$K^+$-nucleon amplitudes in the medium below 800 MeV/c

E. Friedman$^{1,*}$

$^1$Racah Institute of Physics, Hebrew University, Jerusalem, Israel

**Abstract.** Simple in-medium meson-nucleon kinematics has been applied recently in calculations of strong interaction effects in kaonic atoms, pionic atoms and elastic scattering of low energy pions by nuclei. More sensitive tests of this approach are possible with $K^+$-nucleus interactions below 800 MeV/c because of the superior penetration of kaons into nuclei. The energy dependence of calculated reaction and total cross sections for $K^+$ on $^6$Li, C, Si and Ca agrees better with experiment when this approach is used. Calculated absolute scales are in full agreement with experiment for the very low density nucleus $^6$Li whereas calculations are $23\pm4\%$ too low for C, Si and Ca, thus quantifying phenomenologically the enhancement in the nuclear medium observed before.

1 Introduction

The present note deals with $K^+$ meson-nucleus interaction at low energies where the total $K^+$-nucleon cross sections are small and the kaon penetrates well into the nucleus. The prime motivation for this work was to test recent approach to the calculation of optical potentials based on in-medium kinematics, used so far in analyses of pionic and kaonic atoms and of elastic scattering of low energy $\pi^\pm$ by nuclei [1]. With the deep penetration of $K^+$ mesons into the nucleus at momenta below 800 MeV/c it was possible to test this approach with greater sensitivity than previously done with $K^-$ and pions.

Previous analyses of experimental $K^+$-nucleus integral cross sections showed that the in-medium $K^+$-nucleon amplitude is enhanced compared to the free interaction in this energy range. The present work reaffirms quantitatively the enhancement.

2 Analysis

Applying the in-medium approach to $K^+$ mesons, the kaon-nucleon scattering amplitude is presented in terms of the Mandelstam variable $s$ in the nuclear medium

$$s = (E_K + E_N)^2 - (\vec{p}_K + \vec{p}_N)^2,$$

where $E_K = m_K + E_{\text{lab}}, E_N = m_N - B_N$. $E_{\text{lab}}$ is the laboratory kinetic energy of the kaon and $B_N$ is an average binding energy of a nucleon. In the nuclear medium $\vec{p}_K + \vec{p}_N \neq 0$ where $\vec{p}_K$ is determined by the beam energy and the optical potential, $\vec{p}_N$ is determined by the nuclear

*Correspondence: elifried@cc.huji.ac.il

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (http://creativecommons.org/licenses/by/4.0/).
environment. Averaging over angles, \((\vec{p}_K + \vec{p}_N)^2 \to (p_K)^2 + (p_N)^2\). For the meson \((p_K)^2\) is obtained by substituting locally
\[
p_K^2/2m_K \to E_{\text{lab}} - \text{Re} \, V_{\text{opt}}^K - V_c,
\]
with \(V_{\text{opt}}^K\) the kaon-nucleus optical potential and \(V_c\) the Coulomb potential. For the nucleon \((p_N)^2\) is taken from the Fermi gas model.

Defining \(\delta \sqrt{s} = \sqrt{s} - E_{\text{th}}\) with \(E_{\text{th}} = m_K + m_N\), then to first order in \(B/E_{\text{th}}\) and \((p/E_{\text{th}})^2\) one gets [2, 3]
\[
\delta \sqrt{s} - \xi_N E_{\text{lab}} = -B_N \rho/\bar{\rho} - \xi_N T_N (\rho/\bar{\rho})^{2/3} + \xi_K \text{Re} \, V_{\text{opt}}^K + V_c (\rho/\rho_0)^{1/3},
\]
with \(\xi_N = m_N/(m_N + m_K)\), \(\xi_K = m_K/(m_N + m_K)\) and \(T_N\) the average kinetic energy of a bound nucleon. \(\rho, \bar{\rho}\) and \(\rho_0\) are the local, average and central nuclear densities, respectively. This value of \(\sqrt{s}\) serves as the argument of the in-medium kaon-nucleon amplitude in constructing the kaon-nucleus optical potential. In another version a further Coulomb potential is subtracted from \(\delta \sqrt{s}\) due to ‘minimal substitution’ (MS) arguments [4].

In this approach the energy of the \(K^+\)-nucleon interaction in the medium depends on the optical potential (and density) whereas the optical potential depends on the amplitude, within a ‘\(t\rho\)’ approach. Therefore a self-consistent solution is required. Good convergence is usually achieved after 4-5 iterations. For \(K^+\) interaction with nucleons in the momentum range of 400-800 MeV/c the relevant energies in the medium are lower than the cm beam energy by up to 30 MeV.

3 Results

![Figure 1](image.png)

Figure 1. Ratios between experimental and calculated reaction (solid lines) and total cross sections (dashed lines) of \(K^+\) on \(^6\)Li. Upper part: using free amplitudes at the relevant cm beam energy. Lower part: using in-medium amplitudes at energies given by Eq. (3), including MS term.

Total cross sections for \(K^+\) on D, \(^6\)Li, C, Si and Ca were measured in the early 1990s at four momenta below 800 MeV/c [5–8]. The same set of transmission measurements was later reanalysed to determine not only total cross sections but also reaction cross sections, that are less dependent
on applied corrections [9]. Comparing total cross sections of $K^+$ on $^6$Li with three times the cross sections on deuteron, it was found that within 2% there are no indications for medium effects in the results for $^6$Li, where the average density is about half of the average density for other nuclei [10, 11]. Consequently in what follows we use $^6$Li as a reference.

Fig. 1 shows ratios between experimental and calculated reaction (solid lines) and total cross sections (dashed lines) of $K^+$ on $^6$Li. Calculations used the isospin-averaged forward scattering amplitudes from the SAID software package [12]. The upper part is for calculations based on using the $K^+$-nucleon forward scattering amplitudes at the cm beam energy. In the lower part the energy of that amplitude depended on the local nuclear density, as obtained from iterative self-consistent solution of Eq. (3) including MS term. It is evident that the absolute scale is reproduced quite well by calculations in both cases, but the dependence on beam momentum is not reproduced by the calculations at the upper part. In contrast, using Eq. (3) (lower part) the dependence on the beam momentum is reproduced at the level of 2-3% as is also the absolute scale of the cross sections. Including corrections due to Pauli correlations made no difference in the case of $^6$Li, presumably because of the low density of this nucleus.

Fig. 2 shows ratios between experimental and calculated reaction and total cross sections of $K^+$ on C, Si, and Ca. Upper panels are for cm beam energies, lower panels are for energies given by Eq. (3), including MS term

**Figure 2.** Ratios between experimental and calculated reaction (left panels) and total cross sections (right panels) of $K^+$ on C, Si, and Ca. Upper panels are for cm beam energies, lower panels are for energies given by Eq. (3), including MS term.
improvement is achieved when Pauli corrections too are included. We note that the results for Si have been scattered more than the results for C and Ca also in previous analyses of the same data. However, the ratios for C and Ca are very close to each other and are essentially independent of momentum to within 2-3%. Averaging these, an enhancement of reaction and total cross sections by 23±4% is established.

4 Discussion

The enhancement of experimental total cross sections on C was already noted in 1985 by Siegel, Kaufmann and Gibbs [14] on the basis of very limited data. They showed that traditional calculations failed to produce the enhancement, thus suggesting more exotic mechanisms, see [3] for additional references. The present data was also analysed in 1997 [10, 11], separately at the different energies, arriving at similar conclusions. Peterson [15] showed in 1999 that the enhancement is reproduced by using in medium an increased average $K^+$-nucleon $S$-wave phase-shift compared to free space, suggesting 'swelling' of bound nucleons. A possible link between the additional reactive content and $K^+$ interactions with two nucleons was explored in [16, 17].

In the present work we were able to handle the four energies together thanks to the energy-dependence of the model Eq. (3). The 23±4% enhancement of cross sections could be reproduced phenomenologically by 40±2% increase of the imaginary part of the in-medium amplitude [3]. This is more than wave function renormalization effects due to partial restoration of chiral symmetry, as suggested recently by Jido [18]. Clearly the observed enhancement is an open problem.

I wish to thank A. Gal for fruitful discussions.

References

[1] A. Gal et al., EPJ Web of Conferences 81, 01018 (2014) and references therein