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Hadronic inputs to the $(g-2)_\mu$ puzzle

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Abstract. There is a long standing discrepancy of 3 – 4 standard deviations between the direct measurement and the Standard Model prediction of the anomalous magnetic moment of the muon $(g-2)_\mu$. While new direct measurements have been proposed to clarify the situation, theory calculations are completely limited by the accuracy of the hadronic contributions to $(g-2)_\mu$. In order to increase their precision, experimental information can be used as input. In this presentation we will discuss recent and future measurements of relevant hadronic cross sections and transition form factors.

1 Introduction

The anomalous magnetic moment of the muon, or rather the muon anomaly $a_\mu = (g-2)_\mu/2$, is one of the most precisely measured observables of the Standard Model. It has been measured directly in storage ring experiments since the 1960s with continuously increasing accuracy. The latest measurement was performed in Brookhaven and yielded $a_\mu^{exp} = (11659208.9 \pm 6.3) \times 10^{-10}$ [1]. In the experiment, a beam of longitudinally polarized muons was circulated in a storage ring with a precisely known, constant magnetic field. The polarization of the muons is achieved through the parity violating decay $\pi^+ \to \mu^+\nu_\mu$ of previously produced pions. The muons circulate in the magnetic field $\vec{B}$ of the storage ring with the cyclotron frequency $\vec{\omega}_c$ and their spin processes with the frequency $\vec{\omega}_s$, resulting in the anomalous precession frequency

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m_\mu}\left[a_\mu\vec{B} - \left(a_\mu\frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right].$$

The dependence on $\vec{\omega}_a$ on the electrical field $\vec{E}$ can be canceled by the choice of the value of $\gamma = 29.3$, the so called "magic momentum" of the muons of 3.094 GeV/c. The muons eventually decay through their weak decay $\mu^+ \to e^+\bar{\nu}_\mu\nu_e$, where the direction of the emitted positron is correlated with the spin direction of the muon. Thus, the e^+ rate is modulated with $\vec{\omega}_a = -a_\mu\frac{q\vec{B}}{m_\mu}$ and is used to determine a_μ.

Two new measurements have been proposed at Fermilab, Chicago, USA [2] and at J-PARC, Tokai, Japan [3], which aim at a fourfold improvement of the current experimental precision. While the Fermilab experiment is a continuation of the BNL experiments with major improvements to the apparatus, the J-PARC approach does not rely on the "magic momentum", but avoids the need of focusing electric fields by using an ultra-cold muon beam. Thus, the two experiments are fully complementary.

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The motivation for new direct measurements comes from a difference of three to four standard deviations between the direct measurement and the Standard Model prediction of a_μ, which might be considered as a hint for New Physics. The theory prediction of the anomalous magnetic moment of the muon consists of three parts. The largest part comes from the QED contribution a_μ^{QED}. It has been determined with high accuracy in perturbation theory up to the 10^{th} order [4]. Another contribution comes from the weak interaction a_μ^{weak}. It is small and its uncertainty is well under control by means of perturbation theory [5]. The third contribution stems from the strong interaction a_μ^{had}. At the relevant energies, perturbative approaches cannot be applied to determine the strong contribution. Thus, though its absolute value is comparably small, the uncertainty of a_μ^{had} completely dominates the total uncertainty of the Standard Model prediction of a_μ [6, 7]. However, experimental information can be used as input to the calculations in order to improve the prediction.

Currently, the largest contribution to the absolute value and to the uncertainty of a_μ^{had} comes from the hadronic vacuum polarization. It can be related to hadronic cross sections measured in e^+e^- annihilations, which can be used to systematically improve the prediction. The second largest contribution to a_μ^{had} comes from the hadronic Light-by-Light scattering. In contrast to the hadronic vacuum polarization, it cannot be directly related to experimental observables. So far, hadronic models have been used to evaluate the contribution, using experimental data on transition form factors of light mesons for validation. The absolute value of the hadronic Light-by-Light contribution is comparably small. Its uncertainty is model dependent and can be as large as the uncertainty due to the hadronic vacuum polarization.

2 The hadronic Vacuum Polarization

The hadronic vacuum polarization can be directly related to the experimentally measurable hadronic cross sections $\sigma_{e^+e^-\to had}$ at e^+e^- colliders. The contribution to the anomalous magnetic moment of the muon can be calculated in leading order with the dispersion integral

$$a_\mu^{hVP,LO} = \int ds K(s)\sigma_{e^+e^-\to had}(s),$$

where $K(s)$ is the Kernel function, which is proportional to $1/s$ [8]. The energy dependence of the hadronic cross sections and of the Kernel function make cross sections measured at low center of mass energies most important for $a_\mu^{hVP,LO}$ [7]. In fact, the evaluation of the dispersion integral shows that $a_\mu^{hVP,LO}$ is determined to more than 75% by the hadronic cross sections measured below 1 GeV/c^2, which are dominated by the ρ-resonance and its decay into two pions. For the determination of the uncertainty $\delta a_\mu^{hVP,LO}$ the cross sections of higher pion multiplicities, like $e^+e^- \to \pi^+\pi^-\pi^0$ or $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$, and the production of kaon pairs play an almost equally important role. Thus, the measurement of these hadronic cross sections with high accuracy allows to systematically improve the theory prediction of a_μ^{had} with experimental input.

Two methods have been established to measure hadronic cross sections at e^+e^- machines. Traditionally, the energy dependence of hadronic cross sections is studied in energy scan experiments, where the accelerators have to be adjusted for the measurement of every individual data point. Alternatively, events with Initial State Radiation (ISR) can be exploited. The irradiated photon reduces the effective center-of-mass energy, which is available for the reaction and, thus, allows to study processes at energies different than the nominal energy of the machine. This method has proven to be very effective at high luminosity machines, which run at fixed energies, like ϕ, τ-charm, and B factories.

As the cross section $\sigma(e^+e^- \to \pi^+\pi^-)$ dominates $a_\mu^{hVP,LO}$, precise knowledge is mandatory. In the past, it has been measured by several experiments over a wide energy range. Reference [7] provides a compilation of the relevant measurements. The most precise measurements with systematic uncertainties of less than 1% have been provided from the BaBar collaboration [9, 10], the KLOE

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collaboration [11–13], and the CMD, CMD-2, and SND collaborations [14–19]. Since the accuracy of the Novosibirsk experiments is currently limited by statistics, the world average is dominated by the BaBar and KLOE results. However, even though both experiments claim a total uncertainty of less than 1%, their results differ by more than 5% at and above the ρ-peak region. The difference reflects directly in the uncertainty of $a_\mu^{hVP,LO}$.

Recently, the BESIII collaboration measured $\sigma(e^+e^- \to \pi^+\pi^-)$ in the mass region between 600 and 900 MeV/c^2[20]. As the mass region contains the dominating ρ-peak, it makes up for 70% of the total $\pi^+\pi^-$ contribution and determines $a_\mu^{hVP,LO}$ to 50%. The BESIII measurement is based on 2.9 fb^{-1} of data taken at the $\psi(3770)$ resonance and exploits the ISR technique to determine the cross section at lower energies. The dominating background from radiative di-muon production is successfully suppressed by applying an artificial neural network for particle identification. The careful evaluation of all systematic uncertainties results in a total uncertainty of the cross section measurement of 0.9%. Fig. 1 shows the comparison of $a_\mu^{hVP,LO}$ evaluated in the mass range of this measurement with the corresponding results from BaBar and KLOE. Good agreement with the KLOE measurements is observed, while the BaBar result is shifted to higher values of $a_\mu^{hVP,LO}$. The BESIII result confirms the deviation between the direct measurement of a_μ and its Standard Model prediction on the level of three – four standard deviations.

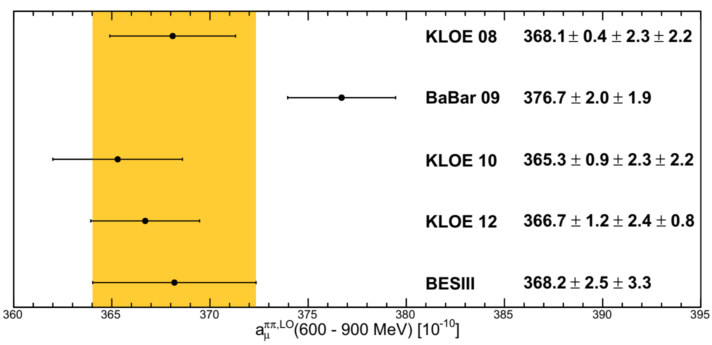

Figure 1. Comparison of $a_\mu^{\pi\pi,LO}(600 - 900\text{MeV})$ from the measurements of KLOE, BaBar, and BESIII. The yellow band shows the error of the BESIII result. Error bars include statistical and systematical errors. Figure is taken from Ref. [20].

3 The hadronic Light-by-Light scattering

Improving the accuracy of the hadronic Light-by-Light scattering contribution to the muon anomaly a_μ^{hLbL} is a more complex endeavour. It cannot be related directly to a measurable quantity. The interactions of real and virtual photons with virtual mesons, which are part of a_μ^{hLbL} can be described at lowest energies by means of chiral perturbation theory and at highest energies by means of perturbative QCD. The relevant energy scale for the muon anomaly, however, is the intermediate energy regime, where perturbative approaches fail.

The classic approaches to evaluate a_μ^{hLbL} are based on hadronic models. The two calculations mostly referred to are the so-called "Glasgow Consensus" $a_\mu^{hLbL} = (10.5 \pm 2.6) \times 10^{-10}$ [21] and the more conservative estimate of Jegerlehner and Nyffeler $a_\mu^{hLbL} = (11.6 \pm 4.0) \times 10^{-10}$ [6]. The difference in the uncertainties δa_μ^{hLbL} is model dependent. Experimental input is used for the validation of the models. The relevant observables are transition form factors (TFF), which describe the coupling of photons and hadrons.

The information for which mesons the TFF should be measured in which energy range can be deduced from theoretical evaluations of the hadronic Light-by-Light term. A counting scheme proposed by de Rafael [22] shows that pion loops and pseudoscalar meson exchange are the dominating contributions to a_μ^{hLbL}. Thus, data on the TFF of single π^0 and of pion pairs are needed. Knecht and Nyffeler provide a two-dimensional integral representation of the π^0 exchange contribution to the hadronic Light-by-Light term [23]. In the representation, the term factorizes into so-called universal weighting functions $w_i(Q_1, Q_2)$, which contain the model independent parts of a_μ^{hLbL,π^0}, and functions $f_i(Q_1, Q_2)$, which contain the form factor information and, thus, the model dependence of different approaches. From the momentum dependence of the $w_i(Q_1, Q_2)$, it has been shown for VMD-like models that for the determination of a_μ^{hLbL} it is most relevant to determine the TFF of π^0 for space-like momentum transfers between $0.05 \leq Q^2[\mathrm{GeV}^2] \leq 1.5$.

Experimentally, space-like momentum transfers are accessible in two-photon reactions at e^+e^- colliders. In contrast to the annihilation reaction, the exchange of two photons in the collision allows for the direct production of pseudoscalar, scalar, and tensor states. The cross section for the reaction is directly proportional to the TFF of the produced mesons. The measurement of two-photon reactions is challenging since the cross section is peaked towards small scattering angles of the lepton. To study TFFs at arbitrary values of momentum transfers would require special tagging detectors, which are installed close to the beam pipes. In conventional detector setups at e^+e^- machines these detectors are not available. A trade-off can be found in single-tag measurements, where only one of the two scattered leptons and the produced meson are reconstructed. By energy and momentum conservation the unmeasured lepton is required to be scattered along the beam direction and, thus, to have very small momentum transfer. Instead of a TFF depending on two arbitrary momentum transfers $F(Q_1^2, Q_2^2)$, the dependence on only one momentum transfer $F(Q^2)$ is studied. The information is still valuable for a_μ^{hLbL}, as the TFF should factorize for small energies as $F(Q_1^2, Q_2^2) \sim F(Q_1^2, 0) \times F(0, Q_2^2)$.

Currently, the world data base on the space-like TFF of the π^0 is dominated by the recent measurements of the B-factories, BaBar and Belle [24, 25]. The results cover a wide range of momentum transfers of $4 \leq Q^2[\mathrm{GeV}^2] \leq 40$, and show a discrepancy, which is often referred to as "Belle-BaBar Puzzle". For the evaluation of the hadronic Light-by-Light scattering these data are, however, less important as the provided information is well outside of the relevant region of momentum transfer. At lower momentum transfers only the results from CELLO [26] and CLEO [27] are available. They provide information of the TFF down to $Q^2 = 0.5$ GeV^2. However, they suffer from low statistics.

New measurements of space-like TFF have been announced by the BESIII and KLOE-2 collaborations. At BESIII, 2.9 fb^{-1} of data taken at $\sqrt{s} = 3.773$ GeV/c^2 are used to measure the TFF of π^0 at momentum transfers of $0.3 \leq Q^2[\mathrm{GeV}^2] \leq 3.1$ [28, 29]. Estimates based on Monte Carlo studies suggest an unprecedented statistical accuracy for $Q^2 > 1.5$ GeV^2 and competitive results at higher momentum transfers. The KLOE-2 experiment has been equipped with special tagging detectors, which allow to measure the space-like TFF for $0.01 \leq Q^2[\mathrm{GeV}^2] \leq 0.1$ [30]. The ongoing data taking aims at collecting 5 fb^{-1} at the peak of the ϕ resonance, which allows for a statistical accuracy of the TFF measurement of 6%.

Recently, the Belle collaboration reported the first single-tag measurement of neutral pion pairs [31]. The $\pi^0\pi^0$ system was studied for momentum transfers between $3 \leq Q^2[\mathrm{GeV}^2] \leq 30$, for invariant masses of the pion pairs between $0.5 \leq m_{\pi^0\pi^0}[\mathrm{GeV/c}^2] \leq 2.1$, and with complete coverage of the helicity angle. In a partial wave analysis the $|D_0|^2, |D_2|^2$, and $|S|^2$ wave amplitudes have been determined, and the TFF of the $f_0(980)$ resonance and the three possible helicity states of the $f_2(1270)$ resonance were measured and compared to different predictions [32, 33]. Similar to the measurement of the π^0 TFF by the Belle collaboration, this measurement does not provide information in a Q^2 region relevant for a_μ^{hLbL}. The measurement of the $\pi^+\pi^-$ and $\pi^0\pi^0$ pair production in $\gamma\gamma$ collisions announced by the BESIII collaboration [29] will provide information at $0.2 \leq Q^2[\text{GeV}^2] \leq 2$, for values of $m_{\pi^0\pi^0}$ threshold to 2 GeV/c^2, and with full coverage of the helicity angle.

Despite of the existing and upcoming measurements there remains the issue of model dependence in the determination of a_μ^{hLbL}. In the recent years data driven approaches have been proposed in order to reduce the model dependence. Escribano, Masjuan and colleagues proposed to parameterize the TFFs with Padé approximants and fit the free parameters to data [34, 35]. The method provides an estimate of the systematic uncertainty and allows for space-like and time-like data to be used in order to obtain an analytic description of the TFF, which can be used in the calculations of a_μ^{hLbL}.

The development of another approach has recently been started by theory groups in Bern [36–39] and Mainz [40, 41]. The aim is to provide a data driven estimate of a_μ^{hLbL} based on dispersion relations. The approaches of the two groups are fully complementary in the way they relate the dominating contributions of a_μ^{hLbL} to measurable quantities. The goal is to achieve a reliable error estimate in the order of 10% to 20%. The input from experiments needed in the approaches are the space-like TFF of π^0 for arbitrary virtualities $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ and the partial waves of the two-pion system in $\gamma^*\gamma^* \to \pi\pi$. Since the currently available data cannot provide the required information, the Bern group proposed a scheme to reconstruct the TFF and the partial waves dispersively from available time-like information of hadronic and Dalitz decays of vector mesons and hadronic cross sections in e^+e^- reactions [37]. Among the various available experimental results, which can be evaluated in this reconstruction, also the recent A2 result on $\omega \to \pi^0 e^+e^-$ [42] and the KLOE-2 result on $\phi \to \pi^0 e^+e^-$ [43] become relevant. The latter does not only provide a significant improvement on the precision of the branching ratio, but also provides the first measurement of the time-like TFF.

4 Summary

The current limitation of the Standard Model prediction of a_μ comes from the uncertainties of the hadronic contributions. The dispersive approach to systematically improve the contribution of the hadronic vacuum polarization a_μ^{hVP} requires precision data on hadronic cross sections. The recent measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ by BESIII has yet to be included into the world average. Further high precision measurements of the relevant hadronic cross sections will be provided by the BaBar, KLOE, BESIII, and CMD-3 collaborations.

In order to achieve a reliable error estimate for the hadronic Light-by-light contribution, dispersive approaches have been proposed, which relate the dominating contributions to a_μ^{hLbL} to experimental results on the TFF of π^0 and $\pi\pi$ partial waves. New measurements by KLOE, BESIII, and Belle will provide the data. In the meantime a procedure has been proposed to dispersively reconstruct the information from other, already available experimental inputs. In view of the announced precision of the new direct measurements of a_μ similar efforts have to be made for the contributions of the η and η' meson exchange in a_μ^{hLbL}, which are of the same size [23].

It is considered as a realistic outcome of the ongoing efforts to reduce the uncertainty of the Standard Model prediction of a_μ by a factor two in the near future [44].

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