found from dimensional and chirality considerations:

$$\frac{\delta^3\Gamma}{\delta\bar{c}_x^{*A}\delta V_y^B\delta c_z^C} = -\frac{ie_0}{16}f^{ABC}\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\Big(f(p,q)\partial^2\Pi_{1/2} - F_\mu(p,q)(\gamma^\mu)_{\dot a}{}^b\bar{D}^{\dot a}D_b + F(p,q)\Big)_y \times\Big(D_x^2\delta^8_{xy}(q+p)\,\bar{D}_z^2\delta^8_{yz}(q)\Big), \tag{12}$$

where $\partial^2\Pi_{1/2} \equiv -D^a\bar{D}^2D_a/8$ is the supersymmetric transversal projection operator and

$$\delta^8_{xy}(p) \equiv \delta^4(\theta_x-\theta_y)e^{ip_\alpha(x^\alpha-y^\alpha)}. \tag{13}$$

To calculate the ghost correlator entering in the Slavnov–Taylor identity, we introduce the chiral source $\mathcal{J}$ and add the term

$$-\frac{e_0}{2}\int d^4x\,d^2\theta\,f^{ABC}\mathcal{J}^A c^B c^C + \text{c.c.} \tag{14}$$

to the action. Then from the dimensional and chirality considerations we find

$$\frac{\delta^2}{\delta c_z^C\delta c_w^D}\left\langle\delta c_y^B\right\rangle = -i\varepsilon\frac{\delta^3\Gamma}{\delta c_z^C\delta c_w^D\delta\mathcal{J}_y^B} = -\frac{ie_0\varepsilon}{4}f^{BCD}\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}H(p,q)\bar{D}_z^2\delta^8_{zy}(q+p)\bar{D}_w^2\delta^8_{yw}(q), \tag{15}$$

where $[H(p,q)] = 1$ and, by construction, $H(p,q) = H(p,-q-p)$.

Substituting the explicit expressions for the Green function (12) and the ghost correlator (15) into the Slavnov–Taylor identity we can rewrite it in the form

$$G_c(q)F(q,p) + G_c(p)F(p,q) = 2G_c(q+p)H(-q-p,q), \tag{16}$$

where we use the compact notation $G_c(-q^2/\Lambda^2) \to G_c(q)$. The identity (16) will be used below for proving finiteness of the three-point ghost-gauge vertices.

However, let us first prove that the function $H(p,q)$ is finite. H is contributed by diagrams in which one leg corresponds to the chiral source $\mathcal{J}$ and two other legs correspond to the chiral ghost superfields c. They contain the expression

$$\int d^4y\,d^2\theta_y\,\mathcal{J}_y^A\cdot\frac{\bar{D}_y^2D_y^2}{4\partial^2}\delta^8_{y1}\cdot\frac{\bar{D}_y^2D_y^2}{4\partial^2}\delta^8_{y2} = -2\int d^4y\,d^4\theta_y\,\mathcal{J}_y^A\cdot\frac{D_y^2}{4\partial^2}\delta^8_{y1}\cdot\frac{\bar{D}_y^2D_y^2}{4\partial^2}\delta^8_{y2}. \tag{17}$$

Therefore, the considered contribution can be presented as an integral over the whole superspace, which includes the integration

$$\int d^4\theta = -\frac{1}{2}\int d^2\theta\bar{D}^2 + \text{total derivatives in the coordinate space.} \tag{18}$$

Therefore, two left supersymmetric derivatives should act to the chiral external lines. Consequently, a non-trivial result can be obtained only if two right supersymmetric derivatives also act to the external lines. Thus, the result should be proportional to the second degree of the external momenta and is finite in the UV region. Thus, the function $H(p,q)$ is UV finite.

Then we proceed to proving the non-renormalization theorem for the three-point ghost-gauge vertices. For this purpose the Slavnov–Taylor identity is multiplied by the renormalization constant

Z_c (such that $(G_c)_R = Z_c G$ is finite). Then we differentiate the result with respect to $\ln\Lambda$ and take the limit $\Lambda \to \infty$. Then, due to the finiteness of the functions $(G_c)_R$ and H

$$\Big((G_c)_R(q)\frac{d}{d\ln\Lambda}F(q,p) + (G_c)_R(p)\frac{d}{d\ln\Lambda}F(p,q)\Big)\Big|_{\Lambda\to\infty} = 0. \qquad (19)$$

Setting $p = -q$ in this identity we obtain $dF(-q,q)/d\ln\Lambda\Big|_{\Lambda\to\infty} = 0$. This implies that the corresponding renormalization constant is finite,

$$\frac{d}{d\ln\Lambda}(Z_\alpha^{-1/2}Z_c Z_V) = 0. \qquad (20)$$

Consequently, the function $F(p,q)$ is finite. Therefore, all three-point gauge-ghost vertices are also finite, because they also have the renormalization constant $Z_\alpha^{-1/2}Z_c Z_V$.

4 One-loop calculation

The two-point ghost Green function in the one-loop approximation (in the Euclidean space after the Wick rotation) is

$$G_c(p) = 1 + e_0^2 C_2 \int \frac{d^4k}{(2\pi)^4}\Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big)\Big(- \frac{1}{6k^4} + \frac{1}{2k^2(k+p)^2} - \frac{p^2}{2k^4(k+p)^2}\Big) + O(e_0^4, e_0^2\lambda_0^2), \qquad (21)$$

where $R_k \equiv R(k^2/\Lambda)$ and $K_k \equiv K(k^2/\Lambda^2)$. Evidently, this function is divergent in the ultraviolet region (for infinite Λ).

The three-point ghost-gauge Green functions in the one-loop approximation have been calculated in [6]. Here we present only results for the functions F:

$$F(p,q) = 1 + \frac{e_0^2C_2}{4}\int\frac{d^4k}{(2\pi)^4}\Big\{ - \frac{(q+p)^2}{R_k k^2(k+p)^2(k-q)^2} - \frac{\xi_0\, p^2}{K_k k^2(k+q)^2(k+q+p)^2}$$
$$+ \frac{\xi_0\, q^2}{K_k k^2(k+p)^2(k+q+p)^2} + \Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big)\Big(- \frac{2(q+p)^2}{k^4(k+q+p)^2} + \frac{2}{k^2(k+q+p)^2}$$
$$- \frac{1}{k^2(k+q)^2} - \frac{1}{k^2(k+p)^2}\Big)\Big\} + O(\alpha_0^2, \alpha_0\lambda_0^2); \qquad (22)$$

$$(23)$$

We see that this expression is finite in the ultraviolet region. This fact is in agreement with the non-renormalization theorem discussed above. Expressions for the functions F_μ and f are much larger and can found in [6]. They are also finite, but in the one-loop approximation this result is trivial and follows from simple dimensional considerations. It was also verified in [6] that the other Green functions of the considered structure are also finite.

The one-loop expressions for the function H has the form

$$H(p,q) = 1 - \frac{e_0^2C_2}{4}\int\frac{d^4k}{(2\pi)^4}\Big\{\frac{p^2}{R_k k^2(k+q)^2(k+q+p)^2} + \frac{(q+p)^2}{k^4(k+q+p)^2}\Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big)$$
$$+ \frac{q^2}{k^4(k+q)^2}\Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big)\Big\} + O(e_0^4, e_0^2\lambda_0^2). \qquad (24)$$

It is finite in the ultraviolet region and proportional to the second degree of the external momenta. (This fact was very important for proving finiteness of the three-point ghost-gauge vertices.)

Finally, we note that correctness of the one-loop calculations has been confirmed by verifying the Slavnov–Taylor identities, and in particular, Eq. (16).

5 NSVZ β-function and non-renormalization of the ghost-gauge vertices

In $\mathcal{N} = 1$ supersymmetric theories the β-function is related with anomalous dimension of the matter superfields by the equation [2, 4, 5]

$$\beta(\alpha) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)}{2\pi(1 - C_2\alpha/2\pi)}, \tag{25}$$

which has originally been found from various general arguments, such as instantons, anomalies, etc. For $\mathcal{N} = 1$ SQED, regularized by higher derivatives, the NSVZ relation [27, 28] was derived by explicit summation of supergraphs [19, 20]. To generalize this result to the case of the dimensional reduction is a complicated and so far unsolved problem [29–32].

Qualitatively, the NSVZ β-function in the Abelian case appears as follows [16]. Let us consider a supergraph without external lines. By attaching two external lines of the background gauge superfield we obtain a contribution to the β-function. From the other hand, cutting the matter line in original supergraph we obtain a contribution to the anomalous dimension of the matter superfields. It has been shown in [19, 20] that two these contributions coincide up to a multiplicative factor.

Let us equivalently rewrite the NSVZ β-function for the non-Abelian case in a different form, which admits similar qualitative interpretation. This can be done using the non-renormalization of the three-point ghost-gauge vertices proved above. First, we present the NSVZ relation (for the renormalization group functions defined in terms of the bare coupling constant) in the form

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{3C_2 - T(R) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r}{2\pi} + \frac{C_2}{2\pi}\cdot\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0}. \tag{26}$$

The β-function in the right hand side can be expressed in terms of the renormalization constant Z_α,

$$\beta(\alpha_0,\lambda_0) = \frac{d\alpha_0(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\alpha_0\frac{d\ln Z_\alpha}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}}. \tag{27}$$

Then, using the equation $d(Z_\alpha^{1/2}Z_V Z_c)/d\ln\Lambda = 0$ we obtain

$$\beta(\alpha_0,\lambda_0) = -2\alpha_0\frac{d\ln(Z_c Z_V)}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = 2\alpha_0\Big(\gamma_c(\alpha_0,\lambda_0) + \gamma_V(\alpha_0,\lambda_0)\Big), \tag{28}$$

where γ_c and γ_V are anomalous dimensions of the Faddeev–Popov ghosts and of the quantum gauge superfield (defined in terms of the bare coupling constants), respectively. Substituting this expression into the right hand side of the NSVZ relation (26) we obtain

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\gamma_c(\alpha_0,\lambda_0) - 2C_2\gamma_V(\alpha_0,\lambda_0) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r\Big).$$

From this form of the NSVZ β-function we see that the matter and ghost contributions to the right hand side look very similar.

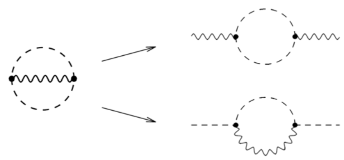

Figure 1. Graphical interpretation of the new form of the NSVZ relation in the non-Abelian case

Let us assume that, similarly to the Abelian case, the NSVZ relation is valid for the RG functions defined in terms of the bare coupling constants in the case of using the higher covariant derivative regularization. The RG functions defined in terms of the renormalized coupling constant are scheme dependent and satisfy the NSVZ relation only in a certain subtraction scheme. In the Abelian case the NSVZ scheme with the higher derivative regularization has been constructed in [33–35]. One can similarly try to construct the NSVZ scheme in the non-Abelian case. Repeating argumentation of [33–35] it is possible to verify that in the non-Abelian case the RG functions defined in terms of the bare coupling constant coincide with the ones defined in terms of the renormalized coupling constants if the boundary conditions

$$Z_\alpha(\alpha,\lambda,x_0) = 1; \qquad (Z_\phi)_i{}^j(\alpha,\lambda,x_0) = \delta_i{}^j; \qquad Z_c(\alpha,\lambda,x_0) = 1, \tag{29}$$

where x_0 is a fixed value of $\ln\Lambda/\mu$, are imposed on the renormalization constants. (For example, it is possible and convenient to choose $x_0 = 0$.) We also assume that the renormalization constants satisfy the equation

$$Z_V = Z_\alpha^{1/2} Z_c^{-1}, \tag{30}$$

If our assumption (that the RG functions defined in terms of the bare coupling constants satisfy the NSVZ relation in the case of using the higher covariant derivative regularization) is true, then the conditions (29) and (30) give the NSVZ scheme with the higher covariant derivative regularization.

6 Conclusion

For $\mathcal{N} = 1$ SYM theories the three-point vertices with two ghost legs and one leg of the quantum gauge superfield are finite. This has been proved using the Slavnov–Taylor identities in all orders and has been verified by an explicit one-loop calculation. Due to the non-renormalization of the triple ghost-gauge vertices the renormalization constants can be chosen so that $Z_\alpha^{-1/2} Z_V Z_c = 1$. The corresponding anomalous dimensions can also be related in all orders.

The NSVZ β-function can be rewritten in terms of the anomalous dimensions of the quantum gauge field, of the Faddeev–Popov ghosts, and of the matter superfields. Then the ghost contribution has a structure similar to the matter superfield contribution. The resulting expression for the NSVZ β-function has a very simple qualitative interpretation.

Using the above results it is possible to suggest a simple prescription giving the NSVZ scheme in the non-Abelian case, if the higher covariant derivative method is used for regularization.

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