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Electromagnetic transition form factor and radiative corrections in decays of neutral pions

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Abstract. We briefly summarize experimental and theoretical results on the rare decay $\pi^0 \to e^+e^-$. The notorious $3.3\,\sigma$ discrepancy between the SM prediction and the experimental value provided by KTeV collaboration is discussed in the view of a complete set of NLO QED radiative corrections. We also present the Two-Hadron Saturation (THS) scenario for the PVV correlator and apply it to the decay under discussion. The discrepancy then reduces down to $1.8\,\sigma$.

1 Introduction

The rare decay of a neutral pion, i.e. the process $\pi^0 \to e^+e^-$, drew attention of theorists during last years due to the precise measurement of its branching ratio done by KTeV experiment at Fermilab [1]

$$B^{\text{KTeV}}(\pi^0 \to e^+e^-(\gamma),\, x_{\text{D}} > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,. \tag{1}$$

Soon after it has been shown [2] that the prediction based on the Standard Model (SM) is in $3.3\,\sigma$ disagreement with this measurement. One can immediately think about the possibility that this is a sign for a new physics. But our strategy is to look first properly for a more conventional solution. In what follows we will investigate in detail the radiative corrections and suggest a new model for the electromagnetic transition form factor of a neutral pion.

2 Rare decay $\pi^0 \to e^+e^-$ and radiative corrections

The pure-QED two-loop virtual radiative corrections were calculated in [3]. Since the compensation of the IR divergences is necessary, also the exact bremsstrahlung contributions were additionally taken into account in [4]. The overall size of the corrections was found to be

$$\delta(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%\,, \tag{2}$$

which differs significantly from previous approximate calculations done in [5] and [6], where for $\delta(0.95)$ the authors obtained -13.8 % and -13.3 %, respectively. After taking into account a proper calculation of the radiative corrections in the QED sector, the original discrepancy between SM and KTeV experiment reduces to the $2\,\sigma$ level or less. In what follows we would like to check if using a more sophisticated model for the pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ helps the current situation.

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3 Two-Hadron Saturation for the PVV correlator

Within a family of large-N_c motivated resonance-saturation models for the PVV correlator in the chiral limit, we consider the following ansatz [7]:

$$\Pi^{\mathrm{THS}}(r^2;p^2,q^2) = \frac{B_0F^2}{r^2(r^2-M_P^2)}\frac{P(r^2;p^2,q^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\,. \tag{3}$$

We have taken into account two multiplets in both vector and pseudoscalar channel. To our knowledge such an approach was not used in the literature before. Together with the constraints we apply further we call this approach the Two-Hadron Saturation (THS). In the numerator there stands a general polynomial which is symmetric in p^2 and q^2. Since the correlator must drop at large momenta, we can, in the very beginning, restrict the number of free parameters to 22, so the polynomial looks as follows:

$$P(r^2;p^2,q^2) = c_0p^2q^2 + c_1[(p^2)^3q^2+(q^2)^3p^2] + c_2(r^2)^2p^2q^2 + \ \ldots \tag{4}$$

To minimize the number of unknown parameters, we apply high- as well as low-energy theoretical constraints. First we demand satisfying of the leading-order OPE constraints [8] by our ansatz. Next, we define the πVV correlator:

$$\mathcal{F}_{\pi VV}(p^2,q^2) \equiv \frac{1}{\mathcal{Z}_\pi}\lim_{r^2\to 0} r^2\Pi(r^2;p^2,q^2)\,, \tag{5}$$

where $\mathcal{Z}_\pi = B_0F$ denotes the overlap between the pion field and the pseudoscalar quark current. Instead of involving subleading orders in the high-energy expansion we in addition apply the Brodsky–Lepage (B-L) constraint [9] on the resulting πVV correlator. Finally, after matching at the photon point to the chiral anomaly we are left with a correlator that appears to have only one free dimensionless parameter κ:

$$\begin{aligned}\mathcal{F}^{\mathrm{THS}}_{\pi VV}(p^2,q^2) = &-\frac{N_c}{8\pi^2F}\frac{M_{V_1}^4M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\\ &\times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_cM_{V_1}^2M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2M_{V_2}^2}\right]\right\}.\end{aligned} \tag{6}$$

We determine the value of this parameter from the fit to the ω-π transition form factor measurements with the result $\kappa = 21 \pm 3$. Note that our result is independent of the mass M_P of the first pseudoscalar excitation. This happens due to the fact that at the end of the day we could conveniently rescale the only free parameter left. For the mass of the first vector-meson multiplet M_{V_1} we use the physical masses of ρ or ω mesons. To account for the uncertainty given by taking only two multiples out of the whole tower of resonances, we vary the mass of the second multiplet M_{V_2} within a range given by physical masses of the first and second vector-meson excitations, i.e. $M_{V_2} \in [1400, 1740]$ MeV.

By construction, our approach satisfies all the theoretical LO constraints given by QCD and might be compared with other models, like Vector-Meson Dominance (VMD), which violates OPE, or Lowest-Meson Dominance (LMD), which strongly violates B-L.It is worth to note that in the singly off-shell regime, the correlator does not depend on κ, so we have a full predictive power for associated quantities.

4 Results

Taking into account the complete set of NLO radiative corrections and THS model, we get for the theoretical prediction of the quantity measured by KTeV experiment $B^{\mathrm{THS}} = (5.8 \pm 0.2) \times 10^{-8}$. If we recall the experimental value $B^{\mathrm{KTeV}} = (6.44 \pm 0.33) \times 10^{-8}$, we find that the disagreement is only at the level of only $1.8\,\sigma$. Thus, if the KTeV result will be confirmed by future experiments, two scenarios are conceivable. Either there are indeed some aspects of the THS approach, which are not well-suited for $\pi^0 \to e^+e^-$, or the beyond-Standard Model physics influences the rare pion decay significantly. But under the present circumstances the current discrepancy is inconclusive. Actually, the quantity measured by KTeV was a ratio of branching ratios, where the Dalitz decay was used as the normalization channel. This is why it is important to have the latter process under control.

5 Dalitz decay of a neutral pion

Corrections to the Dalitz plot in the form of a table of values were calculated in the classical work [10]. The new investigation [11] of this topic was motivated by needs of NA48/NA62 experiments at CERN, the aim of which is, among other goals, to measure the slope a of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$. Unlike before, we have added 1γIR contribution, which was for long time considered to be negligible. Additionally, no approximation considering masses of the particles involved was used during the calculation, so the results are applicable for related decays like $\eta \to \ell^+\ell^-\gamma$. Note though that new contributions are needed in the case of η decays. C++ code was developed, which returns the correction for any given x and y and which became a part of the simulation software of NA62 experiment.

6 Conclusions

The complete set of NLO radiative corrections in the QED sector for both discussed decays, the Dalitz decay and the rare decay $\pi^0 \to e^+e^-$, is now available. We showed on the case of KTeV experiment that their use in future experimental analyses should be essential. We also presented THS model for electromagnetic transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2)$, which satisfies all main theoretical constraints and is phenomenologically successful. Taking all the pieces together, we get reasonable SM prediction for the pion rare decay branching ratio, which differs from the KTeV result by inconclusive $1.8\,\sigma$.

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