

Thermodynamics of Resonant Scalars in AdS/CFT and implications for QCD^{*}

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Abstract. We explore the thermodynamics of a simple 5D Einstein-dilaton gravity model with a massive scalar field, with asymptotically AdS behavior in the UV. The holographic renormalization is addressed in details, and analytical results are obtained at high temperatures. We study the power corrections predicted by the model, and compare with lattice data in the deconfined phase of gluodynamics. Finally, it is discussed the role played by the conformal anomaly for integer values of the dimension of the condensate dual to the scalar field.

1 Introduction

The physics of the quark-gluon plasma has turned out to be more difficult than initially expected. Quantum Chromodynamics (QCD) is asymptotically free at high energies and temperatures, and this allows us to use perturbative methods to study its thermodynamic properties in this regime. Nevertheless close, and above, the phase transition non perturbative effects become very important, and usual perturbative approaches are not applicable. Lattice calculations at finite temperature show that there is a first order phase transition in gluodynamics for $N_c \geq 3$ [1, 2] and a crossover in QCD [3]. In addition, they seem to indicate the existence of power corrections in temperature in the deconfined phase that cannot be explained within a naive perturbative approach, see e.g. [4–6].

New techniques have appeared recently that can help in a true understanding of the underlying mechanism of strongly coupled gauge theories. The postulated gauge/gravity duality between a strongly coupled super Yang-Mills theory and a weakly coupled supergravity theory in $\text{AdS}_5 \times S^5$ space [7], became a powerful tool in recent years to understand the non perturbative dynamics of QCD close to the phase transition. While a rigorous top-down approach is still far from giving any experimental prediction, several bottom-up models that can quantitatively reproduce many results of QCD in the low energy limit have been proposed in the literature. The core of the bottom-up scenario is to find a reasonable non-conformal metric of the AdS_5 space which mimics the physical properties of QCD, see e.g. [8–10]. In this work we use this approach, and explore the thermodynamics of a simple holographic model based on 5D Einstein-dilaton gravity with a massive scalar field. Some related works in the literature are e.g. [11–15].

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2 Power corrections as a signal of non perturbative effects in the thermodynamics of QCD

Gluedynamics is invariant under scale and conformal transformations at the classical level. This classical invariance is, however, broken by quantum corrections due to the necessary regularization of the UV divergences. The divergence of the dilatation current equals the trace of the energy-momentum tensor T_{μ}^{μ} [16] yielding the *trace anomaly* [17]. At finite temperature, the energy density ε and the pressure P enter as [18]

$$\langle T_{\mu}^{\mu} \rangle = -\varepsilon + 3P = -\frac{\beta(g)}{2g} \langle (F_{\mu\nu}^a)^2 \rangle, \quad (1)$$

where $F_{\mu\nu}$ is the field strength tensor and $\beta = \mu \partial g / \partial \mu$ is the beta function. While in the conformal limit, i.e. at very high temperatures, one has $\varepsilon = 3P$, a non-vanishing value of the trace anomaly provides a measure of the interaction. This means that $\varepsilon - 3P$ is crucial to understand the deconfinement process, as the non perturbative nature of QCD plays a prominent role in this regime.

The equation of state of QCD has been studied for a long time by using different methods. A naive weak coupling expansion turns out to be poorly convergent in the regime of temperatures close to the phase transition, see e.g. [19]. It has been proposed in the literature several methods to resum the perturbative expansion, one of the most popular being the Hard Thermal loop (HTL), currently computed up to 3-loops order [20]. However, all these methods fail to reproduce the lattice data for the trace anomaly in the regime $T_c \lesssim T \lesssim (2.5 - 3)T_c$, which corresponds to a strongly interacting quark-gluon plasma picture, see Fig. 1 (left).

It was studied in [2, 5, 6, 21] the possible existence and characterization of power corrections in the equation of state of gluedynamics. We show in Fig. 1 (right) a plot of the lattice data of the trace anomaly of gluedynamics $(\varepsilon - 3P)/T^4$ as a function of $(T_c/T)^2$. A clear linear behavior appears for temperatures $T \gtrsim 1.13T_c$. This behavior contradicts perturbation theory which contains no powers but only logarithms in the temperature, a feature shared by HTL and other resummation techniques. Basically, this result can be summarized in the formula

$$\Delta(T) \equiv \frac{\varepsilon - 3P}{T^4} = a_{\Delta}(\mu_T) + b_{\Delta} \left(\frac{T_c}{T} \right)^2, \quad (2)$$

where $a_{\Delta}(\mu_T) \sim 1/\log T$ is the perturbative contribution. A fit of the lattice data with this formula, using $a_{\Delta}(\mu_T) = \Delta_{\text{HTL}}(\mu_T)$, leads to $b_{\Delta} = (3.57 \pm 0.28)$, see [21]. This motivates the need of new methods to account for non perturbative effects.

3 Thermodynamics of AdS/QCD

The gauge/gravity duality is nowadays a powerful tool to study the strong coupling properties of gauge theories, and in particular of QCD, either at zero or finite temperature. One of the most important applications of this duality is the physics of strongly coupled plasmas. In particular, we can study the thermodynamics of a field theory (or QCD) from the classical computation of the thermodynamics of black holes in the gravity dual. This duality can be expressed in the form

$$S_{\text{Black Hole}}(T) = \frac{A(r_{\text{horizon}})}{4G_D} \longleftrightarrow S_{\text{QCD}}(T). \quad (3)$$

The entropy of a black hole can be obtained classically from the famous Bekenstein-Hawking entropy formula, where $A(r_{\text{horizon}})$ is the area of the black hole horizon, see e.g. [12, 13, 22, 23]. In conformal AdS₅ the metric has a horizon in the bulk space at $r_T = \pi \ell^2 T$ where ℓ is the size of the AdS space, and the entropy scales like $S_{\text{Black Hole}} \propto r_T^3 \propto T^3$. However, in order to have a reliable extension of this duality to SU(N_c) Yang-Mills theory, the first task is to control the breaking of conformal invariance.

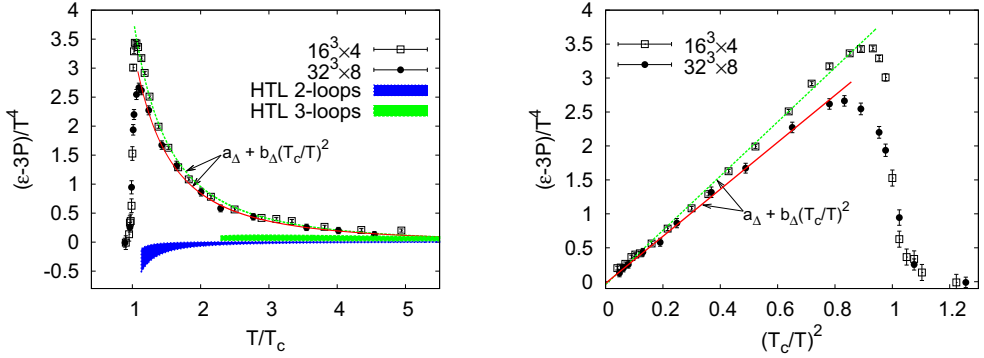


Figure 1. Trace anomaly density $(\varepsilon - 3P)/T^4$ as a function of T (left) and $1/T^2$ (right) (in units of T_c). Lattice data are from [1] for gluodynamics with $N_c = 3$. The fits use Eq. (2) with $\mu_T \sim T$ and b_Δ as adjustable parameters. See [6, 21] for further details.

3.1 The 5D Einstein-dilaton model at finite temperature

The bottom-up approach turns out to be quite useful to study the thermodynamics of QCD in the strongly coupling regime. It is based on the building of a gravity dual of QCD, including the main properties of QCD. One of the most successful models within this approach is the 5D Einstein-dilaton model, with the Euclidean action [12]

$$S = \frac{1}{2\kappa^2} \int d\rho d^4x \sqrt{G} (-R + G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + 2V(\Phi)) - \int_{\rho=v} d^4x \sqrt{\gamma} 2K + S_{\text{boundary}}, \quad (4)$$

where κ^2 is the 5D Newton constant related to the 5D Planck mass by $\kappa^2 = 1/(2M^3)$, Φ is a scalar field, and v is a cut-off surface near the boundary. Taking the limit $v \rightarrow 0$, one takes the surface to the AdS_5 boundary. The first boundary term is the usual Gibbons-Hawking term, while the second one S_{boundary} might depend on $\Phi'(v)$. The introduction of a scalar breaks conformal invariance, and the form of the scalar potential $V(\Phi)$ is phenomenologically adjusted to describe some observables of QCD, like for instance the trace anomaly, see Fig. 1. There are in the literature many different proposals for the dilaton potential, but in this work we will restrict to the much simpler form

$$V(\Phi) = -\frac{6}{\ell^2} + \frac{1}{2} m^2 \Phi^2, \quad m^2 \ell^2 = \Delta(\Delta - 4), \quad 0 \leq \Delta \leq 4, \quad (5)$$

where ℓ is the radius of AdS_5 , and $m^2 < 0$ is a tachyonic mass. A black hole solution of the form ¹

$$ds_{\text{Black Hole}}^2 = G_{\mu\nu} dx^\mu dx^\nu = \frac{\ell^2}{4\rho^2} d\rho^2 + \frac{\ell^2}{\rho} g_{\tau\tau}(\rho) d\tau^2 + \frac{\ell^2}{\rho} g_{xx}(\rho) d\vec{x}^2, \quad (6)$$

with an horizon in the extra coordinate $g_{\tau\tau}(\rho_h) = 0$, allows to describe the deconfined phase of the field theory dual. From the *AdS/CFT dictionary*, the model is the holographic dual of a Conformal Field Theory (CFT) with a deformation which breaks conformal invariance, i.e.

$$\mathcal{L} = \mathcal{L}^{\text{CFT}} + \lambda \mathcal{O}, \quad (7)$$

where \mathcal{O} is an operator dual of the scalar field with $\dim \mathcal{O} = \Delta$, and λ is the source of the operator with $\dim \lambda = 4 - \Delta$. In the following we will focus on the so-called *Resonant Scalars*, i.e. the cases $\Delta \in \mathbb{Z}$, as they have some special properties.

¹We work in Fefferman-Graham coordinates, as this allows for a clean treatment of the renormalization. See e.g. [24].

3.2 Holographic Renormalization

In order to study the holographic renormalization of the model, one should perform a near boundary expansion of the fields. If one considers the case $\Delta \notin \mathbb{Z}$, then the expansion of the scalar field reads

$$\Phi(\rho) = \phi_0 \cdot \rho^{(4-\Delta)/2} + \phi_{(2\Delta-4)} \cdot \rho^{\Delta/2} + \dots \quad \rho \rightarrow 0. \quad (8)$$

When $\Delta > 2$ then ϕ_0 is interpreted as the source λ , and $\phi_{(2\Delta-4)}$ as the condensate $\langle \mathcal{O} \rangle$ in Eq. (7).² From now on we will focus on the cases $\Delta = 1$ and 3, as their physical implications for QCD turn out to be highly relevant. In this case the expansion of the scalar field and the metric read

$$\Phi(\rho) = \chi \cdot \rho^{1/2} + \frac{\chi^3}{6} \rho^{3/2} \log \rho + \psi \cdot \rho^{3/2} + \dots, \quad (9)$$

$$g_{xx}(\rho) = 1 - \frac{\chi^2}{6} \rho + g_{(4)xx} \rho^2 - \frac{\chi^4}{24} \rho^2 \log \rho + \dots, \quad (10)$$

$$g_{\tau\tau}(\rho) = 1 - \frac{\chi^2}{6} \rho - 3g_{(4)xx} \rho^2 - \chi \psi \rho^2 + \frac{\chi^4}{9} \rho^2 - \frac{\chi^4}{24} \rho^2 \log \rho + \dots, \quad (11)$$

where, roughly speaking, χ is interpreted as the source and ψ as the condensate when $\Delta = 3$ (the opposite when $\Delta = 1$). The $\log \rho$ contributions appear as a consequence of a resummation of higher orders in Eq. (8) that become dominant in the limit $\Delta \rightarrow 1$ or 3. Regularity at the horizon $\rho = \rho_h$ allows to fix the coefficients ψ and $g_{(4)xx}$. Without loss of generality we can choose $S_{\text{boundary}} = 0$, and then the counterterms needed to regularize the action read

$$S_{\text{ct}} = \frac{1}{\kappa^2 \ell} \int_{\rho=v} d^4x \sqrt{\gamma(v)} \left(3 + \frac{\Phi^2(v)}{2} + \frac{\Phi^4(v)}{12} \log(v\mu_0^2) \right), \quad (12)$$

where μ_0 is an arbitrary renormalization scale. The variation of the regularized action $S_{\text{reg}} = S_{\text{on-shell}}[v]$ reads

$$\delta S_{\text{reg}} = -\frac{1}{\kappa^2 \ell} \int_{\rho=v} d^4x \sqrt{\gamma} 2v \Phi'(v) \delta \Phi(v) + \dots, \quad (13)$$

and from here one can compute the condensate as the variation of the action with respect to the source,

$$\langle \mathcal{O}_3 \rangle = \frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta \chi} = \lim_{v \rightarrow 0} \left(\frac{1}{v^{-1/2}} \frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta \Phi(v)} \right) = \frac{\ell^3}{\kappa^2} \left(-2\psi - \frac{\chi^3}{3} + \frac{\chi^3}{3} \log \mu_0^2 \right). \quad (14)$$

For $\Delta = 1$ the computation is the same but the interpretation is different. In this case $\dim \chi = 1$ so that χ should be the condensate, i.e. $\langle \mathcal{O}_1 \rangle = \chi$. This means that $\frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta \chi}$ is interpreted as the renormalized source of the operator \mathcal{O}_1 .

3.3 Conformal Anomaly

By using the previous ingredients, one can obtain general formulas for the thermodynamics of a CFT deformed by $\int d^4x \chi \mathcal{O}_3$. The energy density and pressure are obtained from the $T^{\tau\tau}$ and T^{xx} components of the energy-momentum tensor. A straightforward computation from the regularized action leads to [25]

$$\varepsilon = \langle T^{\tau\tau} \rangle = -2 \frac{\delta S_{\text{ren}}}{\delta g_{\tau\tau}(v)} = \frac{\ell^3}{\kappa^2} \left(6g_{(4)xx} + \chi \psi - \frac{5}{24} \chi^4 + \frac{\chi^4}{12} \log \mu_0^2 \right), \quad (15)$$

$$P = \langle T^{xx} \rangle = -2 \frac{\delta S_{\text{ren}}}{\delta g_{xx}(v)} = \frac{\ell^3}{\kappa^2} \left(2g_{(4)xx} + \chi \psi - \frac{\chi^4}{72} - \frac{\chi^4}{12} \log \mu_0^2 \right). \quad (16)$$

²In the case $\Delta < 2$ the interpretation of these quantities is the inverse: ϕ_0 is the condensate and $\phi_{(2\Delta-4)}$ is the source.

From these expressions one can easily check that the following Ward identity for the trace anomaly

$$-\varepsilon + 3P = \langle T_\mu^\mu \rangle = (\Delta - 4)\chi \langle \mathcal{O}_\Delta \rangle + \mathcal{A}, \quad (17)$$

is fulfilled when $\Delta = 3$, with $\mathcal{A} = -\frac{\ell^3}{6\kappa^2}\chi^4$. We have checked that Eq. (17) is also valid when $\Delta = 1$ and 2. The term \mathcal{A} is the holographic conformal anomaly, it is a zero temperature contribution and it appears only when $\Delta \in \mathbb{Z}$.³ This anomaly has been studied also in [26] in a resonant theory in AdS₄.

3.4 Solution for the Equation of State

It is possible to obtain some analytical results for the equation of state by making a high temperature expansion. The temperature and entropy density are obtained from the metric of Eq. (6) as

$$T = \frac{1}{2\pi} \sqrt{2\rho_h g''_{\tau\tau}(\rho_h)}, \quad s = \frac{2\pi}{\kappa^2} \left(\frac{\ell^2}{\rho_h} g_{xx}(\rho_h) \right)^{3/2}, \quad (18)$$

respectively. We will focus on the case $\Delta = 3$. From the equations of motion one can easily observe that the general dependence of the entropy density is $s = T^3 \sigma(\chi/T)$, where σ is some specific function. By applying the standard thermodynamic relations one gets that the pressure can be written as $P = T^4 H_1(\chi/T) + \chi^4 H_2(\chi/\mu_0)$, where H_2 is a zero temperature contribution. From a computation of $-\varepsilon + 3P$ with this formula and its comparison with Eq. (17), one can make the identification

$$\frac{\partial P}{\partial \chi} = -\langle \mathcal{O}_3 \rangle, \quad -\frac{\chi^5}{\mu_0} H_2' \left(\frac{\chi}{\mu_0} \right) = \mathcal{A}. \quad (19)$$

The dependence in μ_0 arises only from the conformal anomaly. It is possible to make a computation of $H_1(\chi/T)$ at high temperature as a power series expansion in χ/T . The result for the trace anomaly is⁴

$$\varepsilon - 3P = \frac{\pi^4 \ell^3 T^4}{2\kappa^2} \left[-4c_3 \left(\frac{\chi}{\pi T} \right)^2 + \mathcal{O}(\bar{\chi}^4 \log \bar{\chi}) \right] - \frac{2\ell^3}{3\kappa^2} \chi^4 \log \frac{\chi}{\mu_0}, \quad \bar{\chi} \equiv \chi/T, \quad (20)$$

where $c_3 = -\Gamma[\frac{3}{4}]^2 / (\Gamma[\frac{1}{4}] \Gamma[\frac{5}{4}]) \approx -0.457$ with $\Gamma[n]$ the Euler gamma function. We find that the thermodynamics of a deformation $\int d^4x \chi \mathcal{O}_3$ leads to a power series expansion in T^2 which can explain very precisely the lattice data of the trace anomaly in the deconfined phase of gluodynamics, as shown in Fig. 1. From the fit of Eq. (2) one gets $\chi = \frac{\kappa}{\ell^{3/2}} (0.629 \pm 0.025) T_c$.⁵

4 Conclusions and outlook

The non perturbative behavior of gluodynamics/QCD near and above T_c is characterized by power corrections in T^2 . While perturbative methods fail to reproduce the non perturbative regime of QCD at finite temperature, AdS/CFT serves as a powerful tool to study this regime by using much simpler classical gravity techniques. In this work we have studied a simple holographic model of conformal

³The expressions of \mathcal{A} for other values of Δ will be presented in [25].

⁴If one chooses the Newton constant to reproduce the Stefan-Boltzmann limit of gluodynamics at high temperatures, $P_{\text{gluons}} \rightarrow (N_c^2 - 1) \frac{\pi^2}{45} T^4$, then one has $\frac{\ell^3}{\kappa^2} = (N_c^2 - 1) \frac{2}{45\pi^2}$.

⁵While these power corrections in T^2 were studied in a similar model in [14], we have performed here an analysis that shows the consistency of the result with general formulas like Eq. (17). Our analysis conveniently accounts for the holographic conformal anomaly, and this is only possible with a careful holographic renormalization when $\Delta \in \mathbb{Z}$.

symmetry breaking in 5D based on dilatons that allows to account for power corrections in the deconfined phase of QCD in an elegant way. The model is dual to a CFT deformed by an operator \mathcal{O} . We find that the power corrections in the equation of state can be conveniently described with a deformation of $\dim \mathcal{O} = 3$. Finally, we have studied in details the holographic renormalization of the theory, and the role of the holographic conformal anomaly when $\dim \mathcal{O} \in \mathbb{Z}$.

We plan to extend the computation of the equation of state of the model for any value of Δ in the interval $0 \leq \Delta \leq 4$. This will be performed numerically in the whole regime of temperatures, as well as analytically at high and low temperatures. There remain also some open questions, like the physical interpretation of the condensate of dimension 3. These and other issues will be addressed in [25].

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