EPJ Web of Conferences **126**, 04046 (2016) DOI: 10.1051/epjconf/201612604046
ICNFP 2015

The magnetic polarizabilities and g-factor of the neutral and charged ρ mesons in a strong magnetic field on the lattice

O.E. Solovjeva[1,2,a], E.V. Luschevskaya[1,2,4], and O.V. Teryaev[3,2]

[1] *State Scientific Center of the Russian Federation - Institute for Theoretical and Experimental Physics , Bolshaya Cheremushkinskaya 25, 117218 Moscow, Russia*
[2] *National Research Nuclear University "MEPhI", Kashirskoe highway 31, 115409 Moscow, Russia*
[3] *Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[4] *School of Biomedicine, Far Eastern Federal University, 690950 Vladivostok, Russia*

Abstract. Study of the ground state energies of neutral and charged vector meson was performed in $SU(3)$ lattice gauge theory in the strong abelian magnetic field. Our method allows us to distinguish the ground state energy of mesons for various spin projections on the axis of the external magnetic field . We are observe the splitting the ground state energy of ρ mesons depending on spin projections: for neutral ρ^0 meson with zero spin projection on the axis of the field ground state energy decreases, while the energies with non-zero spins increase with the field value. For charged $\rho^{\pm}$ mesons we observe the agreement between Landau level picture and our data at magnetic field less then 0.6GeV^2. The magnetic polarizabilities of vector mesons were found for the lattice volume 18^4 fitting the ground state energies. We found the g-factor of $\rho^{\pm}$ in the chiral limit. This value is compatible with experimental determination and theoretical value from relativistic quark model.

1 Introduction

Magnetic polarizabilities and g-factors are fundamental characteristics of particles. This parameters are interesting for understanding the internal structure of hadrons and effects which may appear in electromagnetic field. Conception of polarizability was first used for the nuclear matter in the analysis of the scattering of low energy gamma quanta by atomic nuclei by Migdal [1]. For hadrons the notion of polarizability was discussed in papers [2]-[4]. New notion of polarizability assumes process of the Compton scattering on particle. The electric and magnetic polarizabilities determine the response of hadron to two-photon interactions.
In this work we will discuss the behaviour of the ground state energy of ρ mesons in the abelean strong magnetic field from the $SU(3)$ lattice gauge theory. The g-factor and magnetic polarizabilities are parameters which determine the behaviour of the ground state energy.

[a] e-mail: oesolovjeva@gmail.com

2 Details of calculations

The detailed description of the technical details our calculations are presented in [8],[9]. We generate $200-300$ $SU(3)$ statistically independent lattice gauge configurations for lattice volumes 16^4, 18^4 and lattice spacings $a = 0.105$ fm, 0.115 fm and 0.125 fm. The $U(1)$ external magnetic field is included only into the Dirac operator which is used for the calculation of eigenfunctions ψ_k and eigenvectors λ_k of a test quark in a background gauge field A_μ.

This field is a sum of non-abelian $SU(3)$ gluonic field and $U(1)$ abelian constant magnetic field.

$$A_{\mu ij} \to A_{\mu ij} + A^B_\mu \delta_{ij}, \tag{1}$$

where

$$A^B_\mu(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}). \tag{2}$$

The twisted boundary conditions are imposed because it is necessary to take into account periodic boundary conditions for fermions. [5].

The value of the magnetic field in finite volume is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \quad k \in \mathbb{Z}, \tag{3}$$

where $q = -1/3\, e$. It follows to a minimal value of magnetic field $(eB)^{1/2} = 372$ Mev for lattice volume 20^4 and lattice spacing $a = 0.115$ fm.

One should note here that our parameters of lattice for the following values of magnetic fields corresponding to the lattice spacing cut-off: $2.5\mathrm{GeV}^2$ for $a = 0.125$ fm, $2.9\mathrm{GeV}^2$ for lattice spacing $a = 0.115$ fm and $eB \sim 3.5$ GeV^2 for $a = 0.105$ fm etc.
To determine the ground state energy is necessary to calculate the correlation function interpolation operators in Euclidean space :

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A, \tag{4}$$

where interpolation operator is $\psi^\dagger(x)O_1\psi(x)$, analogue creation operator in quantum electrodynamics, which creates a particle with certain quantum numbers : for ρ meson $O_1, O_2 = \gamma_\mu, \gamma_\nu$, $\mu, \nu = 1, .., 4$ are Lorentz indices. This correlation function were calculated separately for every value of external magnetic field. For an isovector states we can express the correlation function through the Dirac propagator:

$$\langle \bar\psi O_1 \psi \bar\psi O_2 \psi \rangle_A = -\mathrm{Tr}\,[O_1 D^{-1}(x, y) O_2 D^{-1}(y, x)] \tag{5}$$

We calculated the Dirac propagator using its eigenvectors and eigenvalues

$$D^{-1}(x, y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}. \tag{6}$$

where $M = 50$ is the number of the lowest eigenmodes.

On the other hand one can make 3-dimentional Fourier transformation of correlation function.For the ground energy state we consider zero momentum $\mathbf{p} = 0$, because in this case $E_0 = m_0$ in zero magnetic field. The correlation function is expand to exponential series.

$$\tilde C(n_t) = \langle \psi^\dagger(\mathbf{0}, n_t) O_1 \psi(\mathbf{0}, n_t) \psi^\dagger(\mathbf{0}, 0) O_2 \psi(\mathbf{0}, 0) \rangle_A = \sum_k \langle 0|O_1|k\rangle\langle k|O_2^\dagger|0\rangle e^{-n_t a E_k} \tag{7}$$

 DOI: 10.1051/epjconf/201612604046

where a is the lattice spacing, n_t is the number of nodes in the time direction, E_k is the energy of the state with quantum number k. If we consider large n_t the main contribution in (7) gives ground state. After taking into account the periodic boundary conditions on the lattice the main contribution to the ground state has the following form

$$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} = 2A_0 e^{-N_T a E_0/2} \cosh((\frac{N_T}{2} - n_t) a E_0) \tag{8}$$

where A_0 is a constant, E_0 is the energy of the ground state. Using this formula we fit our data of correlators (4) and find the ground state energy as a fit parameter. In order to minimize the errors and exclude the contribution of excited states we take various values of n_t from the interval $5 \leq n_t \leq N_T - 5$. We also use a smeared gaussian source and point sink for our calculations.
The correlation functions for various spatial directions are given by the following relations

$$C_{ij}^{VV} = \langle \bar{\psi}(\mathbf{0}, n_t)\gamma_i \psi(\mathbf{0}, n_t)\bar{\psi}(\mathbf{0}, 0)\gamma_j \psi(\mathbf{0}, 0)\rangle, \tag{9}$$

where i,j=1,2,3 for x,y,z components respectively. The form of the density matrix for vector particle with spin $s = 1$ gives the formulas for energies of meson with various spin projections on the axis of the external magnetic field.

For the $s_z = 0$ one can obtain the energy of the ground state from the C_{zz}^{VV} correlator. The combinations of correlators

$$C^{VV}(s_z = \pm 1) = C_{xx}^{VV} + C_{yy}^{VV} \pm i(C_{xy}^{VV} - C_{yx}^{VV}). \tag{10}$$

gives the energies of mesons with $s_z = +1$ and $s_z = -1$.

3 Results

3.1 Neutral ρ meson

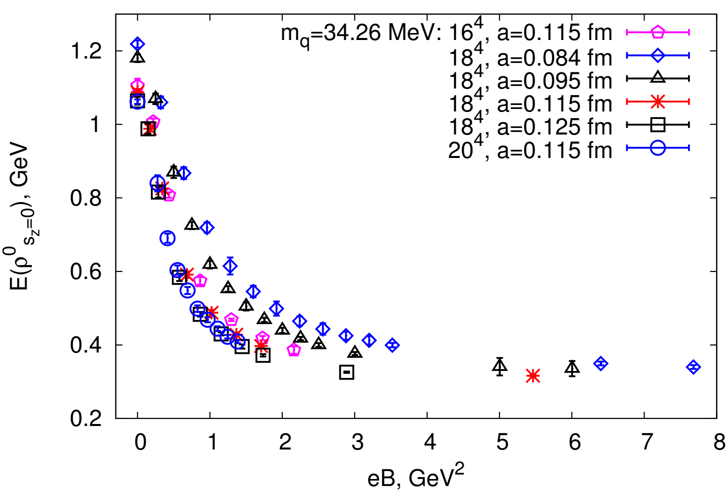

Figure 1. The ground state energy of neutral ρ^0 meson with $s_z = 0$

In the case of neutral ρ^0 meson, we observe a quadratic dependence on the magnetic field. Term with magnetic polarizability gives the contribution to the energy of ρ meson.

We describe the energy of the neutral ρ^0 meson by the following formula:

$$E = E(B = 0) - 2\pi\beta_m(eB)^2, \tag{11}$$

where we use "natural" units $\hbar = c = 1$, but $e^2 = 1/137$ in Gaussian units.
We calculate the energy for every values of magnetic field and find $E(B = 0)$ and β_m from the fit.

In Fig. 1 we show the ground state energy of neutral ρ^0 meson with $s_z = 0$ as a function of the magnetic field for various lattice spacings and volumes. We had a fit of the interval $(eB)^2 \in [0, 0.3\ \mathrm{GeV}^4]$ by the quadratic function (11). This interval allow doesn't correct for a term $\sim (eB)^4$ in the result. From this figure one can see that the ground state energy of ρ^0 meson with $s_z = 0$ decreases with the value of magnetic field. The magnetic polarizability was found as a parameter of the fit by formula 11: $\beta_m^{s_z=0}(\rho^0) = (0.47 \pm 0.03)\ \mathrm{GeV}^{-3}$

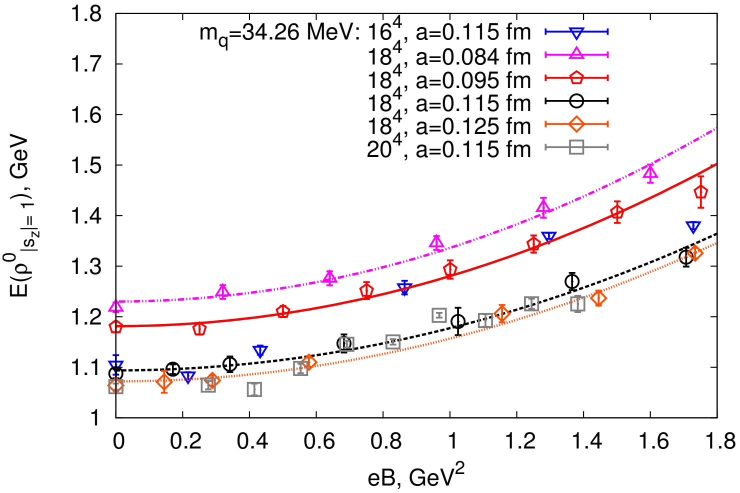

Figure 2. The ground state energy of neutral ρ^0 meson with $|s_z| = 1$

Fig. 2 presents the energy of ρ^0 with nonzero spin projections on the axis of the external magnetic field. This ground state energy increases with the value of magnetic field for the all sets of lattice data. It is worth noting that the ground state energy ρ^0 mesons with $s = -1$ and $s_z = +1$ are equal this follows from the vanishing terms iC_{xy}^{VV} and iC_{yx}^{VV} in (10) for neutral particles. We found the magnetic polarizability and observe the dependence of the results on the lattice spacing so we make an extrapolation to the continuum limit. After extrapolation the polarizability value is $\beta_m^{|s|=1}(\rho^0) = (-0.0235 \pm 0.0023)\ \mathrm{GeV}^{-3}$ for the lattice volume 18^4 and bare quark mass $m_q = 34\ Mev$. One can see that the magnetic polarizability at $s_z = 0$ has another sign from the case of nonzero spin. This phenomena to be the result of the anisotropy created by the magnetic field.

DOI: 10.1051/epjconf/201612604046

3.2 Charged $\rho^{\pm}$ meson

Well-known formula of Landau level describing the energy levels of free charged pointlike particle in a background magnetic field it is not suitable for strong magnetic field.

$$E^2 = |qB| - gs_z qB + m^2(B = 0), \tag{12}$$

here g-factor characterizes magnetic properties of the particle, q is the charge of the particle, $m^2(B = 0)$ is the particle mass at $B = 0$.

We have to take into account the magnetic polarizabilty when particle is not pointlike. In the relativistic case the energy levels has the following form

$$E^2 = |qB| - gs_z qB + m^2(B = 0) - 4\pi m\beta(qB)^2, \tag{13}$$

where β is the magnetic polarizability

We consider ρ^- mesons while ρ^+ corresponds to the reversal of the direction of magnetic field. From the Fig. 3 one can see the energy of the charged vector ρ meson with $s_z = 0$. Our data obey

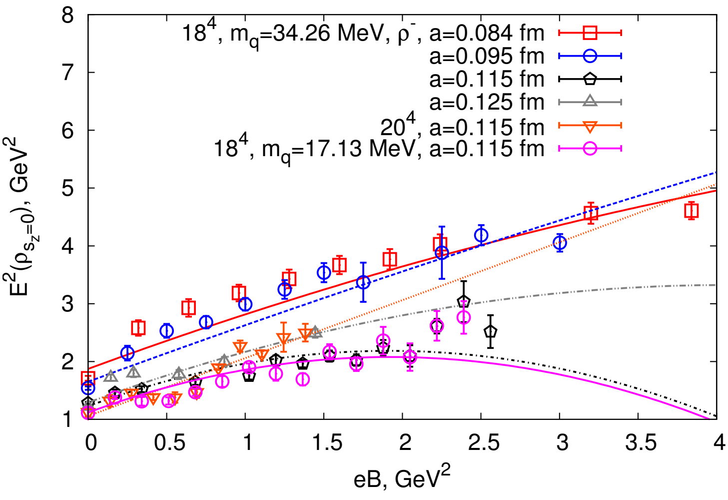

Figure 3. The ground state energy of charged $\rho^{\pm}$ meson with $s_z = 0$

formula (12) at $eB \in [0, 0.6\ \mathrm{GeV}^2]$. We fitting our data by the formula (13) for varios lattice volume and lattice spacing . We represent the energies of the ρ^- with spin projections $s_z = -1$ and $s_z = +1$ in Figs. 4 and 5 correspondingly. The energy of the ρ^- ground state with $s_z = -1$ decreases with the field value and the data agree with the fit Landau levels at $eB \in [0, 0.6\ \mathrm{GeV}^2]$. It is noticeable that we didn't observe the appearance of the tachyonic mode. If we consider strong magnetic field then we have to take into account magnetic dipole polarizabilty, megnetic hyperpolarizability and so on. It is follows, that energy does not equal zero.

From the data obtained one can estimate of g-factor of the $\rho^{\pm}$ meson.It is possible at small magnetic when our data described by the Landau level. We evaluate value of g-factor from the following relation:

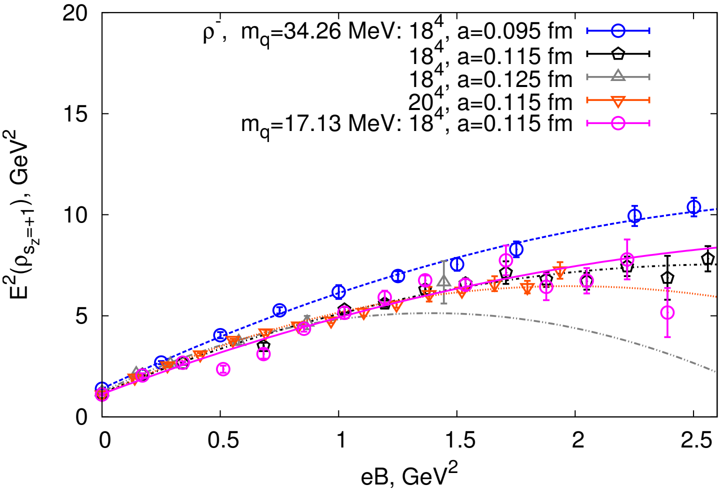

Figure 4. The ground state energy of charged $\rho^{\pm}$ meson with $s_z = +1$

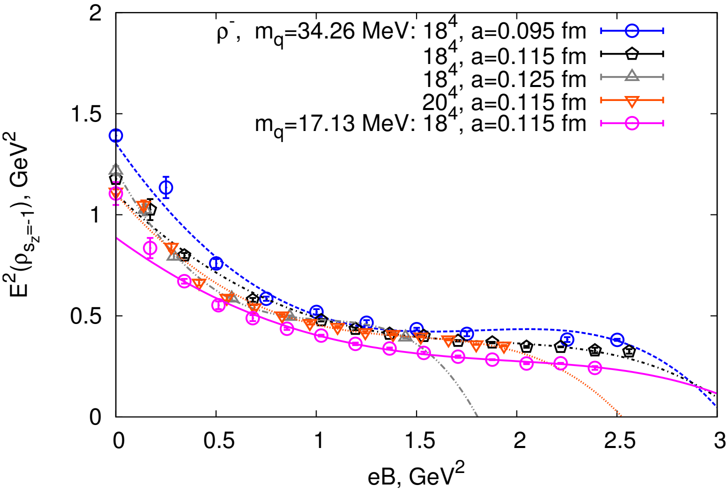

Figure 5. The ground state energy of charged $\rho^{\pm}$ meson with $s_z = -1$

$$g = \frac{E^2(s=+1) - E^2(s=-1)}{2(eB)}, \tag{14}$$

To clarify the value we made extrapolation to $m_q = 0$ and obtain $g = 2.4 \pm 0.2$ for the lattice volume 18^4 and lattice spacing $a = 0.115$ fm. It is clear that this value is compatible with recent experimental determination [6]. From theoretical prediction[7] $g \approx 2.37$, this value close to our result. In following work we plan to increase accuracy of the g-factor of the $\rho^{\pm}$ meson.

4 Conclusions

In this work we present the study of the ground state energies of neutral and charged vector meson in the strong abelian magnetic field from the $SU(3)$ lattice gauge theory. The energy of ρ^0 meson with zero spin projection $s_z = 0$ on the axis of the external magnetic field decreases, while the energies with non-zero spins $s_z = -1$ and $+1$ increase with the value of the magnetic field. Magnetic polarizability were calculated for neutral ρ^0 meson. It should be noted that magnetic polarizabilities with spin projections directed along magnetic field have different sign unlike the case another spin projections. This is due to the anisotropy of the magnetic field created
In the case of charged $\rho^{\pm}$ mesons we observe the agreement between Landau level picture and our data at small magnetic fields . The magnetic polarizabilities of neutral vector mesons were found for the lattice volumes 18^4 fitting the ground state energies. The g-factor of $\rho^{\pm}$ mesons is found from its the ground state energies with various spins. After chiral extrapolation we obtain the value $g = 2.4 \pm 0.2$ for the lattice volume 18^4 and lattice spacing $a = 0.115$ fm.

Acknowledgements

The authors are grateful to FAIR-ITEP supercomputer center where these numerical calculations were performed. This work is completely supported by a grant from the Russian Science Foundation (project number 16-12-10059).

References

[1] A.B. Migdal, J.Phys. USSR **8**, 331 (1944)
[2] A. Klein, Phys. Rev. **99**, 998 (1955)
[3] A.M. Baldin, Nucl. Phys.**18**, 310 (1960)
[4] V.A. Petrun'kin, JETP **13**, 804 (1961)
[5] M.H. Al-Hashimi and U.J. Wiese, Annals Phys. **324**, 343 (2009), arXiv: 0807.0630 [quant-ph].
[6] D. G. Gudino and G. T. Sanchez (2015), arXiv: 1305.6345 [hep-ph]
[7] A. M. Badalian, Yu. A. Simonov Phys. Rev. D **87**, 074012 (2013)
[8] E.V. Luschevskaya, O.E. Solovjeva, O.E. Kochetkov and O.V. Teryaev, Nucl. Phys. B **898**, 627 (2015).
[9] E.V. Luschevskaya, O.E. Kochetkov, O.V. Teryaev and O.E. Solovjeva,JETP Letters **101**, Issue 10, 674 (2015).