The multidimensional nucleon structure

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Abstract. We discuss different kinds of parton distributions, which allow one to obtain a multidimensional picture of the internal structure of the nucleon. We use the concept of generalized transverse momentum dependent parton distributions and Wigner distributions, which combine the features of transverse-momentum dependent parton distributions and generalized parton distributions. We show examples of these functions within a phenomenological quark model, with focus on the role of the spin-spin and spin-orbit correlations of quarks.

1 Introduction

During the past two decades, a lot of attention has been paid to generalized parton distributions (GPDs) [1–7] and to transverse-momentum dependent parton distributions (TMDs) [8–12]. Those objects are of particular interest because they describe the three-dimensional parton structure of hadrons — the distribution of the parton's longitudinal momentum and transverse position in the case of GPDs, and the distribution of the parton's longitudinal momentum and transverse momentum in the case of TMDs. Even though GPDs and TMDs already are quite general entities, the maximum possible information about the (two-) parton structure of strongly interacting systems is encoded in GTMDs [13–15]. GTMDs can reduce to GPDs and to TMDs in certain kinematical limits, and therefore they are often denoted as *mother distributions*. Further limits/integrations reduce them to collinear parton distribution functions and form factors. The Fourier transform of GTMDs can be interpreted as Wigner distributions [16–18], the quantum-mechanical analogue of classical phase-space distributions. A classification of GTMDs for quarks (through twist-4) for a spin-0 target was given in Ref. [13], followed by a corresponding work for a spin- $\frac{1}{2}$ target [14]. In Ref. [15], the counting of quark GTMDs was independently confirmed, and a complete classification of gluon GTMDs was provided as well. Gluon GTMDs appear when describing high-energy diffractive processes like vector meson production or Higgs production [19–21] in the so-called k_T -factorization in Quantum Chromodynamics. Recently, it was also suggested that the Wigner distributions of small- x gluons are closely related to the color dipole scattering amplitude [22].

For a comprehensive review of both the actual status and future developments in the theoretical and experimental investigation of GPDs, TMDs and Wigner functions we refer to a series of papers collected in a "Topical issue on the 3-D structure of the nucleon" [23]. Here, we review some aspects

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related to modelling and imaging of the nucleon structure [24]. In Sect. 2, we revisit the definition of the Wigner distributions obtained by Fourier transform of the GTMDs to the impact-parameter space. We identify all the possible correlations between target polarization, quark polarization and quark orbital angular momentum (OAM) encoded in these phase-space distributions, and we show results for a few selected examples. In Sect. 3 and 4, we discuss some of the complementary information encoded in GPDs and TMDs, in particular with regards to the information on the quark OAM. Concluding remarks are given in the final section.

2 Wigner distributions

The quark GTMD correlator is defined as [14, 15]

$$W_{\Lambda'\Lambda}^{ab} \equiv \int dk^- \int \frac{d^4z}{(2\pi)^4} e^{ik \cdot z} \langle P + \frac{\Lambda}{2}, \Lambda' | \bar{\psi}_b(-\frac{z}{2}) \mathcal{W} \psi_a(\frac{z}{2}) | P - \frac{\Lambda}{2}, \Lambda \rangle, \quad (1)$$

where $\mathcal{W}$ is an appropriate Wilson line ensuring color gauge invariance, k is the quark average four-momentum conjugate to the quark field separation z , and $|p, \Lambda\rangle$ is the spin-1/2 target state with four-momentum p and light-front helicity Λ . A proper definition of the GTMDs should include also a soft-factor contribution [25]. However, it is not relevant for the following multipole analysis. At leading twist, one can interpret

$$W_{\vec{S}\vec{S}^q} = \frac{1}{8} \sum_{\Lambda', \Lambda} (1 + \vec{S} \cdot \vec{\sigma})_{\Lambda\Lambda'} \text{Tr}[W_{\Lambda'\Lambda} \Gamma_{\vec{S}^q}] \quad (2)$$

with $\Gamma_{\vec{S}^q} = \gamma^+ + S_L^q \gamma^+ \gamma_5 + S_T^{qj} i\sigma_T^{j+} \gamma_5$, as the GTMD correlator describing the distribution of quarks with polarization $\vec{S}^q$ inside a target with polarization $\vec{S}$ [26].

The corresponding phase-space distribution is obtained by an appropriate Fourier transform [18]

$$\rho_{\vec{S}\vec{S}^q}(x, \mathbf{k}_T, \mathbf{b}_T; \hat{P}, \eta) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\Delta_T \cdot \mathbf{b}_T} W_{\vec{S}\vec{S}^q}(P, k, \Delta) \Big|_{\xi=0}, \quad (3)$$

where $x = k^+/P^+$ and $\mathbf{k}_T$ are, respectively, the longitudinal fraction and transverse component of the quark average momentum, $\mathbf{b}_T$ is the quark average impact parameter conjugate to the transverse-momentum transfer Δ_T , $\xi = -\Delta^+/2P^+$ is the fraction of longitudinal momentum transfer, and $\eta = +1$ ($\eta = -1$) indicates a future- (past-) pointing Wilson line. This phase-space distribution can be interpreted semi-classically as giving the quasi-probability of finding a quark with polarization $\vec{S}^q$, transverse position $\mathbf{b}_T$ and light-front momentum (xP^+ , $\mathbf{k}_T$) inside a spin-1/2 target with polarization $\vec{S}$ [18]. Thanks to the hermiticity properties of the GTMD correlator (2), these phase-space distributions are always real-valued [27], consistently with their quasi-probabilistic interpretation. Considering the various polarization configurations, one finds that there are 16 Wigner distributions just like there are 16 possible GTMDs [14, 15]. By construction, the real and imaginary parts of these GTMDs have opposite behavior under naive time-reversal transformation. Similarly, each Wigner distribution can be separated into naive T-even and T-odd contributions, $\rho_{\vec{S}\vec{S}^q} = \rho_{\vec{S}\vec{S}^q}^e + \rho_{\vec{S}\vec{S}^q}^o$, with $\rho_{\vec{S}\vec{S}^q}^{e,o}(x, \mathbf{k}_T, \mathbf{b}_T; \hat{P}, \eta) = \pm \rho_{\vec{S}\vec{S}^q}^{e,o}(x, \mathbf{k}_T, \mathbf{b}_T; \hat{P}, -\eta) = \pm \rho_{\vec{S}\vec{S}^q}^{e,o}(x, -\mathbf{k}_T, \mathbf{b}_T; -\hat{P}, \eta)$. We can interpret the T-even contributions as describing the intrinsic distribution of quarks inside the target, whereas the naive T-odd contributions describe how initial- and final-state interactions modify these distributions. The relativistic phase-space distribution is linear in $\vec{S}$ and $\vec{S}^q$

$$\rho_{\vec{S}\vec{S}^q} = \rho_{UU} + S_L \rho_{LU} + S_L^q \rho_{UL} + S_L S_L^q \rho_{LL} + S_T^i (\rho_{TiU} + S_L^q \rho_{TiL}) + S_T^{qi} (\rho_{UTi} + S_L \rho_{LTi}) + S_T^i S_T^{qj} \rho_{TiTj}, \quad (4)$$

Table 1. Correlations between target polarization (S_L, S_T), quark polarization (S_L^q, S_T^q) and quark OAM (ℓ_L^q, ℓ_T^q) encoded in the various phase-space distributions ρ_X , with $X = UU, LU, \dots$

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

and can further be decomposed into two-dimensional multipoles in both $\mathbf{k}_T$ and $\mathbf{b}_T$ spaces [28]. While there is no limit in the multipole order, parity and time-reversal give constraints on the allowed multipoles. It is therefore more appropriate to decompose the Wigner distributions ρ_X as follows

$$\rho_X(x, \mathbf{k}_T, \mathbf{b}_T; \hat{\mathbf{P}}, \eta) = \sum_{m_k, m_b} \rho_X^{(m_k, m_b)}(x, \mathbf{k}_T, \mathbf{b}_T; \hat{\mathbf{P}}, \eta), \quad (X = UU, LU, \dots), \quad (5)$$

$$\rho_X^{(m_k, m_b)}(x, \mathbf{k}_T, \mathbf{b}_T; \hat{\mathbf{P}}, \eta) = B_X^{(m_k, m_b)}(\hat{\mathbf{k}}_T, \hat{\mathbf{b}}_T; \hat{\mathbf{P}}, \eta) C_X^{(m_k, m_b)}[x, \mathbf{k}_T^2, (\mathbf{k}_T \cdot \mathbf{b}_T)^2, \mathbf{b}_T^2], \quad (6)$$

where $B_X^{(m_k, m_b)}$ are the basic (or simplest) multipoles allowed by parity and time-reversal symmetries, multiplied by the coefficient functions $C_X^{(m_k, m_b)}$ which depend on P and T-invariant variables only. The couple of integers (m_k, m_b) gives the basic multipole order in both $\mathbf{k}_T$ and $\mathbf{b}_T$ spaces. All the contributions ρ_X can be understood as encoding all the possible correlations between target and quark angular momenta, see Table 1.

In order to obtain a two-dimensional representation of the Wigner functions, we integrate these phase-space distributions over x and discretize the polar coordinates of $\mathbf{b}_T$. We also set $\eta = +1$ and choose $\hat{\mathbf{P}} = \vec{e}_z = (0, 0, 1)$ so that $\hat{\mathbf{b}}_T = (\cos \phi_b, \sin \phi_b, 0)$ and $\hat{\mathbf{k}}_T = (\cos \phi_k, \sin \phi_k, 0)$. The resulting transverse phase-space distributions are then represented as sets of distributions in $\mathbf{k}_T$ -space

$$\rho_X(\mathbf{k}_T | \mathbf{b}_T) = \int dx \rho_X(x, \mathbf{k}_T, \mathbf{b}_T; \hat{\mathbf{P}} = \vec{e}_z, \eta = +1) |_{\mathbf{b}_T \text{ fixed}} \quad (7)$$

with the origin of axes lying on circles of radius $|\mathbf{b}_T|$ at polar angle ϕ_b in impact-parameter space. In particular, we choose to represent only eight points in impact-parameter space lying on a circle with radius $|\mathbf{b}_T| = 0.4$ fm and $\phi_b = k\pi/4$ with $k \in \mathbb{Z}$. Also, for a better legibility, the $\mathbf{k}_T$ -distributions are normalized to the absolute maximal value over the whole circle in impact-parameter space.

In the following, we will show results from the light-front constituent quark model (LFCQM) [29], which has been extensively applied to calculate form factors [30], TMDs [31–34] and GPDs [35–38], showing a typical accuracy of 30%. Since our purpose is simply to illustrate the multipole structure, we computed only the naive T-even contributions in this model. The naive T-odd contributions are obtained by extracting the coefficient functions from the naive T-even part and multiplying them by the appropriate basic multipoles. Therefore, the global sign of these naive T-odd contributions has been chosen arbitrarily. Only a proper calculation including initial- and/or final-state interactions can determine these global signs.

As an example, we discuss here the Wigner distribution for unpolarized quark in a longitudinal polarized target. We find only two phase-space distributions

$$\rho_{LU}^e = \rho_{LU}^{(1,1)}, \quad \rho_{LU}^o = \rho_{LU}^{(2,2)}, \quad (8)$$

Figure 1. Naive T-even (left) and T-odd (right) contributions to the transverse phase-space distribution ρ_{LU} . Light and dark regions represent, respectively, positive and negative domains of the phase-space distributions.

which are represented in Fig. 1. The corresponding basic multipoles are

$$S_L^q B_{UL}^{(1,1)}(\hat{k}_T, \hat{b}_T; \hat{P}, \eta) = -S_L^q \epsilon_T^{ij} D_k^i D_b^j = S_L^q (\hat{b}_T \times \hat{k}_T)_L, \quad (9)$$

$$S_L^q B_{UL}^{(2,2)}(\hat{k}_T, \hat{b}_T; \hat{P}, \eta) = -\eta S_L^q \epsilon_T^{ij} Q_k^i Q_b^j = \eta S_L^q (\hat{b}_T \times \hat{k}_T)_L (\hat{k}_T \cdot \hat{b}_T). \quad (10)$$

None of these survive integration over $\mathbf{k}_T$ or $\mathbf{b}_T$. Both therefore represent completely new information which is not accessible *via* GPDs or TMDs at leading twist. The $\mathbf{k}_T$ -dipole in $\rho_{LU}^{(1,1)}$ signals the presence of a net longitudinal component of quark OAM correlated with the target longitudinal polarization S_L . By reversing the target longitudinal polarization S_L , one reverses also the orbital flow. The coefficient function $C_{LU}^{(1,1)}$ then gives the amount of longitudinal quark OAM in a longitudinally polarized target $\langle S_L \ell_L^q \rangle$ [18]. Similarly, the contribution $\rho_{LU}^{(2,2)}$ gives the difference of radial flows between quarks with opposite OAM $\langle S_L \ell_L^q \rangle$, with the coefficient function $C_{LU}^{(2,2)}$ representing in some sense the strength of the $\langle S_L \ell_L^q \rangle$ -dependent part of the force felt by the quark due to initial- and final-state interactions. As a matter of fact, the quark OAM can be obtained from the ρ_{LU} distribution as [18, 27, 39]

$$l_z^q = \int d\mathbf{x} d^2\mathbf{k}_T d^2\mathbf{b}_T (\mathbf{b}_T \times \mathbf{k}_T)_z \rho_{UL}(x, \mathbf{k}_T, \mathbf{b}_T; \hat{P} = \vec{e}_z, \eta). \quad (11)$$

Since we are working here with a staple-like gauge link, l_z^q is the canonical version of quark OAM [40]. On the other side, using a straight gauge link, Eq. (11) gives the kinetic OAM appearing in the Ji decomposition, as discussed in Sect. 4. From Fig. 1, we also note the ρ_{LU}^o cannot contribute to the quark OAM, and hence the quark OAM is η -independent [41–43].

3 Transverse Momentum Dependent Parton distributions

The forward limit ($\Delta = 0$) of the correlator (1) corresponds to a transverse momentum dependent quark-quark correlator that can be parametrized in terms of 8 TMDs, depending on the possible polarizations for the nucleon and quark (see Table 2). They are a natural extension of collinear parton

Table 2. Twist-2 TMDs, for different quark (columns) and nucleon (rows) polarizations.

		quark pol.			
		U	T_x	T_y	L
nucleon pol.	U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	0
	T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
	T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
	L	0	$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

distributions from one to three dimensions in momentum space, being functions of both the longitudinal quark momentum fraction x and the square of the transverse momentum k_T . The knowledge of TMDs allows us to build tomographic images of the inner structure of the nucleon in momentum space. The multipole pattern in k_T is clearly visible in Table 2. The TMDs f_1 , g_{1L} and h_1 give the strength of monopole contributions and correspond to matrix elements without a net change of helicity between the initial and final states. The TMDs $f_{1T}^\perp$, g_{1T} , $h_1^\perp$, and $h_{1L}^\perp$ give the strength of dipole contributions and correspond to matrix elements involving one unit of helicity flip, either on the nucleon side ($f_{1T}^\perp$ and g_{1T}) or on the quark side ($h_1^\perp$ and $h_{1L}^\perp$). Finally, the TMD $h_{1T}^\perp$ corresponds to matrix elements where both the nucleon and quark helicities flip, but in opposite directions. Conservation of total angular momentum tells us that helicity flip is compensated by a change of OAM [31], which manifests itself by powers of k_T/M . The corresponding quadrupole structure in k_T is shown in Fig. 2, as obtained from a LFCQM [31]. The function $h_{1T}^\perp$ has attracted a particular interest, since it has been suggested, on the basis of some quark-model calculations [44, 45], that it may be related to the quark OAM:

$$\mathcal{L}_z^q = - \int dx d^2 k_T \frac{k_T^2}{2M^2} h_{1T}^{\perp q}(x, k_T^2). \quad (12)$$

Figure 2. Quark density in the transverse-momentum plane for transverse polarization of quarks and nucleon in perpendicular directions. Left (right) panel is for up (down) quarks.

However, Eq. (12) is not a rigorous expression, and holds only in a restricted class of quark models [26, 46]. In general, no direct quantitative relations between OAM and TMDs should actually be expected as the former represents a correlation between parton position and momentum, whereas the latter only provide information about the momentum distribution. Nevertheless, TMDs do provide some indirect information about the OAM content of the nucleon, for example, by constraining the nucleon light-front wave functions that are eigenstates of the OAM [27].

4 Generalized Parton Distributions

When integrating the correlator (1) over $\mathbf{k}_T$, one obtains an off-diagonal quark-quark correlator that can be parametrized in terms of combinations of 8 GPDs (see Table 3). Comparing the entries in Tables 3 and 2, one notices that the same multipole pattern appears, where the role of $\mathbf{k}_T$ in Table (2) is played by Δ_T in Table (3). Although direct links between GPDs and TMDs cannot exist [14, 47], this correspondence leads us to expect correlations between signs or similar orders of magnitude [48, 49]. The GPDs are functions of x , ξ , and $t = -\Delta^2$. Just like GTMDs and Wigner distributions are connected by Fourier transform at $\xi = 0$, one also finds a direct connection between GPDs at $\xi = 0$ and parton distributions in impact-parameter space. The multipole structure in $\mathbf{b}_T$ is the same as in Table 3, with the only difference that there are no polarization effects in the impact-parameter distributions for longitudinally polarized quarks in a transversely polarized proton and vice versa, since they are forbidden by time reversal. For example, the distribution of unpolarized quarks in a transversely polarized target reads as

$$q_X(x,\mathbf{b}_T) \;=\; \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\Delta_T\cdot\mathbf{b}_T} H(x,0,-\Delta_T^2) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\Delta_T\cdot\mathbf{b}_T} E(x,0,-\Delta_T^2).$$

This quark density is given by the sum of a nucleon spin-independent contribution related to the GPD H and a nucleon spin-dependent contribution from the GPD E , corresponding to monopole and dipole distributions in impact-parameter space, respectively. The dipole contribution introduces a large distortion perpendicular to both the nucleon spin and the momentum of the proton, with opposite sign for u and d quarks, as shown in Fig. 3. While details of the GPD E are not known, its x -integral is equal to the Pauli form factor F_2 , which allows one to constrain the average deformation model-independently to the contribution from each quark flavor q to the nucleon anomalous magnetic moment.

Even before 3D imaging was introduced, there was great interest in GPDs due to their connection with the form factors of the energy-momentum tensor and thus to the angular momentum (spin plus

Table 3. Combinations of twist-2 GPDs, as defined in Ref. [26], for different quark (columns) and nucleon (rows) polarizations.

		quark pol.			
		U	T_x	T_y	L
nucleon pol.	U	$\mathcal{H}$	$i\frac{\Delta_y}{2M}\mathcal{E}_T$	$-i\frac{\Delta_x}{2M}\mathcal{E}_T$	0
	T_x	$i\frac{\Delta_y}{2M}\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}$
	T_y	$-i\frac{\Delta_x}{2M}\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}$
	L	0	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

Figure 3. Quark density in the impact parameter space $\mathbf{b}_T$ for unpolarized quarks in a transversely polarized nucleon along the $\hat{x}$ direction, as obtained within a LFCQM [37]. The left (right) panel is for up (down) quark.

orbital) carried by quarks of flavor q as described by the Ji relation [2]

$$J_q = \frac{1}{2} \int_0^1 dx x \left[H_q(x, \xi, 0) + E_q(x, \xi, 0) \right], \quad (13)$$

where the ξ -dependence on the r.h.s. disappears upon x -integration. The quark kinetic OAM is then obtained by subtracting the quark spin contribution, which is given by the first Mellin moment of the quark helicity distribution.

5 Conclusions

We discussed the leading-twist quark Wigner distributions in the nucleon, introducing a multipole analysis in the four-dimensional transverse phase space. In this approach, the multipole structures are constrained by parity and time-reversal symmetries and are multiplied by coefficient functions which depend on P and T-invariant variables only. This representation provides a clear interpretation of all the amplitudes in terms of the possible correlations between target and quark angular momenta in the transverse phase space, and it gives a convenient basis to make a direct connection with TMDs in transverse-momentum space and GPDs in impact-parameter space. In particular, we discussed results for the Wigner distributions of unpolarized quarks in a longitudinally polarized nucleon that allow us to calculate the phase-space average of the quark OAM. Other ways to access information about the quark OAM from TMDs and GPDs have been also discussed, showing results within a light-front constituent quark model.

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