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The Casimir force for 2d sinusoidal gratings

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Abstract. The Casimir free energy for 2d gratings separated by a vacuum slit is expressed in terms of Rayleigh coefficients, a novel general approach valid for arbitrary 2d surface profiles of gratings is outlined. The normal Casimir force in the system of two identical Si gratings with 2d sinusoidal surface profiles separated by a vacuum slit is computed for several amplitudes of surface profiles, distance dependence of the force is studied. A comparison with results for flat boundaries is performed.

1 Introduction

The Casimir effect is a quantum fluctuation effect in the presence of boundaries. It was theoretically predicted by H.Casimir in 1948 [1] who considered two parallel perfectly conducting plates separated by a vacuum slit and derived the result for the attractive force between the plates.

The theory for two dispersive media which are periodic in one direction, translation invariant in the orthogonal direction and separated by a vacuum slit (i.e. systems and gratings with 1d spatial periodicity) was developed in Refs. [2, 3], the Casimir free energy was expressed in terms of Rayleigh coefficients [4]. Both normal and lateral Casimir forces can be measured in experiments with gratings. The theory was found to be in a very good agreement with experimental measurements of the lateral Casimir force between two sinusoidal 1d Au gratings with different amplitudes [5]. Important difference (up to 66%) in lateral force results based on Rayleigh decompositions and results based on Proximity Force Approximation (PFA) exploiting Lifshitz theory for flat geometries [6] was predicted theoretically and observed in experiments [5]. Details of the method can be found in Refs.[7, 8]. A modal approach to Casimir forces in periodic structures was developed in Ref.[9].

Measurements of the Casimir-Polder potential of Rb atom above a grating made of parallel Au stripes on the sapphire substrate in the presence of additional repulsive potential created by an external laser field were performed during diffraction of Bose-Einstein condensates of Rb atoms [10], results of this experiment were found to be in a good agreement with theoretical results based on Rayleigh decompositions. General analytic properties of Rayleigh coefficients were found in Ref.[11]. Existence of repulsive Casimir-Polder potential for an anisotropic atom in the presence of 20 nm thin 1d Au grating was shown in Ref.[11].

In the current paper the 1d formalism is generalized to systems with 2d periodicity. Periods of gratings coincide in a selected spatial direction though they may differ in orthogonal spatial directions.

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Our formalism is applicable to arbitrary surface profiles of 2d gratings. Gratings are assumed to be homogeneous, their material properties are defined by frequency dependent permittivities.

An outline of the paper is the following. In Sec. 2 we express the Casimir free energy of two gratings with coinciding periods in terms of Rayleigh coefficients and outline a general approach valid for arbitrary 2d surface profiles of gratings. In Sec. 3 we evaluate the Casimir energy and the Casimir normal force between two Si gratings with 2d sinusoidal surface profiles for various amplitudes of surface profiles, distance dependence of the force is studied. A comparison with flat case E.Lifshitz result for the force between two semispaces separated by a vacuum slit [6] is performed.

2 General formalism

Consider two 2d gratings which are periodic in orthogonal spatial directions with periods d_x, d_y and separated by a vacuum slit. In general, materials of the gratings are defined by frequency dependent permittivities. For simplicity the condition $\mu = 1$ for magnetic permeability is assumed in the whole space. In our formalism it is possible to evaluate the Casimir free energy and the Casimir force for arbitrary surface profiles of 2d gratings, periods of the gratings in a selected spatial direction coincide though they may differ in orthogonal directions (Fig. 1).

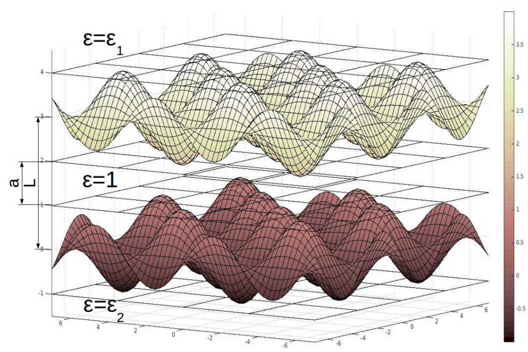

Figure 1. 2d gratings separated by a vacuum slit

The Casimir free energy for 2d gratings separated by a vacuum slit can be written as follows:

$$\mathcal{F} = k_B T \sum_{n=0}^{\infty}{}' \int_{-\pi/d_x}^{\pi/d_x} \frac{dk_x}{2\pi} \int_{-\pi/d_y}^{\pi/d_y} \frac{dk_y}{2\pi} \ln\det\Big(I - R_{1down}(i\omega_n, k_x, k_y) R_{2up}(i\omega_n, k_x, k_y)\Big), \quad (1)$$

here T is the temperature of the system, $\omega_n = \frac{2\pi k_B T n}{\hbar}$ are Matsubara frequencies; R_{1down}, R_{2up} are matrices of reflection coefficients from the lower and the upper gratings correspondingly, the prime means that the zero Matsubara term is taken with the coefficient $1/2$. To obtain reflection matrices for systems with periodic surface profiles or spatially periodic material properties it is convenient to consider Rayleigh decompositions [4].

Consider diffraction of an electromagnetic plane wave on a single grating (Fig. 2). For 2d periodic geometries the solutions of Maxwell equations can be decomposed in Fourier series on variables x and y. One can write exact solutions of Maxwell equations for $z > z_{max}$ in terms of Rayleigh reflection

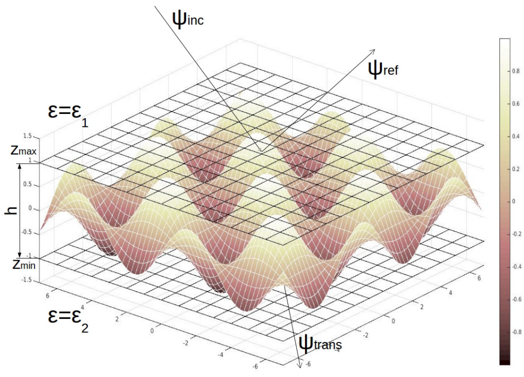

Figure 2. Reflection of the plane wave from the 2d grating

coefficients $R^{(e)}_{np,mq}, R^{(m)}_{np,mq}$ ($e^{-i\omega t}$ factors are omitted):

$$
\begin{aligned}
E^{+}_{y,pq} &= I^{(e)}_{pq}\, e^{i\alpha_p x} e^{i\beta_q y} e^{-i\gamma_{pq} z} + \sum_{n,m=-\infty}^{+\infty} R^{(e)}_{np,mq} e^{i\alpha_n x} e^{i\beta_m y} e^{i\gamma_{nm} z} \\
H^{+}_{y,pq} &= I^{(m)}_{pq}\, e^{i\alpha_p x} e^{i\beta_q y} e^{-i\gamma_{pq} z} + \sum_{n,m=-\infty}^{+\infty} R^{(m)}_{np,mq} e^{i\alpha_n x} e^{i\beta_m y} e^{i\gamma_{nm} z},
\end{aligned}
\tag{2}
$$

and for $z < z_{min}$ in terms of Rayleigh transmission coefficients $T^{(e)}_{np,mq}, T^{(m)}_{np,mq}$:

$$
\begin{aligned}
E^{-}_{y,pq} &= \sum_{n,m=-\infty}^{+\infty} T^{(e)}_{np,mq} e^{i\alpha_n x} e^{i\beta_m y} e^{-i\gamma_{nm} z} \\
H^{-}_{y,pq} &= \sum_{n,m=-\infty}^{+\infty} T^{(m)}_{np,mq} e^{i\alpha_n x} e^{i\beta_m y} e^{-i\gamma_{nm} z},
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
\alpha_p &= k_x + p\frac{2\pi}{d_x} \\
\beta_q &= k_y + q\frac{2\pi}{d_y} \\
\gamma_{pq} &= \sqrt{\varepsilon(\omega)\frac{\omega^2}{c^2} - \alpha_p^2 - \beta_q^2},
\end{aligned}
\tag{4}
$$

d_x, d_y are periods of gratings in orthogonal spatial directions, $k_x \in [-\pi/d_x, \pi/d_x], k_y \in [-\pi/d_y, \pi/d_y], p \in Z, q \in Z$.

Consider the vector of fourier components of longitudinal fields E_x, E_y, H_x, H_y. In numerical calculations it is sufficient to consider fourier components with numbers (n, m) possessing values $n \in (-N, N)$, $m \in (-M, M)$, values of N and M are determined by the accuracy of calculations. Let's denote by $[E_x]$ the vector with $(2N + 1)(2M + 1)$ fourier components of the field E_x, vectors for other components of the electromagnetic field are denoted in analogy.

In the range of values $z_{min} < z < z_{max}$ Maxwell equations can be rewritten in the form of first order differential equations [12]:

$$\partial_z \begin{pmatrix} [E_x] \\ [E_y] \\ [H_x] \\ [H_y] \end{pmatrix} = M \begin{pmatrix} [E_x] \\ [E_y] \\ [H_x] \\ [H_y] \end{pmatrix}, \tag{5}$$

M is a square matrix depending on $i\omega, k_x, k_y$ and grating surface profile in the range $z_{min} < z < z_{max}$. At $z = z_{max}$, $z = z_{min}$ the continuity conditions are imposed on all Fourier components of longitudinal fields E_x, E_y, H_x and H_y. As a result, the system of linear equations on Rayleigh coefficients $R^{(e)}$, $R^{(m)}$, $T^{(e)}$, $T^{(m)}$ can be obtained.

Arranging coefficients $R^{(e)}$, $R^{(m)}$ in the matrix

$$R_{1down} = \begin{pmatrix} R^{(e)}_{ns,ml}(\delta_{ps}\delta_{ql}, 0) & R^{(e)}_{ns,ml}(0, \delta_{ps}\delta_{ql}) \\ R^{(m)}_{ns,ml}(\delta_{ps}\delta_{ql}, 0) & R^{(m)}_{ns,ml}(0, \delta_{ps}\delta_{ql}) \end{pmatrix} \tag{6}$$

we obtain the matrix of reflection coefficients from the lower grating. The arguments of matrix elements in (6) are amplitudes of incident waves $\left(I^{(e)}_{pq}, I^{(m)}_{pq}\right)$.

For evaluation of the Casimir free energy we need matrices of reflection coefficients from the lower and the upper gratings. Suppose one finds the reflection matrix from the lower grating. In a special case when the upper grating is a mirror image of the lower grating it is sufficient to make a change of coordinates $z \to L - z_1, y \to -y_1$ in Rayleigh decompositions for the lower grating (L is a distance between middle planes of corrugation regions of the gratings, Fig.1) to obtain the reflection matrix for the upper grating.

3 Casimir energy and normal force

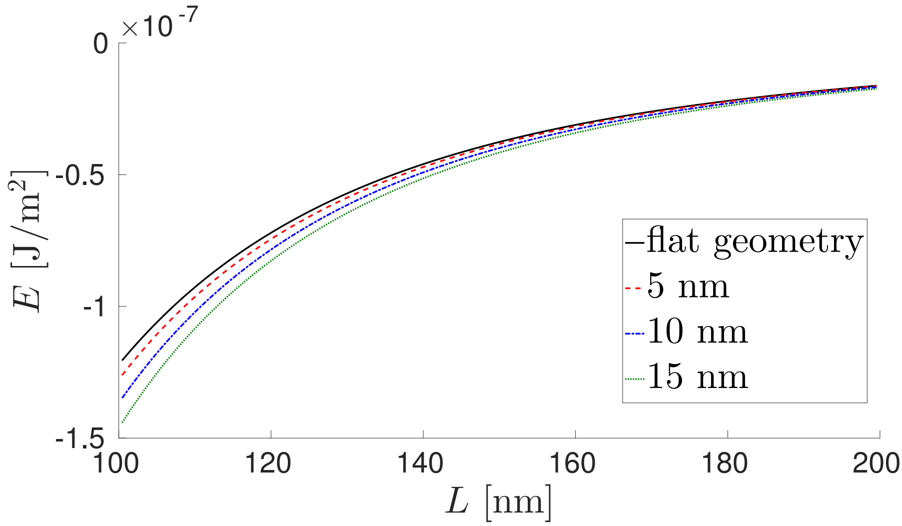

Figure 3. Casimir energy E of two 2d sinusoidal Si gratings with corrugation amplitudes $b = 5, 10, 15$ nm. The Lifshitz energy of two Si semispaces is shown by a black solid line. Grating periods are $d_x = d_y = 100$ nm.

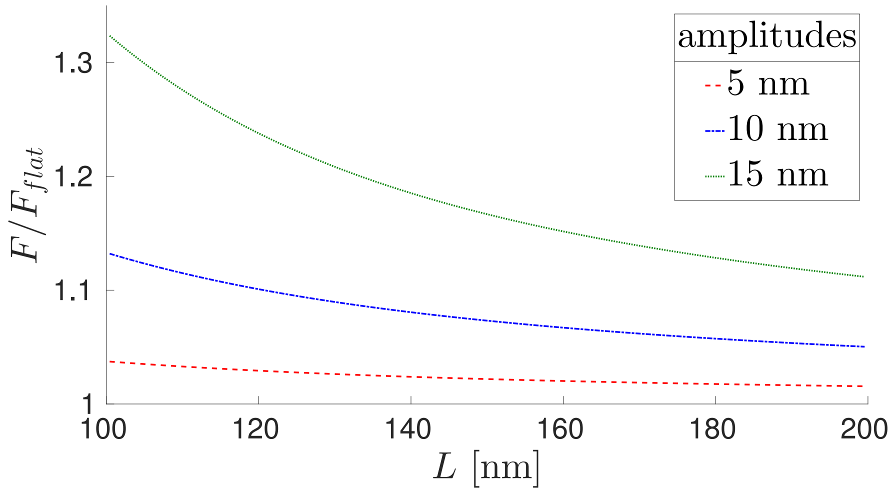

Figure 4. Ratios of the normal force F between two 2d sinusoidal Si gratings to the Lifshitz force F_{flat} between two Si semispaces separated by a distance L.

Consider two homogeneous identical Si gratings with sinusoidal 2d surface profiles separated by a vacuum slit, L is a distance between middle planes of corrugation regions of the gratings (Fig.1). The surface profile of each grating in the corrugation region is defined by a function $f(x, y) = b\cos(2\pi x/d_x)\cos(2\pi y/d_y)$. Note that in this case $z_{max} = -z_{min} = b$. In our calculations we use a frequency dispersion model of Si given by formula (6) in Ref.[13]. We study $T = 0$ case and evaluate Casimir energies E and normal Casimir forces $F = -\partial E/\partial L$ for gratings with periods $d_x = d_y = 100$ nm and amplitudes $b = 5, 10, 15$ nm. Casimir energies of two 2d sinusoidal Si gratings separated by a vacuum slit are evaluated by making use of the $T = 0$ limit of the formula (1), results are shown on Fig.3 where also the Lifshitz energy [6] of two Si semispaces separated by a vacuum slit L is added for comparison. Ratios of the normal Casimir force between two 2d sinusoidal Si gratings separated by a vacuum slit to the Lifshitz force between two Si semispaces separated by a vacuum slit $F_{flat}(L)$ are evaluated in the range $L \in [d_x, 2d_x]$, results are shown on Fig.4.

The normal Casimir force between two sinusoidal gratings is found to be larger than the Lifshitz force $F_{flat}(L)$ between two semispaces (Fig.4). The magnitude of the Casimir force between sinusoidal gratings increases with the increase of the corrugation surface profile amplitude b. This behavior of the Casimir force between gratings is consistent with general expectations: one can construct a sinusoidal grating profile from a dielectric semispace by extracting parts of dielectric bounded by the 2d sinusoidal curve from the semispace in half of the sinusoidal curve period and putting these parts onto the surface of the semispace in another half of the period. This argument explains why the normal force between two sinusoidal gratings should be larger than the force between two semispaces from which these sinusoidal gratings were constructed.

4 Conclusions

In this paper we outline a novel formalism for evaluation of the Casimir free energy in the system of two gratings with 2d periodic surface profiles which are separated by a vacuum slit. The developed formalism is valid for arbitrary 2d surface profiles of gratings. We apply our formalism to evaluate the normal Casimir force in the system of two identical 2d sinusoidal Si gratings separated by a vacuum slit at various separations between the gratings and for several amplitudes of sinusoidal surface profiles. A comparison with Lifshitz force between two semispaces separated by a vacuum slit is performed.

Acknowledgments

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