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PWA with full rank density matrix of the $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ systems at VES setup

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Abstract. Partial Wave Analysis of the $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ final states produced by $29\,GeV/c$ π^- beam on beryllium target is presented. About $42 \cdot 10^6$ events for the first system and $22 \cdot 10^6$ events for the second one are collected with VES setup. The statistics for 3π neutral is known to be the largest in the world while statistics for 3π charged is next to largest. The t' range for the analysis is $0 < |t'| < 0.8\ GeV^2/c^2$. The data are analysed using formalism of full rank density matrix. The comparison of the analysis results for two systems is presented.

1 Preface

Large statistics for the reactions $\pi^- A \to \pi^+\pi^-\pi^- A$ and $\pi^- A \to \pi^-\pi^0\pi^0 A$ is collected with VES setup. Brief description of the upgraded VES setup can be found in [1]. Final states for both reactions should have similar resonant content. Still these states are quite different from the point of view of experimental setup, acceptance and the structure of Dalitz plot. So the comparison of these reactions can be very useful. This permits us to test isospin relations between these states, better understand possible resonances, check for issues in the setup, hardware, methods and programs involved in the analysis.

One should stress that partial wave analysis (PWA) is complex and model dependent procedure. Results of PWA can depend on how the amplitudes are constructed, what parametrization of density matrix is used (rank 1, rank 2, full rank), how good is the description of the acceptance of the experimental setup. To test the stability and physical meaning of PWA results it may also be very useful to compare these results for given PWA model and two final states.

In this article we present first attempt to compare PWA results of $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ final states on large statistics in a wide range of momentum transfer $|t'|$. The statistics used here is somewhat larger than one presented on the conference due to improvements in methods used in offline reconstruction. The same parametrization, namely full density matrix, is used to analyse both states. General properties of the data are shown in Fig. 1 a) – c). Here and below results for $\pi^+\pi^-\pi^-$ final state are drawn in black while results for $\pi^-\pi^0\pi^0$ are drawn in red (grey in b/w figures). The analysis is performed in four t' regions 0–0.03–0.15–0.30–0.80 GeV^2/c^2.

2 Method of the analysis

Our method of the analysis is based on Illinois PWA [2]. Isobar model is used, namely sequential decay of 3π system via $\pi\pi$ subsystems. Partial wave has quantum numbers $J^P LM^\eta$ R where J^P is

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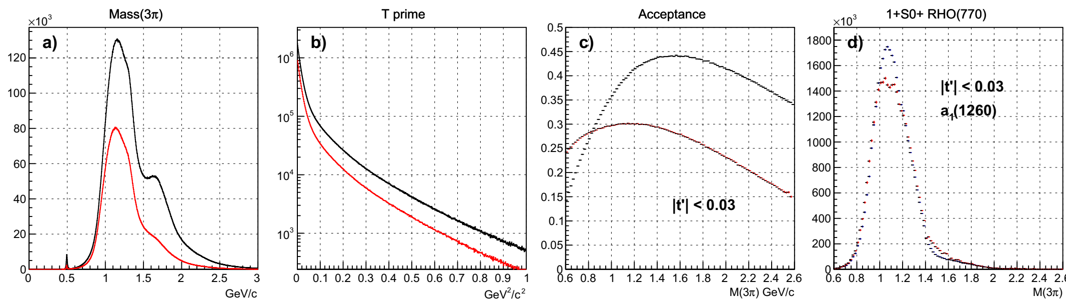

Figure 1. Graph of a) $M(3\pi)$, b) t', c) acceptance, d) intensity of wave $1^+S0^+\rho(770)\pi$ for $\pi^+\pi^-\pi^-$ (black) and $\pi^-\pi^0\pi^0$ (red) final states.

spin and parity for 3π system, M^η is its projection of spin and exchange naturality, R is the known resonance in $\pi\pi$ system, L is orbital momentum in Rπ decay. Isospin and G-parity $I^G = 1^-$ are the same for both 3π final states. Amplitudes are constructed using d-functions. Relativistic corrections are applied according to [3, 4]. Two particle states f_0, ρ, f_2, ρ_3 in $\pi\pi$ subsystems are described by relativistic Breit-Wigners with parameters from PDG [6]. To describe $\pi\pi$ S-wave this is not enough so we use here modified M solution from [5] for broad part of the amplitude and include narrow $f_0(975)$ and $f_0(1500)$ separately. To make M solution broad we drop 4-th order terms and coupling to $K\bar{K}$. We name this pseudo state ε, it should describe $f_0(1370)$ and $f_0(500)$.

We are using positive definite density matrix of unlimited rank and event by event extended log likelihood fit to describe data. No restrictions are imposed on the rank of the matrix. The advantage of this approach is that interference terms can vary freely; the disadvantage is that number of parameters is large which limits total number of waves used. Here we have 37 waves, only some of them are forced to be fully coherent with some others. This is done to reduce total number of parameters, fit has more than 900 of them. GEANT4 based model is used to describe the acceptance of the VES setup.

Let us define coherent part of the density matrix ρ as the largest part of the matrix which has rank one and so behaves like vector of amplitudes. It can be constructed as follows. Let us decompose hermitian matrix ρ with dimension d into its eigenvalues and eigenvectors:

$$\rho_{ij} = \sum_{k=1}^{d} e_k V_k^i V_k^{j+} \quad \text{where} \quad \begin{cases} e_k \text{ is k-}th\text{ eigenvalue} \\ V_k \text{ is k-}th\text{ eigenvector} \end{cases}$$

Let $e_1 > e_2 > \ldots > e_d > 0$. Let us define coherent part of the density matrix as "leading term" with matrix elements $\rho_{ijL} = e_1 V_1^i V_1^{j+}$. If $e_1 \gg e_2$, as is often the case for 3π systems, then matrix elements of this term are stable with respect to small variations of ρ matrix elements. So this decomposition can have physical meaning. Experience shows that resonances tend to concentrate in ρ_L.

3 Fit results

For $\pi^+\pi^-\pi^-$ states f_0, ρ, f_2, ρ_3 are connected to $\pi^+\pi^-$ subsystem while for $\pi^-\pi^0\pi^0$ ρ and ρ_3 are seen in $\pi^-\pi^0$ and f_0, f_2 in $\pi^0\pi^0$. It is known from isospin consideration that if we neglect phase space factors and the difference in Dalitz plot structure for given states we should have

$$R = \frac{\sigma(\pi^-\pi^0\pi^0)}{\sigma(\pi^+\pi^-\pi^-)} = \begin{cases} 1 & \text{for waves with } \rho(770), \rho_3(1690) \\ 1/2 & \text{for waves with } f_0(...), f_2(1270) \end{cases}$$

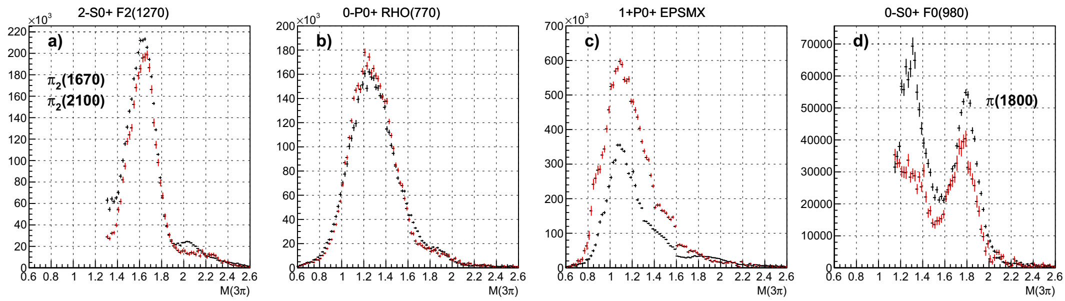

Figure 2. Intensities of waves a) $2^-S0^+f_2(1270)\pi$ b) $0^-P0^+\rho(770)\pi$ c) $1^+P0^+\varepsilon\pi$ d) $0^-S0^+f_0(980)\pi$ for $t' < 0.03$ GeV^2/c^2.

All waves for $\pi^-\pi^0\pi^0$ state coupled to $\pi^0\pi^0$ have factor 1/2 in cross section. To simplify comparison they are scaled by 2 in the following pictures. All intensities produced by PWA program are corrected by acceptance. After this correction and scaling intensities of resonances should be approximately the same for both 3π systems. We apply an additional correction factor 1.10 to $\pi^-\pi^0\pi^0$ intensities to make $2^+D1^+\rho\pi$ signal to be the same in both systems. We choose the $a_2(1320)$ for that as this resonance is narrow and has minimal background. This correction accounts for an error in overall normalization.

We have found that most of large waves are comparable for both final states. This can be seen for $1^+S0^+\rho(770)\pi$, $2^-S0^+f_2(1270)\pi$, $0^-P0^+\rho(770)\pi$, $0^-S0^+f_0(980)\pi$ for $|t'| < 0.030\,GeV^2/c^2$ in Fig. 1 d), Fig. 2 a),b),d). Notable exception is the wave $1^+P0^+\varepsilon\pi$ in Fig. 2 c), which is almost two times larger than expected for $\pi^-\pi^0\pi^0$ system. Excellent coincidence is seen for waves $2^+D1^+\rho(770)\pi$ and $4^+G1^+\rho(770)\pi$ in all t' regions, these waves are shown for $|t'| = 0.03 - 0.15\,GeV^2/c^2$ in Fig. 3 a),b).

For small waves the degree of agreement depends of the wave. For exotic wave with $J^{PC} = 1^{-+}$ and positive exchange naturality, namely the wave $1^-P1^+\rho(770)\pi$, the coincidence is good for $|t'| >$ 0.03 GeV^2/c^2, see Fig. 4 a),b). For this wave in the coherent part of the density matrix, where this wave is 5–20 times smaller than in full density matrix, the coincidence is good in all $|t'|$ regions, see Fig. 5 a),b). But exotic waves with negative exchange naturality, namely $1^-P0^-\rho(770)\pi$ and $1^-P1^-\rho(770)\pi$, are suppressed by 4–5 times in $\pi^-\pi^0\pi^0$ final state, see Fig. 4 c),d).

Some possible resonant states are better observed in $\pi^+\pi^-\pi^-$ final state. It is suggested by some experimental groups that there exists an object $a_1(1420)$ in the wave $1^+P0^+f_0(980)\pi$, see Fig. 3 c). One can see that the bump at $M(3\pi)\sim 1.4\,GeV/c$ is much more significant in $\pi^+\pi^-\pi^-$. The same is true for $\pi_2(2100)$ in Fig. 2 a) and not well established $a_3(1875)$ in the wave $3^+S0^+\rho_3(1690)\pi$ in Fig. 3 d). The same can be stated about possible decay mode $\pi(1800)\rightarrow f_0(1500)\pi$ in the wave $0^-S0^+f_0(1500)\pi$, see Fig. 5 c). Possible $a_2(1700)$ object is not seen in waves $2^+D1^+\rho(770)\pi$ and $2^+P1^+f_2(1270)\pi$ in neither 3π final state in neither $|t'|$ region, see Fig. 3 a), Fig. 5 d).

4 Conclusions

Mass-independent PWA is done for $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ data collected with VES setup. Large waves are mostly consistent with isospin symmetry. Decay modes $\pi(1800) \rightarrow f_0(1500)\pi$, $a_3(1875) \rightarrow \rho_3(1690)\pi$ are possibly seen in 0^-S0^+ and 3^+S0^+ waves in $\pi^+\pi^-\pi^-$ but are less prominent in $\pi^-\pi^0\pi^0$. The same can be said about the bump in $1^+P0^+f_0(980)\pi$ wave at $M \sim 1.4\,GeV/c^2$. State $a_2(1700)$ is not seen in $2^+D1^+\rho\pi$ and $2^+P1^+f_2(1270)\pi$ in neither final state. The wave $1^-P1^+\rho(770)$ with $J^{PC} = 1^{-+}$ is small in both 3π final states in all $|t'|$ regions. Its coherent part is severely smaller.

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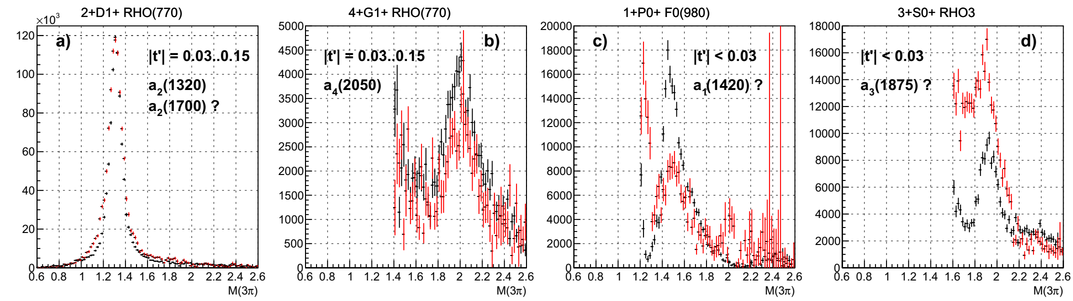

Figure 3. Intensities of waves a) $2^+D1^+\rho(770)\pi$ b) $4^+G1^+\rho(770)\pi$ c) $1^+P0^+f0(980)\pi$ d) $3^+S0^+\rho_3(1690)\pi$.

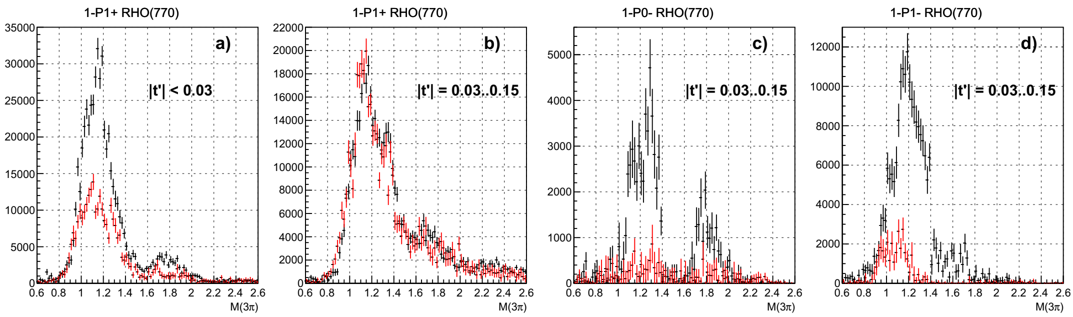

Figure 4. Intensities of waves a), b) $1^-P1^+\rho(770)\pi$, c) $1^-P0^-\rho(770)\pi$, d) $1^-P1^-\rho(770)\pi$.

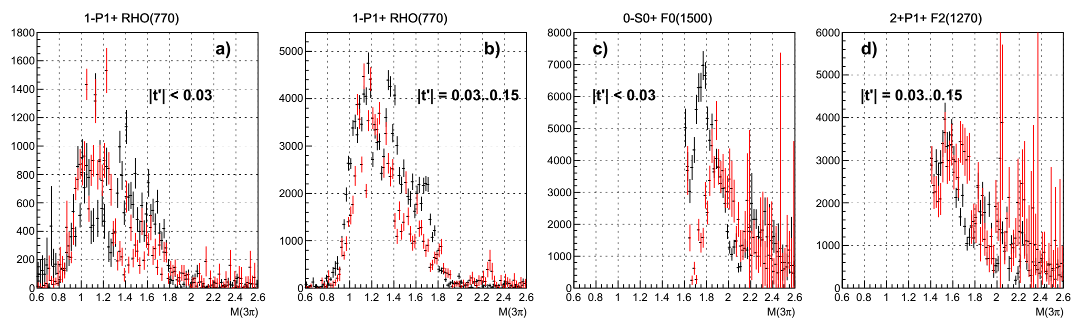

Figure 5. Intensities of waves a), b) $1^-P1^+\rho\pi$, coherent part of the density matrix; c) $0^-S0^+f_0(1500)\pi$, d) $2^+P1^+f_2(1270)\pi$, full density matrix.

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