

# The Feynman-Vernon Influence Functional Approach in QED

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**Abstract.** In the path integral approach we describe evolution of interacting electromagnetic and fermionic fields by the use of density matrix formalism. The equation for density matrix and transitions probability for fermionic field is obtained as average of electromagnetic field influence functional. We obtain a formula for electromagnetic field influence functional calculating for its various initial and final state. We derive electromagnetic field influence functional when its initial and final states are vacuum. We present Lagrangian for relativistic fermionic field under influence of electromagnetic field vacuum.

## 1 Introduction

At the present time the development of non-perturbative methods of non-linear dynamics description is actual for quantum systems interacting with strong electromagnetic field [1–5]. One of these methods is Feynman-Vernon influence functional approach in the path integral formalism [6]. This method development was presented in many papers [7–9]. In the Feynman-Vernon influence functional approach the main point is its calculation for the model of investigated system. In this paper we calculate Feynman-Vernon influence functional of quantum electromagnetic field vacuum on fermionic field.

## 2 The Feynman-Vernon Influence Functional of quantum electromagnetic field on fermionic field

The QED Lagrangian for a spin-1/2 field interacting with the electromagnetic field is given by

$$\mathcal{L}_{full}(x) = \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - j_\mu(x)A^\mu(x) \quad (1)$$

where  $\gamma^\mu$  are Dirac matrices,  $\psi(x)$  is a bispinor field of spin-1/2 particles (e.g. electron-positron field),  $A_\mu(x)$  is electromagnetic four-potential;

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \quad (2)$$

is the electromagnetic field tensor,

$$j_\mu(x) = e\bar{\psi}(x)\gamma_\mu\psi(x) \quad (3)$$

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is four-current. We use natural units system  $c = \hbar = 1$  in formulas (1)–(3) and further.

By the use of the second quantization formalism we present fields as field operators  $\hat{\psi}(\mathbf{x}, t)$ ,  $\hat{\bar{\psi}}(\mathbf{x}, t)$ ,  $\hat{j}_\mu(\mathbf{x}, t)$ ,  $\hat{A}^\mu(\mathbf{x}, t)$  where  $\mathbf{x}$  and  $t$  describe coordinate of 3D space and time moment. Here we introduce the creation and annihilation operators:

$$\hat{A}^\mu(\mathbf{x}, t) = \sum_{\mathbf{k}, \lambda = \pm 1} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}^{(b)}}} \left( \varepsilon_\lambda^\mu \hat{a}_{\mathbf{k}\lambda} e^{i\mathbf{k}\mathbf{x}} + \varepsilon_\lambda^{\mu*} \hat{a}_{\mathbf{k}\lambda}^\dagger e^{-i\mathbf{k}\mathbf{x}} \right) \quad (4)$$

$$\hat{\psi}(\mathbf{x}, t) = \sum_{\mathbf{p}, \sigma = \pm \frac{1}{2}} \frac{1}{\sqrt{2V\omega_{\mathbf{p}}^{(f)}}} \left( \hat{b}_{\mathbf{p}\sigma} u_\sigma(p) e^{i\mathbf{p}\mathbf{x}} + \hat{c}_{\mathbf{p}\sigma}^\dagger u_\sigma(-p) e^{-i\mathbf{p}\mathbf{x}} \right), \quad (5)$$

$$\hat{\bar{\psi}}(\mathbf{x}, t) = \sum_{\mathbf{p}, \sigma = \pm \frac{1}{2}} \frac{1}{\sqrt{2V\omega_{\mathbf{p}}^{(f)}}} \left( \hat{b}_{\mathbf{p}\sigma}^\dagger u_\sigma(p) e^{-i\mathbf{p}\mathbf{x}} + \hat{c}_{\mathbf{p}\sigma} u_\sigma(-p) e^{i\mathbf{p}\mathbf{x}} \right), \quad (6)$$

$$\hat{j}_\mu^+(\mathbf{k}, t) = \int \hat{j}_\mu(\mathbf{x}, t) e^{i\mathbf{k}\mathbf{x}} d\mathbf{x}, \quad \hat{j}_\mu^-(\mathbf{k}, t) = \int \hat{j}_\mu(\mathbf{x}, t) e^{-i\mathbf{k}\mathbf{x}} d\mathbf{x} \quad (7)$$

$$\hat{j}_\mu(\mathbf{x}) = \hat{j}_\mu(\mathbf{x}, \hat{b}_{\mathbf{p}\sigma}, \hat{b}_{\mathbf{p}\sigma}^\dagger, \hat{c}_{\mathbf{p}\sigma}, \hat{c}_{\mathbf{p}\sigma}^\dagger) \quad (8)$$

where  $\hat{a}_{\mathbf{k}\lambda}^\dagger$  and  $\hat{a}_{\mathbf{k}\lambda}$  are creation and annihilation operators of photon with wave vector  $\mathbf{k}$  and polarization (the eigenvalue of the z-component of the photon spin)  $\lambda = \pm 1$ ,  $\varepsilon_\lambda^\mu$  is unitary 4-vector of polarization and  $\varepsilon_\lambda^{\mu*} \varepsilon_{\lambda\mu} = -1$ ;  $\hat{b}_{\mathbf{p}\sigma}$  and  $\hat{b}_{\mathbf{p}\sigma}^\dagger$  are creation and annihilation operators of electron with wave vector  $\mathbf{p}$  and polarization  $\sigma = \pm \frac{1}{2}$ ;  $\hat{c}_{\mathbf{p}\sigma}^\dagger$  and  $\hat{c}_{\mathbf{p}\sigma}$  are creation and annihilation operators of positron with wave vector  $\mathbf{p}$  and polarization  $\sigma = \pm \frac{1}{2}$ ,  $V$  is volume of space.

For evolution of the interacting systems we construct Hamiltonian of full system  $\hat{H}_{full}$  on the base of operators  $\hat{a}_{\mathbf{k}\lambda}^\dagger$ ,  $\hat{a}_{\mathbf{k}\lambda}$ ,  $\hat{b}_{\mathbf{p}\sigma}^\dagger$ ,  $\hat{b}_{\mathbf{p}\sigma}$ ,  $\hat{c}_{\mathbf{p}\sigma}^\dagger$ ,  $\hat{c}_{\mathbf{p}\sigma}$ .

$$\hat{H}_{full} = \hat{H}_{Dirac} + \hat{H}_{field} + \hat{H}_{int} \quad (9)$$

where

$$\hat{H}_{Dirac} = \sum_{\mathbf{p}, \sigma} \omega_{\mathbf{p}}^{(f)} \left( \hat{b}_{\mathbf{p}\sigma}^\dagger \hat{b}_{\mathbf{p}\sigma} + \hat{c}_{\mathbf{p}\sigma}^\dagger \hat{c}_{\mathbf{p}\sigma} \right) \quad (10)$$

— Hamiltonian of Dirac field;

$$\hat{H}_{field} = \sum_{\mathbf{k}, \lambda} \omega_{\mathbf{k}}^{(b)} \hat{a}_{\mathbf{k}\lambda}^\dagger \hat{a}_{\mathbf{k}\lambda} \quad (11)$$

— Hamiltonian of electromagnetic field;

$$\hat{H}_{int} = \sum_{\mathbf{k}, \lambda} \frac{1}{\sqrt{2\omega_{\mathbf{k}}^{(b)} V}} \left( \varepsilon_\lambda^\mu \hat{j}_\mu^+(\mathbf{k}, t) \hat{a}_{\mathbf{k}\lambda} + \varepsilon_\lambda^{\mu*} \hat{j}_\mu^-(\mathbf{k}, t) \hat{a}_{\mathbf{k}\lambda}^\dagger \right) \quad (12)$$

— interaction part of Hamiltonian.

The evolution equation of statistical operator  $\hat{\rho}$  is the following

$$\hat{\rho}(t_f) = \hat{U}(t_f, t_{in}) \hat{\rho}(t_{in}) \hat{U}^\dagger(t_f, t_{in}) \quad (13)$$

where  $\hat{\rho}(t_{in})$  is statistical operator, describing initial state at moment  $t_{in}$  and  $\hat{U}(t_f, t_{in})$  is operator of evolution from initial time moment  $t_{in}$  to final time moment  $t_f$

$$\hat{U}(t_f, t_{in}) = \hat{T} \exp\left[-\frac{i}{\hbar} \int_{t_{in}}^{t_f} \hat{H}_{full}(\tau) d\tau\right]. \quad (14)$$

For density matrix calculation we need to formulate equation (13) in functional representation. We introduce coherent representation  $|a_{\mathbf{k}\lambda}\rangle$  for electromagnetic field by the following way

$$\hat{a}_{\mathbf{k}\lambda}|a_{\mathbf{k}\lambda}\rangle = a_{\mathbf{k}\lambda}|a_{\mathbf{k}\lambda}\rangle, \quad \langle a_{\mathbf{k}\lambda}|\hat{a}_{\mathbf{k}\lambda}^\dagger = \langle a_{\mathbf{k}\lambda}|a_{\mathbf{k}\lambda}^*, \quad (15)$$

$$\langle a'_{\mathbf{k}'\lambda'}|a_{\mathbf{k}\lambda}\rangle = \delta_{\mathbf{k}'\mathbf{k}}\delta_{\lambda'\lambda} \exp\left\{-\frac{1}{2}\left(a'_{\mathbf{k}'\lambda'}a_{\mathbf{k}\lambda}^* + a_{\mathbf{k}\lambda}^*a_{\mathbf{k}\lambda} - a'_{\mathbf{k}'\lambda'}a_{\mathbf{k}\lambda} - a_{\mathbf{k}\lambda}^*a_{\mathbf{k}'\lambda'}\right)\right\}. \quad (16)$$

$$\int |a_{\mathbf{k}\lambda}\rangle\langle a_{\mathbf{k}\lambda}| da_{\mathbf{k}\lambda}^* da_{\mathbf{k}\lambda} = \hat{1}. \quad (17)$$

where  $a_{\mathbf{k}\lambda}$  — complex value, which describe one mode of quantum electromagnetic field with wave vector  $\mathbf{k}$  and polarization  $\lambda$ .

We introduce grassman representation  $|b_{\mathbf{p},\sigma}\rangle$  and  $|c_{\mathbf{p},\sigma}\rangle$  for Dirac field by the following way

$$\hat{b}_{\mathbf{p},\sigma}|b_{\mathbf{p},\sigma}\rangle = b_{\mathbf{p},\sigma}|b_{\mathbf{p},\sigma}\rangle, \quad \langle b_{\mathbf{p},\sigma}|\hat{b}_{\mathbf{p},\sigma}^\dagger = \langle b_{\mathbf{p},\sigma}|b_{\mathbf{p},\sigma}^*, \quad \int |b_{\mathbf{p}\sigma}\rangle\langle b_{\mathbf{p}\sigma}| db_{\mathbf{p}\sigma}^* db_{\mathbf{p}\sigma} = \hat{1}; \quad (18)$$

$$\hat{c}_{\mathbf{p},\sigma}|c_{\mathbf{p},\sigma}\rangle = c_{\mathbf{p},\sigma}|c_{\mathbf{p},\sigma}\rangle, \quad \langle c_{\mathbf{p},\sigma}|\hat{c}_{\mathbf{p},\sigma}^\dagger = \langle c_{\mathbf{p},\sigma}|c_{\mathbf{p},\sigma}^*, \quad \int |c_{\mathbf{p}\sigma}\rangle\langle c_{\mathbf{p}\sigma}| dc_{\mathbf{p}\sigma}^* dc_{\mathbf{p}\sigma} = \hat{1}; \quad (19)$$

$$\langle b'_{\mathbf{p}'\sigma'}|b_{\mathbf{p}\sigma}\rangle = \delta_{\mathbf{p}'\mathbf{p}}\delta_{\sigma'\sigma} \exp\left\{-\frac{1}{2}\left(b'_{\mathbf{p}'\sigma'}b_{\mathbf{p}\sigma}^* + b_{\mathbf{p}\sigma}^*b_{\mathbf{p}\sigma} - b'_{\mathbf{p}'\sigma'}b_{\mathbf{p}\sigma} - b_{\mathbf{p}\sigma}^*b'_{\mathbf{p}'\sigma'}\right)\right\}. \quad (20)$$

$$\langle c'_{\mathbf{p}'\sigma'}|c_{\mathbf{p}\sigma}\rangle = \delta_{\mathbf{p}'\mathbf{p}}\delta_{\sigma'\sigma} \exp\left\{-\frac{1}{2}\left(c'_{\mathbf{p}'\sigma'}c_{\mathbf{p}\sigma}^* + c_{\mathbf{p}\sigma}^*c_{\mathbf{p}\sigma} - c'_{\mathbf{p}'\sigma'}c_{\mathbf{p}\sigma} - c_{\mathbf{p}\sigma}^*c'_{\mathbf{p}'\sigma'}\right)\right\}. \quad (21)$$

where  $b_{\mathbf{p},\sigma}$  and  $c_{\mathbf{p},\sigma}$  — grassman variable.

In holomorphic representation  $|a_{\mathbf{k}\lambda}, b_{\mathbf{p}\sigma}, c_{\mathbf{p}\sigma}\rangle = |a_{\mathbf{k}\lambda}\rangle \otimes |b_{\mathbf{p}\sigma}\rangle \otimes |c_{\mathbf{p}\sigma}\rangle$  we present the evolution equation in the following

$$\begin{aligned} \rho(a_f^*, b_f^*, c_f^*, a_f', b_f', c_f'; t_f) &= \int da_{in}^* da_{in}' db_{in}^* db_{in}' dc_{in}^* dc_{in}' da_{in}^* da_{in} db_{in}^* db_{in} dc_{in}^* dc_{in} \times \\ &\times \langle a_f, b_f, c_f | \hat{U}(t_f, t_{in}) | a_{in}, b_{in}, c_{in} \rangle \rho(a_{in}^*, b_{in}^*, c_{in}^*, a_{in}', b_{in}', c_{in}'; t_{in}) \times \\ &\times \langle a_{in}', b_{in}', c_{in}' | \hat{U}^\dagger(t_f, t_{in}) | a_f', b_f', c_f' \rangle \end{aligned} \quad (22)$$

where the density matrix:

$$\rho(a_f^*, b_f^*, c_f^*, a_f', b_f', c_f'; t_f) = \langle a_f, b_f, c_f | \hat{\rho}(t_f) | a_f', b_f', c_f' \rangle \quad (23)$$

For simplification we relabel variables  $a_{\mathbf{k}\lambda}$  and  $\theta_{\mathbf{p}\sigma}$  to  $\alpha$  and  $\theta$ .

Be the use of amplitude transition properties [10], we present the kernel of evolution operator  $\langle a_f, b_f, c_f | \hat{U}(t_f, t_{in}) | a_{in}, b_{in}, c_{in} \rangle$  as a path integral:

$$\langle a_f, b_f, c_f | \hat{U}(t_f, t_{in}) | a_{in}, b_{in}, c_{in} \rangle = \int \mathcal{D}a^*(\tau) \mathcal{D}a(\tau) \mathcal{D}b^*(\tau) \mathcal{D}b(\tau) \mathcal{D}c^*(\tau) \mathcal{D}c(\tau) \times \exp \left\{ i S_{full} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] \right\}, \quad (24)$$

where full system action is the following

$$S_{full} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = S_f [b^*(\tau), b(\tau), c^*(\tau), c(\tau)] + S_b [a^*(\tau), a(\tau)] + S_{int} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)], \quad (25)$$

where

$$S_f [b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = \int_{t_{in}}^{t_f} \left( \frac{\dot{b}^*(\tau)b(\tau) - b^*(\tau)\dot{b}(\tau)}{2i} + \frac{\dot{c}^*(\tau)c(\tau) - c^*(\tau)\dot{c}(\tau)}{2i} - \omega^{(f)} (b^*(\tau)b(\tau) + c^*(\tau)c(\tau)) \right) d\tau \quad (26)$$

is the action  $S_f$  of fermionic field,

$$S_b [a^*(\tau), a(\tau)] = \int_{t_{in}}^{t_f} \left( \frac{\dot{a}^*(\tau)a(\tau) - a^*(\tau)\dot{a}(\tau)}{2i} - \omega^{(b)} a^*(\tau)a(\tau) \right) d\tau \quad (27)$$

is the action  $S_b$  of bosonic field,

$$S_{int} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = \int_{t_{in}}^{t_f} \left[ \varepsilon_{\mu} \tilde{J}^{\mu+} [b^*(\tau), b(\tau), c^*(\tau), c(\tau)] a(\tau) + \varepsilon_{\mu} \tilde{J}^{\mu-} [b^*(\tau), b(\tau), c^*(\tau), c(\tau)] a^*(\tau) \right] d\tau; \quad (28)$$

is the action  $S_{int}$  of interaction part.

Using (24)–(29) we present equation (22) for density matrix evolution in the following form:

$$\rho(a_f^*, b_f^*, c_f^*, a_f', b_f', c_f'; t_f) = \int da_{in}^* da_{in}' db_{in}^* db_{in}' dc_{in}^* dc_{in}' da_{in} da_{in}' db_{in} db_{in}' dc_{in} dc_{in}' \times \rho(a_{in}^*, b_{in}^*, c_{in}^*, a_{in}', b_{in}', c_{in}'; t_{in}) \times \mathcal{D}a^*(\tau) \mathcal{D}a(\tau) \mathcal{D}b^*(\tau) \mathcal{D}b(\tau) \mathcal{D}c^*(\tau) \mathcal{D}c(\tau) \mathcal{D}a'^*(\tau) \mathcal{D}a'(\tau) \mathcal{D}b'(\tau) \mathcal{D}b'(\tau) \mathcal{D}c'(\tau) \mathcal{D}c'(\tau) \times \exp \left\{ i \left( S_{full} [a^*(\tau), a(\tau), b(\tau), b(\tau)] - S_{full} [a'^*(\tau), a'(\tau), b'(\tau), b'(\tau)] \right) \right\}. \quad (30)$$

We can choose at initial moment  $t_{in}$

$$\rho(a_{in}^*, b_{in}^*, c_{in}^*, a_{in}', b_{in}', c_{in}'; t_{in}) = \rho_f(b_{in}^*, c_{in}^*, b_{in}', c_{in}'; t_{in}) \times \rho_b(a_{in}^*, a_{in}'; t_{in}). \quad (31)$$

This is true for many models.

We consider the quantum transition from pure fermionic field state  $|n\rangle$  at initial time moment  $t_{in}$

$$\rho_{syst}(b_{in}^*, c_{in}^*, b_{in}', c_{in}'; t_{in}) = \psi_n(b_{in}^*, c_{in}^*) \psi_n^*(b_{in}', c_{in}') \quad (32)$$

$$\psi_n^*(b_{in}, c_{in}) = \langle n | b_{in}, c_{in} \rangle \quad \psi_n(b_{in}^*, c_{in}^*) = \langle b_{in}, c_{in} | n \rangle \quad (33)$$

and electromagnetic field from coherent state  $|\zeta\rangle$  at  $t_{in}$

$$\rho(a_{in}^*, a'_{in}; t_{in}) = \zeta(a_{in}^*) \zeta^*(a'_{in}) \quad (34)$$

$$\zeta^*(a'_{in}) = \langle \zeta | a'_{in} \rangle \quad \zeta(a_{in}^*) = \langle a_{in} | \zeta \rangle \quad (35)$$

to pure fermionic field state  $|m\rangle$  at time moment  $t_f$ :

$$\psi_m^*(b'_f, c'_f) = \langle m | b'_f, c'_f \rangle \quad \psi_m(b_f^*, c_f^*) = \langle b_f, c_f | m \rangle \quad (36)$$

and to coherent state  $|\chi\rangle$  at time moment  $t_f$

$$\rho(a_f, a'_f) = \chi(a_f) \chi^*(a'_f) \quad (37)$$

$$\chi^*(a'_f) = \langle \chi | a'_f \rangle \quad \chi(a_f) = \langle a_f | \chi \rangle \quad (38)$$

The transition probability  $P(m, \chi, t_f; n, \zeta, t_{in})$  is determined from (30) by the following formula

$$P(m, \chi, t_f; n, \zeta, t_{in}) = \int \int da_f'^* da_f' db_f'^* db_f' dc_f'^* dc_f' da_f^* da_f db_f^* db_f dc_f^* dc_f \times \\ \times \psi_m^*(b_f, c_f) \chi^*(a_f) \rho(a_f^*, b_f^*, c_f^*; a_f', b_f', c_f') \psi_m(b_f'^*, c_f'^*) \chi(a_f') \quad (39)$$

By the use of (30)-(38) we present the transition probability in the path integral representation

$$P(m, \chi, t_f; n, \zeta, t_{in}) = \int da_f^* da_f da_f'^* da_f' db_f^* db_f db_f'^* db_f' dc_f^* dc_f dc_f'^* dc_f' \times \\ \times \mathcal{D}a'^*(\tau) \mathcal{D}a'(\tau) \mathcal{D}a^*(\tau) \mathcal{D}a(\tau) \mathcal{D}b'^*(\tau) \mathcal{D}b'(\tau) \mathcal{D}b^*(\tau) \mathcal{D}b(\tau) \mathcal{D}c'^*(\tau) \mathcal{D}c'(\tau) \mathcal{D}c^*(\tau) \mathcal{D}c(\tau) \times \\ \times \psi_m^*(b_f, c_f) \chi^*(a_f) \exp \left[ \frac{i}{\hbar} S_{full}[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] \right] \psi_n(b_{in}^*, c_{in}^*) \zeta(a_{in}^*) \times \\ \times \psi_n^*(b'_{in}, c'_{in}) \zeta^*(a'_{in}) \exp \left[ -\frac{i}{\hbar} S'_{full}[a^*(\tau), a'(\tau), b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau)] \right] \psi_m(b_f'^*, c_f'^*) \chi(a_f'). \quad (40)$$

Eq. (40) for quantum transition probability we present in the following form

$$P(m, \chi; t|n, \zeta; 0) = \int \mathcal{D}b'^*(\tau) \mathcal{D}b'(\tau) \mathcal{D}b^*(\tau) \mathcal{D}b(\tau) \mathcal{D}c'^*(\tau) \mathcal{D}c'(\tau) \mathcal{D}c^*(\tau) \mathcal{D}c(\tau) \times \\ \times da_f^* da_f da_f'^* da_f' db_f^* db_f db_f'^* db_f' dc_f^* dc_f dc_f'^* dc_f' \psi_m^*(b_f, c_f) \psi_m(b_f'^*, c_f'^*) \times \\ \times \exp \left\{ \iota \left( S_f[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] - S_f[b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau)] \right) \right\} \times \\ \times F[\chi_f | b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)] \zeta_{in} \psi_n(b_{in}^*, c_{in}^*) \psi_n^*(b'_{in}, c'_{in}) \quad (41)$$

where

$$F[\chi_f | b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)] \zeta_{in} = \int da_{in}^* da_{in} da_{in}'^* da_{in}' \times \\ \times \chi^*(a_f) U_{infl} \left[ a_f^*, b_f^*, c_f^*, t_f, b^*(\tau), b(\tau), c^*(\tau), c(\tau), a_{in}, b_{in}, c_{in}, t_{in} \right] \times \\ \times \zeta(a_{in}^*) \zeta^*(a'_{in}) U_{infl} \left[ a_{in}'^*, b_{in}'^*, c_{in}'^*, t_{in}, b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau), a_f, b_f, c_f, t_f \right] \chi(a_f') \quad (42)$$

is influence functional of electromagnetic field on fermionic subsystems.

$$\begin{aligned}
& U_{infl} [a_f^*, b_f^*, c_f^*, t_f, b^*(\tau), b(\tau), c^*(\tau), c(\tau), a_{in}, b_{in}, c_{in}, t_{in}] = \\
& = \int \mathfrak{D}a^*(\tau) \mathfrak{D}a(\tau) \exp \left\{ i S_{infl} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] \right\}
\end{aligned} \quad (43)$$

is electromagnetic field transition amplitude from initial state  $|a_{in}\rangle$  to final state  $|a_f\rangle$ .

$$\begin{aligned}
& S_{infl} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = \\
& = S_b [a^*(\tau), a(\tau)] + S_{int} [a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)].
\end{aligned} \quad (44)$$

is action of fermionic field under electromagnetic field influence. It describe electromagnetic field influence on fermionic field.

The influence functional (42) present the influence of one mode of electromagnetic field with wave vector  $\mathbf{k}$  and polarization  $\lambda$ . We note it by index  $F_{\mathbf{k},\lambda} [\chi_f | b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau) | \zeta_{in}]$ .

The influence functional of all electromagnetic field modes calculate as production of one mode influence functionals:

$$\begin{aligned}
& F[\chi_f | b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau) | \zeta_{in}] = \\
& = \prod_{\mathbf{k}\lambda} F_{\mathbf{k},\lambda} [\chi_f | b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau) | \zeta_{in}].
\end{aligned} \quad (45)$$

For calculating influence functional by eq. (42) we need to calculate amplitude  $U_{infl} [a_f^*, b_f^*, c_f^*, t_f, b^*(\tau), b(\tau), c^*(\tau), c(\tau), a_{in}, b_{in}, c_{in}, t_{in}]$  and to specify  $\chi(a)$ ,  $\zeta(a)$ .

### 3 Electromagnetic field influence functional calculation

In this section we calculate functional  $U_{infl} [a_f^*, b_f, t_f, b^*(\tau)b(\tau), c^*(\tau)c(\tau), a_{in}, b_{in}, t_{in}]$ . For this we present equation (43) as finite-multiple integral [9].

$$\begin{aligned}
& U_{infl} [a_f^*, b_f, t_f, b^*(\tau)b(\tau), c^*(\tau)c(\tau), a_{in}, b_{in}, t_{in}] = \\
& = \lim_{n \rightarrow \infty} \int_C \dots \int_C da_n^* da_n \dots da_1^* da_1 \exp \left[ i (\Delta S_n^{(infl)} + \dots + \Delta S_1^{(infl)} + \Delta S_0^{(infl)}) \right],
\end{aligned} \quad (46)$$

where

$$\Delta S_k^{(infl)} = -\frac{1}{2t} (|a_{k+1}|^2 + |a_k|^2) + (1 - t\Delta\tau\omega^{(b)}) a_{k+1}^* a_k + \Delta\tau e J_k^{\mu+} \varepsilon_\mu^* a_{k+1}^* + \Delta\tau e J_k^{\mu-} \varepsilon_\mu a_k, \quad (47)$$

time interval  $t$  is splitting on equal intervals  $\Delta\tau$ . So  $n\Delta\tau = t_f$  and  $a_{n+1}^* = a_f^*$ ,  $a_0 = a_{in}$ .

We integrate through intermediate variables  $a_1, \dots, a_n$  and obtain the following result in continual limit  $\Delta t \rightarrow 0$ ,  $n \rightarrow \infty$ :

$$\begin{aligned}
& U_{infl} [a_f^*, b_f, t_f, b^*(\tau)b(\tau), c^*(\tau)c(\tau), a_{in}, b_{in}, t_{in}] = \\
& = \exp \left[ -\frac{1}{2} (|a_f|^2 + |a_{in}|^2) + A a_f^* a_{in} + B a_f^* + C a_{in} + S_{infvac} \right].
\end{aligned} \quad (48)$$

where function  $A = \exp[-i\omega^{(b)}t]$  and functional  $B, C, S_{inf}$

$$B[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = -ie \int_{t_{in}}^{t_f} j^{\mu+}(b^*(\tau), b(\tau), c^*(\tau), c(\tau)) \varepsilon_{\mu}^* e^{-i\omega^{(b)}(t-\tau)} d\tau, \quad (49)$$

$$C[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = -ie \int_{t_{in}}^{t_f} j^{\mu-}(b^*(\tau), b(\tau), c^*(\tau), c(\tau)) \varepsilon_{\mu} e^{-i\omega^{(b)}\tau} d\tau, \quad (50)$$

$$\begin{aligned} S_{infvac}[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = \\ = - \int_{t_{in}}^{t_f} \int_{t_{in}}^{\tau} j^{\mu+}(b^*(\tau), b(\tau), c^*(\tau), c(\tau)) \varepsilon_{\mu}^* j^{\nu-}(b^*(\tau), b(\tau), c^*(\tau), c(\tau)) \varepsilon_{\nu} e^{-i\omega^{(b)}(\tau-\tau')} d\tau d\tau'. \end{aligned} \quad (51)$$

Now we calculate the influence functional of electromagnetic vacuum. For this we use equation (42) and specify initial and final state of electromagnetic field as vacuum:

$$\zeta_v(a_{in}) = \exp[-\frac{|a_{in}|^2}{2}], \quad \chi_v^*(a_f) = \exp[-\frac{|a_f|^2}{2}]. \quad (52)$$

By the use of (42) and (48),(52) we calculate the influence functional of one mode  $\mathbf{k}$  and  $\lambda$  electromagnetic field on one mode  $\mathbf{p}$  and  $\sigma$  fermionic field:

$$F_{vack,\lambda} = \exp[S_{infvack\lambda} + S'_{infvack\lambda}], \quad (53)$$

where functionals  $S_{infvack\lambda}, S'_{infvack\lambda}$  is determined by (51).

We define influence functional of multimode electromagnetic vacuum using (45), (51) and (53).

$$F_{vacall}[b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)] = \exp[S_{infvac} + S'_{infvac}], \quad (54)$$

where

$$S_{infvac} = - \sum_{\mathbf{k},\lambda} \frac{1}{2\omega_{\mathbf{k}}V} \int_{t_{in}}^{t_f} \int_{t_{in}}^{\tau} (\varepsilon_{\lambda}^{\mu} J_{\mu}^{+\sigma}(\mathbf{k}, \tau) \varepsilon_{\lambda}^{*\nu} J_{\nu}^{-\sigma}(\mathbf{k}, \tau') e^{i\omega_{\mathbf{k}}(\tau-\tau')} d\tau d\tau'), \quad (55)$$

functional  $S'_{infvac}$  has the same structure as  $S_{infvac}$  with variables  $b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau)$  i.e.  $J_{\mu}^{+\sigma}, J_{\nu}^{-\sigma}$ .

#### 4 The fermionic field evolution under electromagnetic vacuum influence

We describe the fermionic field evolution by the use of probability transition (41). Using equation (54) for influence functional of multimode electromagnetic vacuum, we obtain equation for probability transition

$$P(m; t_f | n; t_{in}) = K^*(m; t_f | n; t_{in}) K(m; t_f | n; t_{in}), \quad (56)$$

where

$$\begin{aligned} K(m; t_f | n; t_{in}) = \int \mathcal{D}b^*(\tau) \mathcal{D}b(\tau) \mathcal{D}c^*(\tau) \mathcal{D}c(\tau) db_f^* db_f dc_f^* dc_f \times \\ \times \psi_m^*(b_f, c_f) \exp\left\{i \left( S_{fulvac}[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] \right)\right\} \psi_n(b_{in}^*, c_{in}^*), \end{aligned} \quad (57)$$

$K^*(m; t_f|n; t_{in})$  is conjugated transition amplitude,

$$S_{full} [b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = S_f [b^*(\tau), b(\tau), c^*(\tau), c(\tau)] + S_{in vac} [b^*(\tau), b(\tau), c^*(\tau), c(\tau)], \quad (58)$$

In equations (56)–(58) we going from grassman variables  $b^*(\tau), b(\tau), c^*(\tau), c(\tau)$  to functions of spinor field  $\psi(x), \bar{\psi}(x)$ .

In equation (55) we convert summation over discrete  $\mathbf{k}$  to integral over their continuum values and Fourier transform  $j_\mu^+(\mathbf{k}, \tau), j_\nu^-(\mathbf{k}, \tau')$  to function  $j_\mu(\mathbf{x}, \tau), j_\nu(\mathbf{x}', \tau')$  by the use of equations (7) and (8). So, we have

$$S_{in vac}[\psi(x), \bar{\psi}(x)] = \frac{i}{2\pi} \int_{t_{in}}^{t_f} \int_{t_{in}}^{\tau} \int j_\mu(\mathbf{x}, \tau) D^{\mu\nu}(\mathbf{x} - \mathbf{x}', \tau - \tau') j_\nu(\mathbf{x}', \tau') d\mathbf{x} d\mathbf{x}' d\tau d\tau', \quad (59)$$

where

$$D^{\mu\nu}(\mathbf{x} - \mathbf{x}', \tau - \tau') = \frac{1}{(2\pi)^3} \int \frac{2\pi i d\mathbf{k}}{\omega_{\mathbf{k}}} \left( \sum_{\lambda} \varepsilon_{\lambda}^{\mu} \varepsilon_{\lambda}^{*\nu} \right) e^{-i\mathbf{k}(\mathbf{x}-\mathbf{x}') - i\omega(\tau-\tau')} \quad (60)$$

— is the photon propagator [11].

In equation (59) we transform iterated integral to double integral. Then we consider the limit  $t_f \rightarrow \infty, t_{in} \rightarrow -\infty$ . Now we obtain the action  $S_{in vac}[\psi(x), \bar{\psi}(x)]$ :

$$S_{in vac}[\psi(x), \bar{\psi}(x)] = \frac{i}{2\pi} \int \int j_\mu(x) D^{\mu\nu}(x - x') j_\nu(x') d^4 x' d^4 x. \quad (61)$$

We transform equation (58) in the same form. So the effective action (58) for action of fermionic field under electromagnetic field vacuum influence is presenting in the following

$$S_{full vac} [\psi(x), \bar{\psi}(x)] = \int \mathcal{L}_{full vac}(\gamma^\mu \partial_\mu \psi(x), \bar{\psi}(x)) dx, \quad (62)$$

where

$$\mathcal{L}_{full vac} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x) + \frac{i}{2\pi} j_\mu(x) \int D^{\mu\nu}(x - x') j_\nu(x') dx', \quad (63)$$

is density of effective Lagrangian,

$$\mathcal{L}_{in vac} = \frac{i}{2\pi} j_\mu(x) \int D^{\mu\nu}(x - x') j_\nu(x') dx', \quad (64)$$

is density of Lagrangian, which describe interaction of spinor field and vacuum of electromagnetic field.

The transition amplitude (57) is equivalent to S-matrix. In addition it contain contribution of electromagnetic vacuum influence on fermionic field dynamics:

$$K(m|n) = \int \mathcal{D}\bar{\psi}(\tau) \mathcal{D}\psi(\tau) \bar{\psi}_m(x) \exp \left\{ i \left( S_{full vac}[\bar{\psi}(\tau), \psi(\tau)] \right) \right\} \psi_n(x), \quad (65)$$

where action  $S_{full vac}[\bar{\psi}(\tau), \psi(\tau)]$  is determined by equation (62).

## 5 Summary

We consider the evolution of fermionic field interacting with electromagnetic field in path integral approach by the use of electromagnetic field influence functional introduction. We obtain formula for influence functional calculation in specific model of electromagnetic field, interaction with fermionic fields and initial and final states. We calculate the electromagnetic vacuum influence functional. This lead to addition term in Lagrangian of spinor field and modify fermionic field dynamics. The developed method allow to describe interacting fermionic system without perturbation expansions.

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