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QCD sum rules for 70-plet baryons

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Abstract. QCD sum rules for the 70-plet baryons are proposed, using unitary pattern of these rules. As it was previously shown, all correlators of QCD sum rules for the 56-plet depend only on two generalized functions of the F and D types, characteristic for the unitary symmetry. The same is shown to be valid for the 70-plet baryons.

1 Introduction

Magnetic moments of the octet $1/2^-$ baryons attracted some attention recently in quark models as well as in the QCD sum rule formalism. In the quark model these baryons are assigned usually to the 70-plet representation of SU(6), in contrast to the $1/2^+$ baryons of the 56-plet (see [1, 2] and the references therein).

Within the QCD sum rules formalism one treats mostly $1/2^+$ baryons starting from pioneering works of Ioffe et al. [3]. The baryon interpolating currents reflect baryon wave functions of the non-relativistic 56-plet quark model (NRQM). The same reasoning is maintained also for the $1/2^-$ baryons (see, e.g., ref.[4]).

In a series of papers we have shown that QCD sum rules for couplings and magnetic moments of the octet $1/2^+$ baryons reproduce unitary symmetry results in terms of F and D quantities ([5] and references therein). In fact, any QCD sum rule for magnetic moments of the octet baryons exhibits the unitary structure. We take a little old-fashioned QCD sum rule [6] as a simple example. If the correlator is

$$\Pi_B = i\int d^4x e^{ipx}\langle 0|T\{\eta_B(x)\bar{\eta}_B(0)\}|0\rangle_\gamma \ , \tag{1}$$

T being time ordering operator, γ means external electromagnetic field while η_B is interpolating current with the quantum numbers of the B baryon, then, say, for Σ^0 it could be written (we put to shorten formulae $m_s = 0$ disregarding differences between vacuum expectation values of various flavors) as:

$$\begin{aligned}\frac{1}{4}\tilde{\beta}^2_\Sigma\Big[\mu(\Sigma^0) + AM^2\Big]e^{-m^2_\Sigma/M^2} &= (e_u+e_d)\Big\{\frac{M^6}{4L^{4/9}} - \frac{a^2L^{4/9}}{144}[2-(\kappa-2\xi)] \\ &- \frac{\chi a^2}{24L^{4/27}}(M^2 - \frac{m_0^2}{8L^{4/9}}) + \frac{bM^2}{96L^{4/9}}\Big\} + e_s\Big\{-\frac{a^2L^{4/9}}{24} + \frac{bM^2}{192L^{4/9}}\Big\} \\ &= e_u\Pi_1(u,d,s;M^2) + e_d\Pi_1(d,u,s;M^2) + e_s\Pi_2(u,d,s;M^2). \end{aligned} \tag{2}$$

Here M^2 is the Borel parameter, the vev's are $a = -(2\pi)^2\langle\bar{q}q\rangle$=0.45 GeV^3, $b = \langle g_s^2G^2\rangle$=0.5 GeV^4, $m_0^2 = 0.8$ GeV^2, $(\kappa - 2\xi)$=1.1, $\chi = -3.3$, while L=ln(M^2/Λ^2_{QCD})/ln(μ^2/Λ^2_{QCD}) with Λ_{QCD}=100

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MeV and normalization point μ=500 MeV. In the limit of degenerated masses of quarks and vacuum expectation values (vev) it reduces to the relation [7]:

$$\mu_{\Sigma^0} = (e_u + e_d)F + e_s(D - F). \tag{3}$$

Note that this relation goes to the NRQM formula with $F = 2/3$ and $D = 1$ and $e_q \to \mu_q$, and from the other side it reproduces the unitary formula by putting values of e_q's for all octet baryons but Λ. The Λ quantities are reproduced easily with our relation [8].

2 NRQM and quark-diquark model for 70-plet

With the 70-plet octet baryons we loose this correspondence. QCD sum rules remain essentially the same [9, 10] (only with γ_5 inside, to change baryon parity) while NRQM gives $\mu^{70}_{\Sigma^0} = 1/3(\mu_u + \mu_d + \mu_s)$ (see [1] and references therein). We propose a way to arrive at the desired QCD sum rules in accord with the NRQM limit [11] with the help of quark-diquark model which has led previously to Eq.(3). Let us take quark wave functions of the 70-plet baryon [1] (and we take Σ^0) as (subindices 1,2 mean $\uparrow,\downarrow$, correspondingly):

$$|\Sigma^{*0}\rangle = |\Sigma^0\rangle_i + |\Sigma^0\rangle_{ii};$$

$$|\Sigma^0\rangle_i = \frac{1}{2\sqrt{3}}|(u_1s_1d_2 + d_1s_1u_2 - s_1u_1d_2 - s_1d_1u_2 + u_1d_2s_1 + d_1u_2s_1$$
$$-u_1s_2d_1 - d_1s_2u_1 + s_2u_1d_1 + s_2d_1u_1 - u_2d_1s_1 - d_2u_1s_1)\psi^{'}(0^+)\rangle;$$

$$|\Sigma^0\rangle_{ii} = \frac{1}{6}|(u_1s_1d_2 + d_1s_1u_2 + s_1u_1d_2 + s_1d_1u_2 + u_1d_2s_1 + d_1u_2s_1 \tag{4}$$
$$+u_1s_2d_1 + d_1s_2u_1 + s_2u_1d_1 + s_2d_1u_1 + u_2d_1s_1 + d_2u_1s_1$$
$$-2u_1d_1s_2 - 2d_1u_1s_2 - 2s_1u_2d_1 - 2s_1d_2u_1 - 2u_2s_1d_1 - 2d_2s_1u_1)\psi^{"}(0^+)\rangle;$$

($\psi^{'}(0^+)$ and $\psi^{"}(0^+)$ are spatial wave functions [1]). To obtain Λ wave functions, and also to control the formulae, we use relations with the exchange of quarks:

$$|\Sigma^{*0}\rangle|_{ds} + |\Sigma^{*0}\rangle|_{us} = -|\Sigma^{*0}\rangle, \quad |\Sigma^{*0}\rangle|ds - |\Sigma^{*0}\rangle|_{us} = -\sqrt{3}|\Lambda^*\rangle.$$

We follow previous reasoning of our quark-diquark model assuming that photon distinguishes between the diquark of (quasi)similar quarks (qq) and single quark Q [7]. That is, we introduce 4 matrix elements of the new operator $\hat{\omega}_q$:

$$\langle q_\uparrow q_\uparrow, Q_\downarrow|\hat{\omega}_q|q_\uparrow q_\uparrow, Q_\downarrow\rangle = w_{11}, \qquad \langle q_\uparrow q_\downarrow, Q_\uparrow|\hat{\omega}_q|q_\uparrow q_\downarrow, Q_\uparrow\rangle = w_{12},$$
$$\langle q_\uparrow q_\uparrow, Q_\downarrow|\hat{\omega}_Q|q_\uparrow q_\uparrow, Q_\downarrow\rangle = v_{11}, \qquad \langle q_\uparrow q_\downarrow, Q_\uparrow|\hat{\omega}_Q|q_\uparrow q_\downarrow, Q_\uparrow\rangle = v_{12}. \tag{5}$$

For the 56-plet one obtains for $\Sigma^0(ud, s)$ with the modified magnetic moment operator $e_q\hat{\omega}_q\sigma_3^q$ (disregarding $\hat{\omega}_q$ one just returns to NRQM):

$$\mu_{\Sigma^0} = (1/6)[4(e_u + e_d)w_{11} + e_s(4v_{11} - 2v_{12})], \tag{6}$$

which, with $w_{11} = 3/2F$, $w_{12} = D$, $v_{11} = D$, $v_{12} = 3F - D$ [7], yields Eq.(3). Instead, for the 70-plet one obtains:

$$\mu^{70}_{\Sigma^0} = 1/3[(e_u + e_d)w_{11} - e_s(v_{11} - 2v_{12})] = (e_u + e_d + 4e_s)\frac{1}{2}F - e_sD. \tag{7}$$

But now, with $F = 2/3$ and $D = 1$, we return to the 70-plet NRQM result $\mu_{\Sigma^0} = 1/3(\mu_u + \mu_d + \mu_s)$, while putting values of e_q's we arrive at magnetic moments of 70-plet baryons in terms of F and D constants [11]:

$$\mu_{N^{*+}} = \mu_{\Sigma^{*+}} = \frac{1}{3}D, \qquad \mu_{\Sigma^{*-}} = \mu_{\Xi^{*-}} = (-F + \frac{1}{3}D),$$
$$\mu_{N^{*0}} = \mu_{\Xi^{*0}} = -2\mu_{\Sigma^{*0}} = 2\mu_{\Lambda^{*0}} = (F - \frac{2}{3}D). \tag{8}$$

We also note that the transition magnetic moment is equal to zero for the 70-plet, $\langle\Lambda^*|\hat{\mu}^{70}|\Sigma^{*0}\rangle$=0. It can be proved also with the relation in ref.[8].

3 QCD sum rules for 70-plet magnetic moments

We take it as a base for the construction of QCD sum rules for the magnetic moments (here for $1/2^+$),

$$\Pi^{70}(u,d,s) = e_u\tilde{\Pi}_1(u,d,s) + e_d\tilde{\Pi}_1(d,u,s) + e_s\tilde{\Pi}_2(u,d,s),$$

with $\tilde{\Pi}_1(u,d,s) = 1/2\Pi_1(u,d,s)$, $\quad \tilde{\Pi}_2 = \Pi_1(d,u,s)+\Pi_2(u,d,s)$. These formulae yield correct NRQM limit and maintain F, D pattern of the QCD sum rules. We proceed with these QCD sum rules in more detail, and take as an example the QCD sum rules for the baryon magnetic moments [6]. The Eq.(2) according to our formulae Eq.(7) transforms itself for the 70-plet into:

$$\begin{aligned} \frac{1}{4}\tilde{\beta}^2_{\Sigma^*}[\mu(\Sigma^{*0}) + A'M^2]e^{-m^2_{\Sigma^*}/M^2} \quad &= \quad \frac{1}{2}(e_u + e_d + 4e_s) \\ &\quad \Big\{\frac{M^6}{4L^{4/9}} - \frac{a^2L^{4/9}}{144}\left[2 - (\kappa - 2\xi)\right] - \frac{\chi a^2}{24L^{4/27}}\left(M^2 - \frac{m_0^2}{8L^{4/9}}\right) \\ &+ \quad \frac{bM^2}{96L^{4/9}}\Big\} - e_s\Big\{-\frac{a^2L^{4/9}}{24} + \frac{bM^2}{192L^{4/9}}\Big\}. \end{aligned} \tag{9}$$

It is seen that sum rules (2) and (9) differ drastically. Model predictions for both sum rules and also some results for $1/2^-$ baryons are given in Table 1.

4 Conclusions

As a result we have shown the way to construct QCD sum rules for the magnetic moments of the 70-plet octet baryons which have the correct NRQM limit. Octet baryons in the 70-plet are analyzed in a way similar to those of the 56-plet. Magnetic moments are written in terms of the D and F quantities characteristic for octet coupling. The main formulae for the magnetic moments are written in such a way as to obtain the NRQM results as well as unitary symmetry ones. In NRQM in the 70-plet octet Tansition magnetic moment is shown to be zero. Borel QCD sum rules are constructed for the magnetic moments of the 70-plet octet. Comparison with some other results in the field are made.

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Table 1. Magnetic moments of 70-plet and 56-plet $1/2^{\pm}$ baryons.

$\mu(B)$	56-plet $1/2^+$ [6]	This Work Eq.(9)	$1/2^-$[12] Lattice	NCQM [2]	$1/2^-$ [10]	Eq.(8) F=2.5 D=4.5
$\mu(p)$	2.72	0.83	-1.8	1.894	1.4	1.5
$\mu(n)$	-1.65	0.22	-1.0	-1.284	-0.54	-0.5
$\mu(\Sigma^+)$	2.52	0.70	-0.6	1.814	1.8	1.5
$\mu(\Sigma^-)$	-1.13	-1.13	1.0	-0.689	-1.1	-1.0
$\mu(\Sigma^0)$		0.11	0.1	0.820	0.4	0.25
$\mu(\Lambda)$	-0.50	-0.11	-0.1		-0.26	-0.25
$\mu(\Xi^0)$	-0.89	-0.43	-0.5	-0.990	-0.55	-0.5
$\mu(\Xi^-)$	-1.18	-1.18	0.8	-0.315	-1.2	-1.0

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