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Off-shell amplitudes and four-jet production in k_T-factorization

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Abstract. We perform a study of both Single and Double Parton Scattering contributions to the inclusive 4-jet production in the k_T-factorization framework at Leading Order and $\sqrt{s} = 7$ TeV and 13 TeV. We discuss the importance of double parton scattering for relatively soft cuts on the jet transverse momenta and find out that symmetric cuts do not suit well k_T-factorization predictions, because of a kinematic effect suppressing the double parton scattering contribution.

1 Introduction

Using a proper generalisation of the Britto-Cachazo-Feng-Witten recursion relation [1, 2] for on-shell QCD amplitudes, we can recursively compute fully gauge-invariant amplitudes with initial state off-shell particle both analytically [3, 4] and numerically [5] and use them to assess both the Single Parton Scattering and Double Parton Scattering contributions to four-jet production. This allows us to expand the analysis of Ref. [6] and assess the differences between the collinear approach and the high-energy factorization (HEF) (or k_T-factorization) framework.

2 Calculations and comparison to experimental data

2.1 Single-parton scattering production of four jets

The collinear factorization formula for the calculation of the inclusive partonic 4-jet cross section reads

$$\sigma^B_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} x_1 f_i(x_1,\mu_F) x_2 f_j(x_2,\mu_F) \times \frac{1}{2\hat{s}} \prod_{l=1}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} \Theta_{4-jet} (2\pi)^4 \delta\left(x_1 P_1 + x_2 P_2 - \sum_{l=1}^{4} k_l \right) \overline{|\mathscr{M}(i,j \to 4\,\text{part.})|^2}. \quad (1)$$

Here $f_i(x_{1,2},\mu_F)$ are the collinear PDFs for the $i-th$ parton, carrying $x_{1,2}$ momentum fractions of the proton and evaluated at the factorization scale μ_F; the index l runs over the four partons in the

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final state, P is the total initial state partonic momentum, associated to the center of mass energy $\hat{s} = P^2 = (P_i + P_j)^2 = 2P_i \cdot P_j$; the Θ function takes into account the kinematic cuts applied and $\mathscr{M}$ is the partonic on-shell matrix element, which includes symmetrization effects due to the possible identity of the final state particles.

The analogous formula to (1) for HEF is

$$\sigma^B_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} d^2k_{T1} d^2k_{T2}\, \mathscr{F}_i(x_1, k_{T1}, \mu_F)\, \mathscr{F}_j(x_2, k_{T2}, \mu_F)$$
$$\times \frac{1}{2\hat{s}} \prod_{l=1}^{4} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-jet}\, (2\pi)^4\, \delta\left(x_1P_1 + x_2P_2 + \vec{k}_{T\,1} + \vec{k}_{T\,2} - \sum_{l=1}^{4} k_l\right) \overline{|\mathscr{M}(i^*, j^* \to 4\,\text{part.})|^2}\,. \tag{2}$$

Here $\mathscr{F}_i(x_k, k_{Tk}, \mu_F)$ is a transverse momentum dependent (TMD) parton density for a given type of parton. Similarly as in the collinear factorization case, x_k is the longitudinal momentum fraction and μ_F is a factorization scale. The new degree of freedom is introduced via the transverse k_{Tk}, which is perpendicular to the collision axis. The formula is valid when the x's are not too large and not too small and, in order to construct a full set of TMD parton densities, we apply the Kimber-Martin-Ryskin (KMR) prescription [7, 8], which, at the end of the day, amounts to applying the Sudakov form factor onto the PDFs.

$\mathscr{M}(i^*, j^* \to 4\,\text{part.})$ is the gauge invariant matrix element for $2 \to 4$ particle scattering with two initial off-shell legs. We rely on the numerical Dyson-Schwinger recursion in the AVHLIB[1] for its computation. If a complete calculation of 4-jet production in k_T factorization was still missing in the literature, it was mainly because computing gauge-invariant amplitudes with off-shell legs is definitely non trivial. Techniques to compute such amplitudes in gauge invariant ways are by now analytically and numerically well established [3–5].

We use a running α_s provided with the MSTW2008 PDF sets and set both the renormalization and factorization scales equal to half the transverse energy, which is the sum of the final state transverse momenta, $\mu_F = \mu_R = \frac{\hat{H}_T}{2} = \frac{1}{2}\sum_{l=1}^{4} k_T^l$, working in the $n_F = 5$ flavour scheme.

There are 19 different channels contributing to the cross section at the parton-level, of which the dominant ones, contributing together to $\sim 93\%$ of the total cross section, are

$$gg \to gggg\,, gg \to q\bar{q}gg\,, qg \to qggg\,, q\bar{q} \to q\bar{q}gg\,, qq \to qqgg\,, qq' \to qq'gg\,. \tag{3}$$

2.2 Double-parton scattering production of four jets

The SPS contribution is expected to dominate for high momentum transfer because, as it is intuitively clear, it is highly unlikely that two partons from one proton and two from the other one are energetic enough for two very hard scatterings to take place, as the well-known behaviour of the PDFs for large momentum fractions suggests. However, if the cuts on the transverse momenta of the final state are lowered, a window opens to observe significant double parton scattering effects, as often stated in the literature on the subject and recently analysed for 4-jet production in the framework of collinear factorization [6]. Here we perform the same analysis in HEF, with the goal to assess the differences in the predictions.

[1] available for download at https://bitbucket.org/hameren/avhlib

First of all, let us present the standard, phenomenologically motivated formula for the computation of differential DPS cross sections, tailored to a four-parton final state,

$$\frac{d\sigma^B_{4-jet,DPS}}{d\xi_1 d\xi_2} = \frac{m}{\sigma_{eff}} \sum_{i_1,j_1,k_1,l_1;i_2,j_2,k_2,l_2} \frac{d\sigma^B(i_1 j_1 \to k_1 l_1)}{d\xi_1} \frac{d\sigma^B(i_2 j_2 \to k_2 l_2)}{d\xi_2}, \tag{4}$$

where the $\sigma(ab \to cd)$ cross sections are obtained by restricting formulas (1) and (2) to a single channel and the symmetry factor m is $1/2$ if the two hard scatterings are identical, in order to avoid double counting, and is otherwise 1, whereas ξ_1 and ξ_2 denote generic kinematical variables for the first and second scattering, respectively.

The effective cross section σ_{eff} can be loosely interpreted as a measure of the transverse correlation of the two partons inside the hadrons. In this paper we stick to the widely used value σ_{eff} = 15 mb.

We also have to use an ansatz for Double Parton Distribution Functions; for collinear-factorization this is

$$D_{1,2}(x_1,x_2,\mu) = f_1(x_1,\mu)\, f_2(x_2,\mu)\, \theta(1-x_1-x_2), \tag{5}$$

where $D_{1,2}(x_1,x_2,\mu)$ is the Double Parton Distribution Function and $f_i(x_i,\mu)$ are the ordinary PDFs. The subscripts 1 and 2 distinguish the two generic partons in the same proton. Of course this ansatz can be automatically generalised to the case when parton transverse momenta are included by simply including the dependence on the transverse momentum.

Coming to DPS contributions, in principle we must include all the possible 45 channels which can be obtained by coupling in all possible distinct ways the 8 channels for the $2 \to 2$ SPS process, i.e.

$$\#1 = gg \to gg\,, \quad \#2 = gg \to q\bar{q}\,, \quad \#3 = qg \to qg\,, \#4 = q\bar{q} \to q\bar{q}$$
$$\#5 = q\bar{q} \to q'\bar{q}'\,, \quad \#6 = q\bar{q} \to gg \quad \#7 = qq \to qq\,, \quad \#8 = qq' \to qq'$$

We find that the pairs $(1,1)$, $(1,2)$, $(1,3)$, $(1,7)$, $(1,8)$, $(3,3)$ $(3,7)$, $(3,8)$ together account for more than 95 % of the total cross section. This was tested for all the sets of cuts considered in this paper.

2.3 Comparison to the collinear approach and to ATLAS data with hard central kinematic cuts

Our HEF calculation was first tested against the 8 TeV data recently reported by the ATLAS collaboration [9]. The kinematic cuts are $p_T > 100$ GeV for the leading jet and $p_T > 64$ GeV for the first three subleading jets; in addition $|\eta| < 2.8$ is the pseudorapidity cut and $\Delta R > 0.65$ is the constraint on the jet cone radius parameter.

We employ the running NLO α_s coming with the MSTW2008 sets. For such cuts, not much difference is expected between the two approaches and indeed we find none; DPS effects are irrelevant with this kinematics.

2.4 Comparison to CMS data with softer cuts

As discussed in Ref. [6], so far the only experimental analysis of four-jet production relevant for the DPS studies was realized by the CMS collaboration [10]. The cuts used in this analysis are $p_T > 50$ GeV for the first and second jets, $p_T > 20$ GeV for the third and fourth jets, $|\eta| < 4.7$ and the jet cone radius parameter $\Delta R > 0.5$. In the rest of this section, we present our results for such cuts.

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As for the total cross section for the four jet production, the experimental and theoretical LO results are:

$$\begin{aligned} \text{CMS collaboration}: &\quad \sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\,nb \\ \text{LO collinear factorization}: &\quad \sigma_{SPS} = 697\,nb\,, \quad \sigma_{DPS} = 125\,nb\,, \quad \sigma_{tot} = 822\,nb \\ \text{LO HEF } k_T\text{-factorization}: &\quad \sigma_{SPS} = 548\,nb\,, \quad \sigma_{DPS} = 33\,nb\,, \quad \sigma_{tot} = 581\,nb \end{aligned} \tag{6}$$

The LO results need refinements from NLO contributions, much more than it does in the case of the ATLAS hard cuts, as we are of course not that deep into the perturbative region. For this reason, in the following we will always perform comparisons only to data normalised to the total (SPS+DPS) cross sections. We find that this is better than introducing fixed K-factors, whose phase-space dependence is never really under control. What is immediately apparent in the HEF total cross section is the dramatic damping of the DPS contribution with respect to the collinear case. This damping effect is of kinematical nature. The point is that the emission of gluon radiation, which is taken into account via the TMDs in our approach and via the real contribution in a collinear NLO calculation, alters the exact momentum balance of the final state two-jet system, so that a lot of events are not taken into account for the higher transverse momentum just above the cut. In Fig. 1 we compare the predictions

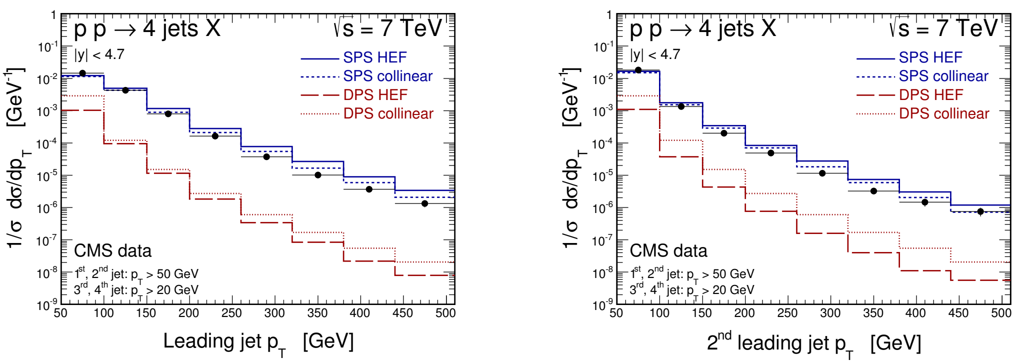

Figure 1. Comparison of the LO collinear and HEF predictions to the CMS data for the 1st and 2nd leading jets.

in HEF to the CMS data for the 1st and 2nd leading jets transverse momenta spectra. Here both the SPS and DPS contributions are normalized to the total cross section, i.e. the sum of the SPS and DPS contributions. In all cases the renormalized transverse momentum distributions agree quite well with the CMS data.

2.5 HEF predictions for a possible set of asymmetric cuts

Now we propose a set of asymmetric cuts. Specifically, we require $p_T > 35$ GeV for the leading jet, $p_T > 20$ GeV for all the other jets and we stick to $|\eta| < 4.7$, $\Delta R > 0.65$ for rapidity and jet size parameter. An experimental analysis with such cuts is not available at the moment. Of course it would be desirable to have such an analysis in the future.

The theoretical total cross sections for these cuts for four-jet production are:

$$\begin{aligned} \text{LO collinear factorization}: &\quad \sigma_{SPS} = 1969\,nb\,, \quad \sigma_{DPS} = 514\,nb \\ \text{LO HEF } k_T\text{-factorization}: &\quad \sigma_{SPS} = 1506\,nb\,, \quad \sigma_{DPS} = 297\,nb \end{aligned} \tag{7}$$

When comparing to (6), it is apparent that now the drop in the total cross section for DPS when moving from LO collinear to HEF approach is considerably smaller, as argued.

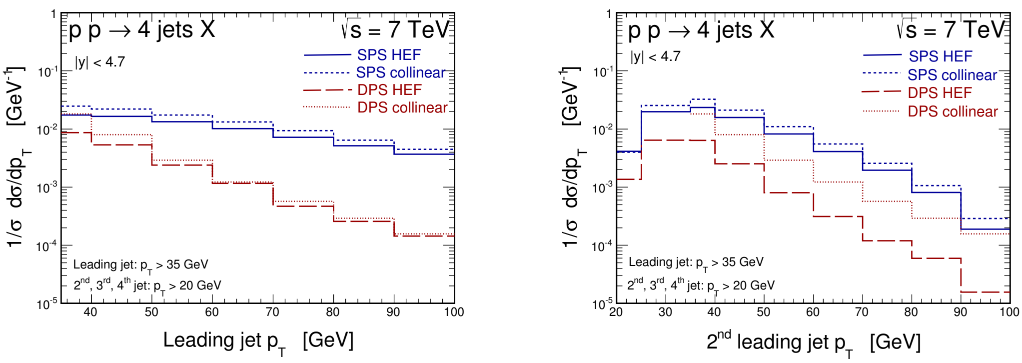

Figure 2. LO collinear and HEF predictions for the 1st and 2nd leading jets with the asymmetric cuts.

In Fig. 2 we show our predictions for the normalized transverse momentum distributions with the new set of cuts.

3 Looking for new variables to enhance DPS

We proposed in [12] that the analysis of cross sections which are differential with respect to three other variables might be interesting in view of the goal of clearly identifying DPS.

The first of such variables was found to be the maximum rapidity distance

$$\Delta Y \equiv max_{\substack{i,j\in\{1,2,3,4\}\\ i\neq j}} |\eta_i - \eta_j| \,. \tag{8}$$

A second candidate is the azimuthal correlations between the jets which are most remote in rapidity

$$\varphi_{jj} \equiv |\varphi_i - \varphi_j| \,, \quad \text{for} \quad |\eta_i - \eta_j| = \Delta Y \,. \tag{9}$$

Finally, we propose that the minimal sum of two azimuthal distances, defined as

$$\Delta\varphi_{3j}^{min} \equiv min_{\substack{i,j,k\in\{1,2,3,4\}\\ i\neq j\neq k}} \left(|\varphi_i - \varphi_j| + |\varphi_j - \varphi_k|\right) \,, \tag{10}$$

can be specially interesting. Lack of space prevents us from presenting a more thorough analysis here, but details can be found in the paper [12].

4 Conclusions

We have compared the perturbative predictions for four-jet production at the LHC in leading-order collinear and high-energy (k_T-)factorization. We find that there is no significant difference between the collinear and HEF approach for hard central cuts, but significant differences show up, especially for DPS, when the cuts on the transverse momenta are lowered. Our approach is able to describe existing CMS data on jet rapidity distributions and we have presented our predictions for differential distributions with respect to other variables as well. We observed that HEF severely tames DPS for symmetric cuts, due to gluon-emission effects encoded in the PDFs which alter the transverse-momentum balance between final state partons. We have found that the damping is sensibly reduces when cuts are not identical.

We find that, for sufficiently small cuts on the transverse momenta, DPS effects are enhanced relative to the SPS contribution when rapidities of jets are large, for large rapidity distances between the most remote jets, for small azimuthal angles between the two jets most remote in rapidity and, finally, for large values of $\Delta\varphi_{3j}^{min}$. In general, the relative effects of DPS in the k_T-factorization approach are somewhat smaller than those found previously in the LO collinear approach. A complete treatment of the subject of this proceeding is extensively given in [11, 12].

Acknowledgments

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