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Charmed Mesons and Charmonia: Three-Meson Strong Couplings

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Abstract. Revisiting the strong couplings of three mesons, each of which involves at least one charm quark, proves clear disaccord between quark-model and QCD sum-rule results.

1 Three-meson strong couplings from relativistic constituent-quark model

We extract the strong couplings of three mesons among which there is, at least, one of the charmonia η_c and J/ψ from the residues of poles in adequate transition form factors for timelike momentum transfer, which, in turn, are inferred from a relativistic dispersion approach based on a constituent-quark model.

2 Definitions of strong couplings, transition form factors, decay constants

Preparatively, let us introduce all the quantities necessary for the formulation of the relation sought, for *pseudoscalar* mesons P, with mass M_P, and *vector* mesons V, with mass M_V and polarization vector ε_μ.

- With momentum transfer $q \equiv p_1 - p_2$, the *strong couplings* $g_{PP'V}$ and $g_{PV'V}$ determine the amplitudes

$$\langle P'(p_2)\, V(q)|P(p_1)\rangle = -\frac{g_{PP'V}}{2}\,(p_1+p_2)^\mu\,\varepsilon^*_\mu(q)\,, \qquad \langle V'(p_2)\, V(q)|P(p_1)\rangle = -g_{PV'V}\,\epsilon_{\varepsilon^*(q)\,\varepsilon^*(p_2)\,p_1\,p_2}\,.$$

- The *transition form factors* $\mathcal{F}(q^2) = F_+^{P\to P'}(q^2)$, $V^{P\to V}(q^2)$ or $A_0^{P\to V}(q^2)$ enter in the two-meson matrix elements of the vector quark current $j_\mu \equiv \bar{q}_1\,\gamma_\mu\,q_2$ and the axial-vector quark current $j^5_\mu \equiv \bar{q}_1\,\gamma_\mu\,\gamma_5\,q_2$

$$\langle P'(p_2)|j_\mu|P(p_1)\rangle = F_+^{P\to P'}(q^2)\,(p_1+p_2)_\mu + \cdots\,, \qquad \langle V(p_2)|j_\mu|P(p_1)\rangle = \frac{2\,V^{P\to V}(q^2)}{M_P+M_V}\,\epsilon_{\mu\,\varepsilon^*(p_2)\,p_1\,p_2}\,,$$
$$\langle V(p_2)|j^5_\mu|P(p_1)\rangle = \mathrm{i}\,q_\mu\,(\varepsilon^*(p_2)\,p_1)\,\frac{2\,M_V}{q^2}\,A_0^{P\to V}(q^2) + \cdots\,. \qquad (1)$$

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- The vector and pseudoscalar *decay constants* $f_{V,P}$ govern the meson–vacuum matrix elements of $j_\mu^{(5)}$:

$$\langle 0|j_\mu|V(q)\rangle = f_V\, M_V\, \varepsilon_\mu(q)\ , \qquad \langle 0|j_\mu^5|P(q)\rangle = \mathrm{i}\, f_P\, q_\mu\ . \tag{2}$$

In terms of the above quantities, the contributions of the *poles* residing at the masses M_{V_R} or M_{P_R} of the relevant vector and pseudoscalar resonances, V_R and P_R, to the form factors introduced in Eqs. (1) read

$$F_+^{P\to P'}(q^2) = \frac{g_{PP'V_R}\, f_{V_R}}{2\, M_{V_R}}\, \frac{1}{1-q^2/M_{V_R}^2} + \cdots\ , \qquad V^{P\to V}(q^2) = \frac{(M_V+M_P)\, g_{PVV_R}\, f_{V_R}}{2\, M_{V_R}}\, \frac{1}{1-q^2/M_{V_R}^2} + \cdots\ ,$$
$$A_0^{P\to V}(q^2) = \frac{g_{PP_RV}\, f_{P_R}}{2\, M_V}\, \frac{1}{1-q^2/M_{P_R}^2} + \cdots\ .$$

3 Dispersion formalism relying on relativistic constituent-quark framework

For the actual theoretical computation of the three transition form factors $\mathcal{F}(q^2)$ lying at the core of our strong-couplings study, past experience leads us to trust in the relativistic *constituent-quark* model [1]. Adhering to this conviction requires us to match our currents to those built up by constituent quarks, Q.

- For heavy-quark currents, this can be easily accomplished by use of corresponding form factors $g_{V,A}$:

$$j_\mu = g_V\, \bar{Q}_1\, \gamma_\mu\, Q_2 + \cdots\ , \qquad j_\mu^5 = g_A\, \bar{Q}_1\, \gamma_\mu\, \gamma_5\, Q_2 + \cdots\ .$$

- For light-quark currents, partial axial-current conservation, e.g., renders this rather cumbersome [2].

Following Ref. [3], we use for our model parameter values the constituent-quark masses of Table 1 and

$$g_V = g_A = 1\ .$$

Table 1. Numerical values of the constituent-quark mass parameters m_Q entering in the present investigation [3].

Quark flavour Q	u	d	s	c
Quark mass m_Q (GeV)	0.23	0.23	0.35	1.45

Then, by application of the relativistic dispersion formalism, we are put in the position to represent the leptonic decay constants $f_{P,V}$ in the form of dispersion integrals of *spectral densities* $\rho_{P,V}(s)$ and the transition form factors $\mathcal{F}(q^2)$ by double dispersion integrals of *double spectral densities* $\Delta_{\mathcal{F}}(s_1,s_2,q^2)$,

$$f_{P,V} = \int \mathrm{d}s\, \phi_{P,V}(s)\, \rho_{P,V}(s)\ , \qquad \mathcal{F}(q^2) = \int \mathrm{d}s_1\, \phi_1(s_1) \int \mathrm{d}s_2\, \phi_2(s_2)\, \Delta_{\mathcal{F}}(s_1,s_2,q^2)\ , \tag{3}$$

involving the *wave functions* of pseudoscalar and vector mesons taking part in the studied reactions [4]

$$\phi_{P,V}(s) = \frac{\pi}{s^{3/4}} \sqrt{\frac{s^2-(m_1^2-m^2)^2}{2\,[s-(m_1-m)^2]}}\, w_{P,V}\!\left(\frac{(s-m_1^2-m^2)^2-4\, m_1^2\, m^2}{4\, s}\right)\ , \qquad \int \mathrm{d}k\, k^2\, w_{P,V}^2(k^2) = 1\ .$$

The spectral densities may be derived from one-loop Feynman graphs, of the kind shown in Fig. 1. For the radial meson wave functions $w_{P,V}(k^2)$, it has become customary to assume simple Gaussian shapes:

$$w_{P,V}(k^2) \propto \exp\left(-\frac{k^2}{2\,\beta_{P,V}^2}\right)\ . \tag{4}$$

The necessary input parameter values, drawn from a variety of related sources, are collected in Table 2.

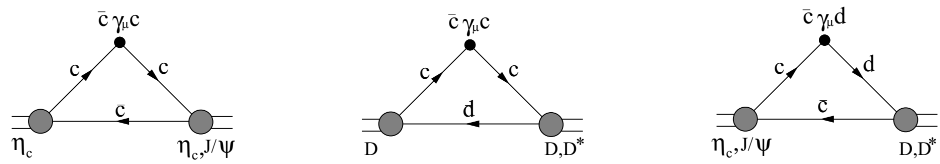


Figure 1. $M_1 > M_2$ meson–meson transitions for $M_{1,2} = \eta_c, J/\psi, D, D^*$, mediated by the *constituent*-quark vector currents $\bar{c}\,\gamma_\mu\, c$ or $\bar{c}\,\gamma_\mu\, d$: Feynman graphs yielding the one-loop contributions to the spectral density $\Delta_{\mathcal{F}}(s_1, s_2, q^2)$.

Table 2. Numerical values of the relevant parameters of the charmed mesons $D_{(s)}^{(*)}$ and the charmonia η_c and J/ψ: meson mass M, leptonic decay constant f defined in Eq. (2), and slope β fixing the width of the Gaussian (4) [5].

Meson	D	D^*	D_s	D_s^*	η_c	J/ψ
M (GeV)	1.87	2.010	1.97	2.11	2.980	3.097
f (MeV)	206 ± 8	260 ± 10	248 ± 2.5	311 ± 9	394.7 ± 2.4	405 ± 7
β (GeV)	0.475	0.48	0.545	0.54	0.77	0.68

In order to extract the strong couplings under discussion, we determine the momentum dependence for the transition form factors $\mathcal{F}(q^2)$ sufficiently far off their resonances R, where $R = V$ for $\mathcal{F} = F_+^{P\to P'}$ and $\mathcal{F} = V^{P\to V}$ but $R = P$ for $\mathcal{F} = A_0^{P\to V}$, *interpolate* our results by means of the simple parametrization

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - q^2/M_R^2\right)\left(1 - \sigma_1\, q^2/M_R^2 + \sigma_2\, q^4/M_R^4\right)}\,, \qquad \operatorname{Res}\mathcal{F}(M_R^2) = \frac{\mathcal{F}(0)}{1 - \sigma_1 + \sigma_2}\,, \tag{5}$$

governed by the three parameters $\sigma_{1,2}$ and $\mathcal{F}(0)$, and extrapolate this momentum dependence of $\mathcal{F}(q^2)$ to the resonance region $q^2 \approx M_R^2$. From the resulting *residues* of the meson poles at $q^2 = M_R^2$, the strong couplings are found by factorizing off all known quantities such as meson masses and decay constants. In case a particular strong coupling shows up in residues of resonance poles of more than one transition form factor, for such multipresent strong coupling an optimized estimate is obtained by a combined fit.

4 Strong couplings among three η_c or J/ψ mesons: $\eta_c\eta_c J/\psi$ and $\eta_c J/\psi J/\psi$

An illustration of such *multipresence* is given by the strong coupling $g_{\eta_c\eta_c\psi}$ [6], with appearance in both

- the residue of $F_+^{\eta_c\to\eta_c}(M_\psi^2)$ arising from the vector current $\bar{c}\,\gamma_\mu\, c$ coupling, with strength f_ψ, to J/ψ and
- the residue of $A_0^{\eta_c\to\psi}(M_{\eta_c}^2)$ from the axial-vector current $\bar{c}\,\gamma_\mu\,\gamma_5\, c$ that couples, with strength f_{η_c}, to η_c:

$$\operatorname{Res} F_+^{\eta_c\to\eta_c}(M_\psi^2) = g_{\eta_c\eta_c\psi}\,\frac{f_\psi}{2\,M_\psi}\,, \qquad \operatorname{Res} A_0^{\eta_c\to\psi}(M_{\eta_c}^2) = g_{\eta_c\eta_c\psi}\,\frac{f_{\eta_c}}{2\,M_\psi}\,.$$

After such detailed preliminaries, the way how to proceed should be pretty plain: We determine the meson wave-function parameters $\beta_{\eta_c,\psi}$ of both charmonia such that the dispersion representation (3) of their decay constants, $f_{\eta_c,\psi}$, reproduces the observed values. With these meson vertices at our disposal, we deduce the strong coupling of interest, for each meson–meson transition where this strong coupling enters (for the case of the $\eta_c\eta_c J/\psi$ coupling, see Fig. 2), from the spectral representation (3) of the form factor corresponding to the respective transition. Thus, our η_c and J/ψ PPV and PVV couplings are [6]

$$g_{\eta_c\eta_c\psi} = 25.8 \pm 1.7\,, \qquad g_{\eta_c\psi\psi} = (10.6 \pm 1.5)\ \text{GeV}^{-1}\,.$$

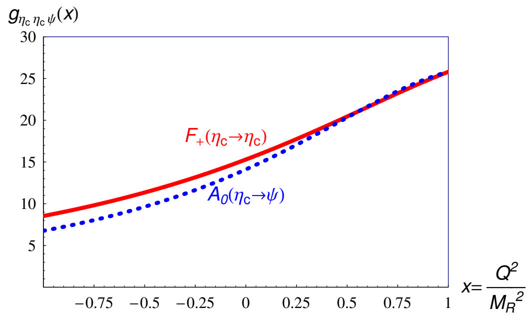
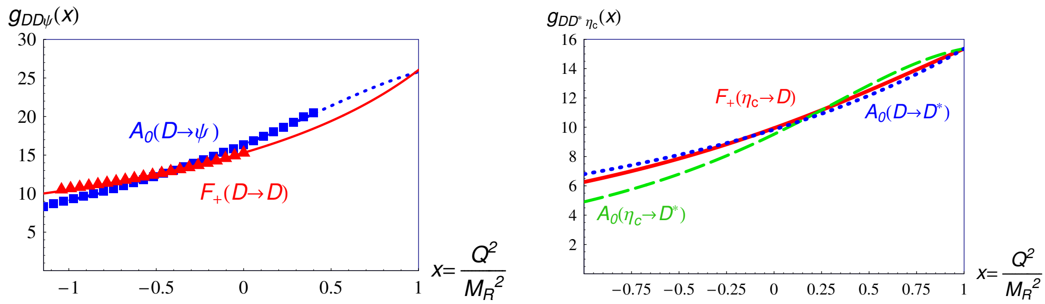


Figure 2. Off-shell strong coupling $g_{\eta_c\eta_c\psi}(x)$ as function of $x \equiv \frac{q^2}{M_R^2}$ for $\eta_c > \eta_c$ (red) or $\eta_c > J/\psi$ (blue) transitions.

5 Strong three-meson couplings of the charmonia J/ψ or η_c to $D_{(s)}$ and $D^*_{(s)}$

Along the same route as in Sec. 4 — and by taking into account, in addition, constituent-quark currents involving one d or s quark — we may likewise discuss the strong couplings of J/ψ or η_c to non-strange ($D^{(*)}$) or strange ($D_s^{(*)}$) charmed mesons. Combining the options sketched in Fig. 3, our findings are [6]

$$\begin{aligned} g_{DD\psi} &= 26.04 \pm 1.43 , & g_{DD^*\psi} &= (10.7 \pm 0.4)\ \text{GeV}^{-1} , \\ g_{DD^*\eta_c} &= 15.51 \pm 0.45 , & g_{D^*D^*\eta_c} &= (9.76 \pm 0.32)\ \text{GeV}^{-1} , \\ g_{D_sD_s\psi} &= 23.83 \pm 0.78 , & g_{D_sD_s^*\psi} &= (9.6 \pm 0.8)\ \text{GeV}^{-1} , \\ g_{D_sD_s^*\eta_c} &= 14.15 \pm 0.52 , & g_{D_s^*D_s^*\eta_c} &= (8.27 \pm 0.37)\ \text{GeV}^{-1} . \end{aligned} \qquad (6)$$



(a) (b)

Figure 3. Off-shell strong couplings $g_{DD\psi}$ and $g_{DD^*\eta_c}$ to $D^{(*)}$ mesons (with resonances indicated by circumflexes): dependences on $x \equiv \frac{q^2}{M_R^2}$ of (a) $g_{D\hat{D}\psi}(x) = \frac{2\,M_\psi}{f_D}\,(1-x)\,A_0^{D>\psi}(q^2)$ (blue) and $g_{DD\hat{\psi}}(x) = \frac{2\,M_\psi}{f_\psi}\,(1-x)\,F_+^{D>D}(q^2)$ (red), got with (lines) and without (symbols) interpolation, and of (b) $g_{D\hat{D}^*\eta_c}(x)$ (red), $g_{DD^*\hat{\eta}_c}(x)$ (blue) and $g_{\hat{D}D^*\eta_c}(x)$ (green).

6 Observations, comparison with findings of different origin, conclusions

Our application of a relativistic dispersion technique, relying on the constituent-quark model, to strong three-meson couplings of quarkonia among each other and to $D_{(s)}^{(*)}$ mesons yields some crucial insights:

1. The interpolation of our numerical transition-form-factor results found at low q^2 by means of the ansatz (5) yields values of the resonance-mass fit parameter M_R close to the experimental meson masses; this can be interpreted as confirmation of the presence of the poles expected at $q^2 \approx M_R^2$.

2. The replacement of the d quark by the s quark (or vice versa) in the quark currents mediating any transition under study enables us to arrive at some estimate of the size of SU(3)-breaking effects. Inspecting Eq. (6), we get a change of the strong couplings under consideration by roughly 10%.

3. Table 3 confronts, for the strong couplings between *charmonia and* $D_{(s)}^{(*)}$ *mesons*, the predictions of our dispersive constituent-quark formalism with (available) corresponding figures from QCD sum rules [7–9]; surprisingly, the latter prove to be smaller than our results [6] by a factor of two.

Table 3. Strong couplings of three mesons which involve one J/ψ: findings of the present relativistic constituent quark-model framework [6], confronted with available results [7–9] extracted from the QCD sum-rule approach.

Coupling	$g_{DD\psi}$	$g_{DD^*\psi}$ (GeV^{-1})	$g_{D_sD_s\psi}$	$g_{D_sD_s^*\psi}$ (GeV^{-1})
Quark model [6]	26.04 ± 1.43	10.7 ± 0.4	23.83 ± 0.78	9.6 ± 0.8
QCD sum rules	11.6 ± 1.8 [7]	4.0 ± 0.6 [7]	11.96 ± 1.34 [8]	4.30 ± 1.53 [9]

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